5th International Conference on Dynamics of Differential Equations

In memory of Jack K. Hale

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DATES	VENUE	CITY
25–29 May 2026	**Instituto de Ciencias Matemáticas (ICMAT)**	**Madrid, Spain**

✦ HALE LECTURER ✦

Rafael de la Llave
Georgia Institute of Technology

PLENARY SPEAKERS

Sue-Ann Campbell
University of Waterloo

Teresa Faria
Universidade de Lisboa

Romain Joly
Université Grenoble Alpes

Rachel Kuske
Georgia Institute of Technology

José A. Langa
Universidad de Sevilla

Daniel Peralta
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Marcone C. Pereira
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James C. Robinson
University of Warwick

Aníbal Rodríguez-Bernal
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Eiji Yanagida
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Welcome

5th International Conference on Dynamics of Differential Equations

In memory of Jack K. Hale

ICMAT, Madrid, May 25–29, 2026

This is the 5th International Conference on Dynamics of Differential Equations – in Memory of Jack K. Hale. The series was established through the joint effort of research groups in the dynamics of differential equations across the Americas, Asia, and Europe, with a scientific focus on the advancement and application of dynamical systems generated by differential equations and their discrete analogs. Previous meetings took place at Georgia Tech, Atlanta, USA (2013), CIRM, Luminy, France (2016), Hiroshima University, Hiroshima, Japan (2018), and Fields Institute, Toronto, Canada (2023).

Each conference in the series features a distinguished Hale Lecture, honoring the legacy of Prof. Jack K. Hale. Previous Hale Lectures were given by Geneviève Raugel (2013), Peter Poláčik (2016), Hiroshi Matano (2018), and John Mallet-Paret (2023). The 5th Hale Lecture will be given by **Rafael de la Llave** (Georgia Tech, USA).

Jack Kenneth Hale

Prof. Hale was one of the most influential figures in modern dynamical systems, shaping the field through his pioneering research, mentorship, and leadership. After earning his Ph.D. in mathematics from Purdue University, he held long-term positions at Brown University, where he was a long-time Director of the Lefschetz Center for Dynamical Systems (LCDS) and at Georgia Tech, where he co-founded and directed the Center for Dynamical Systems and Nonlinear Studies (CDSNS). He authored 15 books and over 200 papers, supervised 48 Ph.D. students and numerous postdocs, and remained deeply committed to the training of young researchers.

Widely recognized for his contributions, he received numerous awards and honors (including the Chauvenet Prize and a Guggenheim Fellowship), an election to several national academies, and several honorary degrees. Beyond his scholarship, Hale was admired for his professionalism, generosity, and dedication to the mathematical community — a legacy honored through these memorial conferences.

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Hale Lecture

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Two directions for the future

Rafael de la Llave
Georgia Institute of Technology

Abstract

I would like to report on some new avenues of research that I hope are open in the future.

The first topic is the effort construct systematically solutions of Functional differential equations. Constructing solutions, gluing them, etc.

An example to keep in mind are the equations of moving charges (charges generate fields by solving Maxwell equations, the particles move in the fields). The resulting equations involve delays/advances which depend implicitly on the solution itself).

The second topic is the efforts to make perturbations of quasi-periodic solutions (KAM theory) into an applied and computational discipline. As it is well known, the basic theory involves number theory, analysis as well as geometry (symplectic, conformally symplectic, variational principles). Computational efforts bring the need for computer assisted proofs and make it necessary to incorporate practical considerations such as modern computer languages, computer architectures, use of AI. The concrete applications (e.g. in celesttial mechancics) bring up other chanllenges.

The numerical calculations, besides concrete applications, open deep mathematical questions such as renormalization group.

Plenary Talks

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Time Delays, Hopf Bifurcation and Synchronization

Sue Ann Campbell
University of Waterloo

Abstract

We consider networks of oscillator nodes with time delayed, global circulant coupling. We first study the existence of Hopf bifurcations induced by the coupling time delay, and then use equivariant Hopf bifurcation theory to determine how these bifurcations lead to different patterns of phase-locked oscillations. We apply the theory to a variety of systems inspired by biological neural networks to show how Hopf bifurcations can determine the synchronization state of the network. We show how the network structure can influence the existence of co-dimension two double Hopf bifurcations which in turn affects whether the nodes synchronize. We consider both discrete and distributed delays and determine how the type of distribution and size of the mean delay affect the results.

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On global dynamics of some classes of nonautonomous delay differential equations

Teresa Faria
Universidade de Lisboa

Abstract

A nonautonomous family of n-dimensional delay differential equations of the form

$$x'(t) = L(t)x_t + f(t, x_t), \quad t \geq 0, \tag{1}$$

where $L : \mathbb{R}^+ \to L(C; \mathbb{R}^n)$ and $f : \mathbb{R}^+ \times C^+ \to (\mathbb{R}^+)^n$ are continuous, will be studied. Here, $C = C([-\tau, 0]; \mathbb{R}^n)$ is the usual phase space of continuous functions with the supremum norm and the bounded linear operator $L(t)$ has diagonal instantaneous negative feedback terms.

We will focus on providing criteria for the persistence and permanence of (1), and, in the case of periodic models, sufficient conditions for the existence of positive periodic solutions.

The results will be illustrated with applications to relevant models from mathematical biology, with emphasis on Nicholson-type systems, for which a result on the global atractivity of positive solutions will also be given. For some of the topics, the more general framework of DDEs with infinite delays set by Hale and Kato in [4] is considered. Some open problems will be presented.

The results in this talk cover some scientific collaborations with J. J. Oliveira (Univ. Minho, Portugal) [3], R. Figueroa (Univ. Santiago de Compostela, Spain) [2], and some ongoing work with Heli Elorreaga (Univ. Bio-Bio, Chile).

References

[1] T. Faria, Permanence and exponential stability for generalised nonautonomous Nicholson systems, *Electron. J. Qual. Theory Differ. Equ.* 2021, pp. 1–19.

[2] T. Faria, R. Figueroa, Positive periodic solutions for systems of impulsive delay differential equations, *Discrete Contin. Dyn. Syst., Ser. B*, 28 (2023), 170-196.

[3] T. Faria, J.J. Oliveira, Persistence for a class of nonautonomous systems of differential equations with unbounded delays, *Math. Biosci.*, 396 (2026), paper 109670, pp. -1-10.

[4] J.K. Hale, J. Kato, Phase space for retarded equations with infinite delay, Funkcial. Ekvac. 21 (1978), 11–41.

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Controlling the Schrödinger equation through domain deformations

Romain Joly
Université Grenoble Alpes

Abstract

In a time-varying domain $\Omega(t) \subset \mathbb{R}^N$, we consider the Schrödinger equation

$$i\partial_t u(t) = -\Delta u(t) \quad \text{on } \Omega(t) \tag{$*$}$$

with Dirichlet boundary conditions. We prove the global approximate controllability of $(*)$ in $L^2(\Omega)$: for any u_0 and u_1 in $L^2(\Omega(0))$ with $\|u_0\|_{L^2} = \|u_1\|_{L^2}$ and for any $\varepsilon > 0$, there exist $T > 0$ and a smooth family of domains $(\Omega(t))_{t\in[0,T]}$ such that $\Omega(0) = \Omega(T)$ and such that the solution of $(*)$ with initial data $u(t = 0) = u_0$ satisfies $\|u(t = T) - u_1\|_{L^2} \leq \varepsilon$.

The proof relies on a combination of adiabatic motions and faster deformations when approaching domains that admit resonances. It also uses classical results, such as the spectrum of dumbbell-shaped domains and the generic simplicity of the eigenvalues of the Laplacian operator.

This is a joint work with Alessandro Duca and Dmitry Turaev.

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Computer-assisted global analysis of energy transfer in vibro-impact configurations

Rachel Kuske
Georgia Institute of Technology

Abstract

We discuss a novel return map approach for studying the global dynamics, developed in the context of a vibro-impact (VI) pair, that is, a ball moving in a harmonically forced capsule. Results are relevant for recent designs of VI-based energy harvesters and nonlinear energy transfer, and hold promise for other non-smooth systems. Computationally efficient short-time realizations are based on "words" that represent key impact sequences and divide the state space according to different dynamics. The word-based maps define surfaces whose characteristics indicate both transients and potential attractors. These perspectives complement the bifurcation structure of the full system and inspire auxiliary maps based on the extreme bounds of the maps, yielding global dynamics of energetically favorable states. Beyond the individual impact pair, we show how the framework is valuable in higher dimensional configurations for shaping the energy transfer, including billiard-type designs, networks of VI-pairs, and multi-cavity VI pairs. In these systems the diagnostics from combined return maps greatly reduce the number of parameters and states of interest. Computational efficiencies follow from maps generated by short sequences of the dynamics, in contrast to traditional dynamical quantities obtained from long-time simulations.

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What global attractors tell us about reality?

José A. Langa
Universidad de Sevilla

Abstract

Since the 1960s, the theory of global attractors has been the subject of intensive research by numerous teams worldwide, giving rise to what may be termed the modern theory of dynamical systems. With a focus on critical review, this talk aims to present several insights derived from this concept regarding the phenomena under study, thereby facilitating a deeper understanding of reality. One of the fundamental contributions of J.K. Hale to the field was the venture beyond mere existence results to analyze the topological and geometric structure of these sets under perturbation. In general, a global attractor consists of a series of minor invariants and the connections between them. In simplified terms, this allows them to be described as directed graphs; at their most complex, they function as authentic information fields that describe both the transient and asymptotic dynamics of observed phenomena. We will demonstrate how these ideas have been formalized to address complex problems in the fields of Ecology and the Neuroscience of human consciousness. In these cases, the structure of the attractors delineates a landscape of profound relationality within the observed subjects, suggesting that this relationality constitutes their most intrinsic and defining nature. The fact that the intrinsic information of these phenomena is organized into structures described by their attractors enables the generation of new metrics for our observations—metrics that were invisible to previous studies in these areas, thus opening the door to an entirely novel analytical perspective. We must credit Hale and other great researchers over these decades for fostering the development of the theory in this direction, enabling promising future interdisciplinary contributions based on their approaches and perspectives.

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Two mathematical challenges in plasma physics

Daniel Peralta-Salas
ICMAT

Abstract

From the mathematical viewpoint, plasmas are described by a combination of the Navier-Stokes equations and Maxwell equations, which results in the so called magnetohydrodynamics (MHD) equations. When the fluid is a perfect conductor, the resistivity of the plasma is neglected and it is described by the ideal MHD equations, which is a good approximation to study solar active regions and plasma confinement near equilibrium configurations. In this talk I will review two classical problems in mathematical plasma physics that remain open. The first one concerns the geometric structure of MHD equilibria (Grad's conjecture, 1967) and the second one is related to the phenomenon of magnetic relaxation and its topological obstructions (Parker, 1972). Recent advances by the speaker and collaborators include the construction of counterexamples of low regularity to Grad's conjecture [1, 2], the proof of Parker's hypothesis in axisymmetric toroidal domains [3] and the discovery of non-symmetric minimizers of the Woltjer's variational principle [4]. The use of modern tools from the theory of dynamical systems (KAM, Newhouse phenomena) is key for some of these results.

References

[1] A. Enciso, A. Luque, D. Peralta-Salas, MHD equilibria with nonconstant pressure in nondegenerate toroidal domains, J. Eur. Math. Soc. 27 (2025) 2251–2291.

[2] A. Enciso, J. Peñafiel-Tomas, D. Peralta-Salas, Steady 3D Euler flows via a topology-preserving convex integration scheme, Preprint (2025).

[3] A. Enciso, D. Peralta-Salas, Obstructions to topological relaxation for generic magnetic fields, Arch. Rat. Mech. Anal. 249 (2025) 6.

[4] D. Peralta-Salas, D. Perrella, D. Pfefferlé, Asymmetry of curl eigenfields solving Woltjer's variational principle, Comm. Math. Phys. 407 (2026) 72.

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Roughness-Induced Effects in a Reaction-Diffusion Equation on a Thin Domain

Marcone C. Pereira
Universidade de São Paulo

Abstract

In this talk, we analyze a reaction-diffusion problem posed in a thin domain endowed with a Robin-type boundary condition, modeling reactions catalyzed at the boundary. Motivated by microfluidic applications, we allow for resonant roughness, in which both the amplitude and the period of the boundary oscillations are of the same order as the domain thickness.

Depending on the strength of the reaction term in the Robin condition, we identify three distinct regimes. In particular, we characterize the critical regime in which the influence of the domain geometry and all physically relevant features of the process are fully captured.

This is joint work with Prof. Igor Pazanin (University of Zagreb) and postdoctoral researcher Jean Carlos Nakasato, and can be partially found in [1, 2].

References

[1] J. C. Nakasato, I. Pazanin and M. C. Pereira, Roughness-induced effects on the convection-diffusion-reaction problem in a thin domain, *Applicable Analysis*, v. 100, p. 1107–1120, 2021.

[2] J. C. Nakasato and M. C. Pereira. Quasilinear problems with nonlinear boundary conditions in higher dimensional thin domains with corrugated boundaries. *Advanced Nonlinear Studies*, v. 23, p. 20230101, 2023.

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The minimal period problem

James C. Robinson
University of Warwick

Abstract

In 1969 Yorke showed that any non-trivial periodic orbit for the ODE $\dot{x} = f(x)$ in $\mathbb{R}^n$ must have period at least $2\pi/L$, when f is Lipschitz with $|f(x) - f(y)| \le L|x - y|$. This result is sharp, as the simple two-dimensional system

$$\dot{x} = -y \qquad \dot{y} = x$$

shows. Lasota & Yorke (1971) proved that the same result ($LT \ge 2\pi$) also holds in an infinite-dimensional Hilbert space H, and that in a general Banach space $LT \ge 4$.

The sharp result in a general Banach space is $LT \ge 6$, which is due to Busenberg, Fisher, & Martelli (1986). However, while this is 'sharp' in that it cannot be improved if the result is to hold for all Banach spaces and all possible f (an example due to the same three authors from 1989), the geometric significance of the 'best constant' $C(X)$ for any particular Banach space (i.e. the constant such that $LT \ge C(X)$ for any L-Lipschitz ODE on X) is unknown, and nor is this best constant known for even such simple cases as $\mathbb{R}^2$ with the ℓ^p norm (when $p \ne 2$).

I will discuss some related problems and some partial results, and highlight a possible connection with the 'girth' introduced by Schäffer (1967), which provides one measure of the 'size' of the unit ball in a Banach space (it is the length of the shortest curve on the surface of the unit ball that joins two antipodal points).

Part of this talk is based on joint work with Alejandro Vidal-López.

References

[1] S. N. Busenberg, D. C. Fisher, & M. Martelli. Better bounds for periodic solutions of differential equations in Banach spaces. *Proc. Amer. Math. Soc.* 98 (1986), no. 2, 376–378.

[2] S. N. Busenberg, D. C. Fisher, & M. Martelli. Minimal periods of discrete and smooth orbits. *Amer. Math. Monthly* 96 (1989), no. 1, 5–17.

[3] A. Lasota & J. A. Yorke. Bounds for periodic solutions of differential equations in Banach spaces. *J. Differential Equations* 10 (1971), 83–91.

[4] J. C. Robinson. *Dimensions, Embeddings, and Attractors.* Cambridge University Press, 2011.

[5] J. J. Schäffer. Inner diameter, perimeter, and girth of spheres. *Math. Ann.* 173 (1967), 59–79.

[6] J. A. Yorke. Periods of periodic solutions and the Lipschitz constant. *Proc. Amer. Math. Soc.* 22 (1969), 509–512.

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How much heat can be lost through the boundary?

Aníbal Rodríguez-Bernal
Universidad Complutense de Madrid and ICMAT

Abstract

In this talk we discuss the problem of quantifying the amount of heat lost through the boundary for the linear diffusion in a solid body under general boundary conditions. We will discuss the case of an exterior domain and analyse the main differences with respect to the bounded domain case. The results depend very strongly on the dimension.

This is a joint work with J. Domínguez de Tena (UCM and ICMAT).

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Computational Dynamics and Data Assimilation

Erik S. Van Vleck
University of Kansas

Abstract

Computational dynamics (CD) seeks to understand the impact of discretization on dynamical systems and increase understanding of the relationship between exact and approximate dynamics. Important areas include preservation of invariants, approximation of invariant sets, and significance (e.g., long time global error analysis, backward error analysis, stability, ...) of numerical solutions. Data assimilation (DA) combines dynamical systems with observational data models to improve predictive abilities and quantify uncertainty in a Bayesian framework. Several issues make understanding CD in DA challenging. These include the use of ensembles/stochastic systems, the impact of observational data models, how the data is incorporated into the dynamics, and imperfect dynamical models. We focus on some recent results for traveling wave solutions under uniform and non-uniform spatial discretization and a posteriori global error estimation for nonlinear dynamical systems based on reduced order modeling and synchronization. These examples are used to motivate recent work on computational approaches for DA. The work focuses on the treatment of the observational data model with adaptive spatial meshes and with reduced order modeling techniques. We conclude with a discussion of some avenues for future investigation.

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Positive solutions of the heat equation with a moving singular potential

Eiji Yanagida
The University of Tokyo

Abstract

This talk is concerned with the heat equation with a time-dependent Hardy-type singular potential. In the subcritical case, it is shown that there exist two types of positive solutions if the motion of the singularity is not so quick (at least 1/2-Hölder continuous). On the other hand, when the singular point moves quickly like a fractional Brownian motion with the Hurst index smaller than 1/2, it is shown that a positive solution exists for a wider range of parameters.

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Stable and unstable manifolds for capillary gravity water waves and a class of nonlinear PDEs

Chongchun Zeng
Georgia Institute of Technology

Abstract

Invariant manifold theory is a fundamental tool in the study of local dynamics near invariant structures in smooth evolution systems. It ensures the existence of nonlinearly invariant structures from linear ones. The theory has been well developed for diffeomorphisms, ODEs, semilinear PDEs, and some quasilinear parabolic PDEs. However, it becomes subtle for quasilinear or more nonlinear PDEs due to regularity issues when there is no smoothing effect. In this talk, we consider a class of nonlinear PDEs whose linearizations satisfy certain energy estimates. We prove that the linear exponential dichotomy implies the existence of local stable/unstable manifolds of the equilibria. In particular the result applies to a class of nonlinear Hamiltonian PDEs including the capillary gravity water waves of one or two fluids, quasilinear wave and Schrödinger equations, KdV type equations, etc., for which the linear analysis is also discussed. Basically, for such systems under certain conditions, spectral instability implies the existence of stable and unstable manifolds, which in particular yields the nonlinear instability in rough Sobolev norms and/or the existence of solutions decaying in high Sobolev norms. This is a joint work with Jalal Shatah.

Invited Speakers

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Sturm Oscillation Theorems for Nonlocal Equations

Peter Bates
Michigan State University

Abstract

Patterns in materials are usually due to oscillations in concentration of a component at the microscopic level. Mathematical models produce these oscillations usually due to spectral changes in a linearization of a nonlinear operator. One of the most useful tools to deduce oscillatory behavior is Sturm's theorem.

In its simplest form, Sturm's comparison result is:
Let $q_1(x) \le q_2(x)$ and not identically equal on $[a,b]$. Let u satisfy $u'' + q_1(x)u = 0$ on $[a,b]$, and v satisfy $v'' + q_2(x)v = 0$ on $[a,b]$. Suppose that $a < b$ are consecutive zeros of u. Then v has at least one zero in (a,b).

This talk addresses the case of linear operators encountered when the interaction within the material has positive range, i.e., the interaction is non-local. There is a discrete version of Sturm's theorem for symmetric tri-diagonal operators with positive off-diagonal terms. However, it does not hold for general symmetric penta-diagonal operators with positive off-diagonal terms, so nonlocal interaction is allowable if it is as short-ranged as possible.

The main point of this talk is to replace the second derivative operator by a bounded integral operator, resulting in a nonlocal diffusion-like operator. First, when u'' is replaced by

$$Lu := \int J(x-y)[u(y) - u(x)]\,dy,$$

where $J \ge 0$ is continuous, even, with compact support and $J(0) > 0$. Then with a scaled version in $\mathbb{R}^n$

$$L_\epsilon u := \int_\Omega \frac{1}{\epsilon^{n+2}} J\left(\frac{x-y}{\epsilon}\right)[u(y) - u(x)]\,dy,$$

for $\epsilon > 0$ small, where $\Omega \subset \mathbb{R}^n$ is smoothly bounded and $J \ge 0$ is radially symmetric with $J(0) > 0$. This operator converges, in some sense, to a multiple of the Laplacian as $\epsilon \to 0$. For $\epsilon > 0$ small the operator should behave like the Laplacian, but it is still long-range, including fine discretizations.

This is joint work with Guangyu Zhao.

Dynamics of infinite-dimensional systems with memory

Tomás Caraballo
Universidad de Sevilla

Abstract

In this talk, the asymptotic behavior of a semilinear heat equation as well as a damped wave equation containing long time memory are analyzed. We set up the problems in the usual framework for dynamical systems generated by differential equations with delay terms. This approach is different from the previous published literature on the long time behavior of heat and wave equations with memory, which were investigated in an extended space after having used the well-known Dafermos transformation. As a consequence, the obtained results provide complete information about the attracting sets for the original problem, instead of the transformed extended one (see [1, 2]).

References

[1] S. Yan, J.H. Xu, T. Caraballo, J. Valero, Dynamics of damped wave equations with long time memory, Commun. Nonlinear Sci. Numer. Simul. 156 (2026), Paper No. 109631

[2] J.H. Xu, T. Caraballo, J. Valero, Asymptotic behavior of a semilinear problem in heat conduction with long time memory and non-local diffusion. J. Differential Equations 327 (2022), 418–447

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Gradient structure of non-autonomous cascade systems

Alexandre N. de Carvalho
Universidade de São Paulo

Abstract

In this talk we will explain the gradient structure of the attractors of autonomous cascade dissipative systems with finitely many equilibria, all of them hyperbolic. We then establish the gradient structure of the skew product attractor associated to a small non-autonomous perturbation of such cascade systems. The results can be applied to cascaded ODEs and PDEs and is a joint work with M. Bortolan, M. Brito and J. Langa (see [1]).

References

[1] M. C. Bortolan, M. C. A. Brito, A. N. Carvalho, J. A. Langa, Gradient structure of dynamics for nonautonomous cascade systems. *J. Dyn. Diff. Equat.* (2026). `https://doi.org/10.1007/s10884-026-10493-3`

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On exponential decay in time for fractional Schrödinger parabolic problems

Jan Cholewa
Uniwersytet Śląski w Katowicach

Abstract

Linearly perturbed fractional diffusion equation is considered in the Lebesgue spaces $L^p(\mathbb{R}^N)$, $1 \leq p \leq \infty$, with a potential from a class of functions satisfying a uniform integrability property in unit balls.

Integral representation of the solutions is used and the relations concerning exponential type of the associated perturbed semigroup are discussed with respect to $1 \leq p \leq \infty$.

The conditions on the potential are given which lead to exponential decay in time of the perturbed semigroup.

These results are jointly developed with Anibal Rodriguez-Bernal and appear in [1].

References

[1] Jan W. Cholewa and Anibal Rodriguez-Bernal. Exponential decay for fractional Schrödinger parabolic problems. Journal of Dynamics and Differential Equations, doi.org/10.1007/s10884-025-10440-8.

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Patterns and local minima for boundary reactions

Neus Cónsul
Universitat Politècnica de Catalunya

Abstract

The Casten-Holland and Matano theorem for interior reactions states that no nonconstant stable solutions exist in convex domains Ω of $\mathbb{R}^n$ under zero Neumann boundary conditions. In this talk we establish that the analogous statement fails for boundary reactions when $n = 2$ (that is, for harmonic functions in Ω with a Neumann reaction term on its boundary $\partial\Omega$). For instance, nonconstant stable solutions exist when Ω is a square, or a smooth strictly convex approximation of it. In regular polygons of many sides, which approach the circle, we can prove the existence of as many nonconstant stable solutions as wished. Instead, in the circle such stable solutions do not exist.

More importantly, we can predict the existence or not of nonconstant stable solutions, as well as the location of its boundary "vortices" (p, q), through the properties of a real function defined on $\partial\Omega\times\partial\Omega$ (the renormalized energy) which depends only on the conformal structure of the domain Ω. This requires the development of a new Ginzburg-Landau theory for real-valued functions and the analysis of the half-Laplacian on the real line.

This is a joint work with Xavier Cabré and Matthias Kurzke.

Transition Routes to Synchronization in the Classical Kuramoto Model

Jia-Yuan Dai
National Tsing Hua University

Abstract

Most forward orbits of the classical Kuramoto model with identical frequencies lead to stable totally synchronized states. Besides, there is a plethora of saddle partially synchronized states. In this talk, we rigorously describe the transition routes from partial synchronization to total synchronization.

By the gradient structure of the classical Kuramoto model, the global attractor consists of (circles of) equilibria and heteroclinic orbits between them. By using the permutation symmetry of indices, we determine when two distinct equilibria can be connected by heteroclinic orbits. This yields a connection graph ordered by inclusion. Moreover, we show that, surprisingly, the invariant manifolds of partially synchronized equilibria are linear.

As a consequence, the classical Kuramoto model is a Morse–Smale system. In particular, the connection graph persists under any small perturbations of the classical model. This is joint work with Bernold Fiedler and Alejandro López Nieto.

References

[1] J.-Y. Dai, B. Fiedler, and A. López Nieto, Transient rebellions in the Kuramoto oscillator: Morse–Smale structural stability and connection graphs of finite 2-shift type, (2025). 10.48550/arXiv.2512.02937.

State space options for delay equations, a comprehensive overview

Odo Diekmann
Universiteit Utrecht

Abstract

To associate a dynamical system with a delay equation, one first has to specify the underlying state space. Since there is a bit of freedom in choosing this space, various frameworks coexist. The aim of this talk is to reveal the relationship between existing approaches, essentially by embedding all of them into the framework of twin semigroups [1]

References

[1] O. Diekmann, S.M. Verduyn Lunel, Twin semigroups and delay equations, Journal of Differential Equations 286 (2021) 332-410

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Center manifold and Hopf bifurcation for semilinear evolution equations in an invariant subspace: An integrated semigroup approach

Arnaud Ducrot
Université Le Havre Normandie

Abstract

We study Hopf bifurcation for a class of partial differential equations of reaction-diffusion type with nonlinear advection and boundary conditions. The presence of the nonlinear advection term prevents a direct analysis in the natural phase space. To overcome this difficulty, we develop an approach based on non-densely defined operators and integrated semigroup theory, which allows us to reformulate the equation so that the advection term appears as a perturbation acting on a subspace Y of more regular functions.
Within this functional framework, we establish the associated spectral theory and construct an invariant spectral decomposition directly in Y, including an extension property for the spectral projectors. These key estimates enable us to prove the well-posedness of the problem and to establish the existence of a smooth local center manifold. As a special case, we derive a Hopf bifurcation result for the original nonlocal and nonlinear reaction–diffusion–advection equation.

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Exploring new bifurcation patterns in nonautonomous d-concave equations via loss of hyperbolicity

Jesús Dueñas
Universidad de Valladolid

Abstract

This talk investigates bifurcation patterns in nonautonomous scalar ordinary differential equations whose derivative with respect to the state variable is concave in measure, the so-called d-concave equations. Exploiting the strong restrictions imposed by d-concavity on the global dynamics, we analyze bifurcation scenarios associated with the loss of hyperbolicity of the upper and lower equilibria of the global attractor.

Using a minimal but multiply ergodic skewproduct base, we construct a jump bifurcation in which two nonhyperbolic compact invariant sets coexist. Under parameter variation, each of these sets disappears abruptly, one for each of the two possible directions of change. This produces a bifurcation structure that departs from the classical saddle-node picture and illustrates how the loss of hyperbolicity at the extremal equilibria of the global attractor can organize nonautonomous dynamics in a nonstandard way.

The study of these bifurcation scenarios is motivated by critical transition theory, where small changes in the inputs of a system may trigger sudden qualitative shifts in its response. From this perspective, the phenomena described here suggest a possible mechanism for critical transitions beyond the better-studied saddle-node setting.

This is joint work with Carmen Núñez and Rafael Obaya.

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Enhancing synchronizability of directed networks

Cinzia Elia
Università degli Studi di Bari

Abstract

We consider a network of identical nonlinear differential equations, coupled through linear nearest neighbor coupling, so that the matrix structure of the network is that of a tridiagonal, possibly reduced, matrix. Each agent has an attractor and we are interested in guaranteeing stability of the synchronous solution in the network. We work with networks associated to connected and simple graphs, and we consider different modelling assumptions.

- Undirected graphs. We give necessary conditions for synchronization.
- Directed graphs with associated unreduced tridiagonal matrix structure. We solve an inverse eigenvalue problem with structure and guarantee synchronization for any number of agents.
- Directed graphs with possibly reduced tridiagonal matrix structures. We construct an optimal network with two leaders that maximizes synchronizability with the minimum cost and the shortest possible transient. The synchronization is guaranteed for any number of agents.

Numerical examples illustrate the theory. This is a joint work with Luca Dieci and Alessandro Pugliese.

References

[1] J.K. Hale. Diffusive Coupling, Dissipation, and Synchronization. Journal of Dynamics and Differential Equations, vol. 9-1, pp. 1-52 (1997).

[2] L. Pecora and T. Carroll. Master Stability Functions for Synchronized Coupled Systems. Physical Review Letters, vol. 80, n. 10, pp. 2109-1112, (1997).

[3] T. Nishikawa and A.E. Motter. Maximum performance at minimum cost in network synchronization. Physica D, vol. 224, pp.77-89, (2006).

[4] L. Dieci, C. Elia and A. Pugliese. On an inverse tridiagonal eigenvalue problem and its application to synchronization of network motion. Linear Algebra and Its Applications, vol. 728, pp. 26–46, (2026).

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Controllability and bifurcation for nonautonomous control systems.

Roberta Fabbri
Università degli Studi di Firenze

Abstract

In the talk controllability properties of nonautonomous control-affine systems with compact control range of the form

$$\dot{x}(t) = f_0(\omega{\cdot}t, x(t)) + \sum_{i=1}^{m} u_i(t) f_i(\omega{\cdot}t, x(t)), \quad u(t) = (u_i(t))_{i=1,\dots,m} \in U,$$

are analyzed. Here $f_i : \Omega \times M \to TM, i = 0, 1, \dots, m$, are continuous maps, Ω is a compact metric space and M denotes a d-dimensional connected smooth (C^∞)-manifold with tangent bundle TM. For every $\omega \in \Omega$, any map $f_i(\omega, \cdot)$ is a C^1-vector field on M. The set $U \subset \mathbb{R}^m$ is compact and convex, and the control functions u are in

$$\mathcal{U} := \{u \in L^\infty(\mathbb{R}, \mathbb{R}^m) \,|u(t) \in U \text{ for almost all } t \in \mathbb{R}\}.$$

Furthermore, (Ω, σ) is a minimal continuous flow with $\sigma : \mathbb{R} \times \Omega \to \Omega$ written as $\sigma(t, \omega) = \sigma_t(\omega) = \omega \cdot t, t \in \mathbb{R}$.

After introducing the concepts of nonautonomous control sets and invariant control sets, and their relations with the nonautonomous equilibria for the uncontrolled system, some bifurcation results for the scalar case ($M = \mathbb{R}, d = 1$) are given. The presented results combine methods from nonautonomous dynamical and control systems.

References

[1] F. Colonius, R. Fabbri, Nonautonomous control systems and skew product flows, J. Dyn. Control Syst. 32 (2026), no.1, Paper No.3, 28 pp.

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On Bifurcation from Infinity: A Compactification Approach

Juliana Fernandes
Universidade Federal do Rio de Janeiro

Abstract

We will introduce a compactification method that has been successfully applied in various contexts, such as in non-dissipative reactiondiffusion equations, where it provides a clear framework for studying unbounded solutions, their stability, and related phenomena like heteroclinic orbits. After discussing the general application of this method in broader settings, we will focus on its specific use in the context of scalar parabolic partial differential equations with nonlinear boundary conditions. In this case, we will examine how the compactification technique aids in understanding bifurcations from infinity, the associated equilibria, and their connection to the Steklov eigenfunctions. This is joint work with P. Lappicy and J. M. Arrieta.

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Dynamics of lattice neural network models with random inputs

Xiaoying Han
Auburn University

Abstract

Lattice neural network models describe neural activity with local or nonlocal interactions across a connected network. Incorporating realistic features such as time delays, adaptive synapses, and stochastic inputs leads to challenging infinite-dimensional dynamical systems. In this talk, we study a class of such models with random inputs and formulate them in an appropriate functional setting. We establish well-posedness and show that the system generates a random dynamical system. Using weighted space techniques and tail estimates, we construct absorbing sets and prove asymptotic compactness, which yields the existence of a random attractor describing the long-term dynamics.

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Global Continuation of the Rotating Eight Families in the Three-Body Problem

Olivier Hénot
National Taiwan University

Abstract

The $\Gamma_1, \Gamma_2, \Gamma_3$ families [1] in the equal-mass three-body problem consist of relative periodic solutions in which the three bodies chase one another along a closed orbit. All three families bifurcate from the figure-eight choreography [2], with the parameter being the angular frequency of a rotating frame about one of three possible axes.

The family Γ_1 was first described by C. Marchal in the 1990s [3]. In his work, this branch was shown to bifurcate from the Lagrange triangle. Following the discovery of the figure-eight solution in 1993, Marchal conjectured that this branch (later identified with Γ_1) extends all the way to the figure-eight. This conjecture has recently been resolved by means of a computer-assisted proof [4].

In this talk, I will present the Γ_i families and explain how modern a posteriori validation methods can be used to settle their global fate. The resolution of Marchal's conjecture will serve as a guiding example. There are two key ingredients to the proof: (*i*) a formulation of the continuation problem that allows one to pass through the bifurcations along the branch, (*ii*) an infinite-dimensional analytical framework in which the entire Γ_1 family is realized as a single isolated fixed-point of an operator.

On the other hand, the global behavior of the Γ_2 family was previously unclear. I will show that it, unexpectedly, continues into a family connecting two figure-eight solutions lying in orthogonal planes, and intersects the Γ_1 family at an intermediate point. If time permits, I will conclude with the planar family Γ_3, for which a collision of the bodies is conjectured and remains an open problem.

References

[1] A. Chenciner, J. Féjoz, and R. Montgomery, Rotating eights I: the three Γ_i families, *Nonlinearity*, 18(3):1407–1424, 2005.

[2] A. Chenciner and R. Montgomery, A remarkable periodic solution of the three-body problem in the case of equal masses, *Annals of Mathematics*, 152:881–901, 2000.

[3] C. Marchal, The family P_{12} of the three-body problem — the simplest family of periodic orbits, with twelve symmetries per period, *Celestial Mechanics and Dynamical Astronomy*, 78:279–298, 2000.

[4] R. Calleja, C. García-Azpeitia, O. Hénot, J.-P. Lessard, and J. D. Mireles James, From the Lagrange triangle to the figure eight choreography: proof of Marchal's conjecture, to appear in *Transactions of the American Mathematical Society*, 2026.

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Typical periodic optimization for dynamical systems: Symbolic dynamics

Wen Huang
University of Science and Technology of China

Abstract

This talk mainly reviews and introduces recent progress on the typical optimization of dynamical systems: the symbolic systems case. The contents include: the theory of maximizing sets in dynamical systems, for the study of ergodic optimization in systems with weak hyperbolicity, but where the Mañé cohomology lemma does not hold. The theory yields a structural theorem that isolates the part of the system responsible for any robust non-periodic optimization. The structural theorem is developed further in the setting of symbolic dynamics: given any shift space, for typical Lipschitz functions, the maximizing measure is shown to be either periodic or supported on the Markov boundary of the shift space.

It follows that Contreras' Typical Periodic Optimization theorem for shifts of finite type can be extended to a wide class of shift spaces, including every sofic shift. This is joint work with Oliver Jenkinson, Leiye Xu, and Yiwei Zhang.

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Rigorous Construction of Stop-and-Go Waves in the Optimal Velocity Model via a differential-difference equation

Kota Ikeda
Meiji University

Abstract

Traffic congestion is characterized by the self-organized propagation of density waves, a phenomenon that can be reproduced by the optimal velocity (OV) model on a circuit [1]:

$$x_n'' = a(V(\Delta x_n) - x_n'), \quad n = 1, \dots, N. \tag{1}$$

Here, x_n represents the position of the n-th vehicle, $a > 0$ is the driver's sensitivity parameter, and V is the OV function. The variable $\Delta x_n \equiv x_{n+1} - x_n$ denotes the headway distance between the n-th vehicle and the $(n+1)$-th vehicle ahead of it, with the periodic boundary condition $\Delta x_N = L + x_1 - x_N$, where L is the length of the circuit. To investigate the solutions corresponding to the congestion phase in the OV model, we introduce a periodic function $u = u(t)$ with period N and a wave speed $c > 0$. Specifically, the function u is defined such that $u(ct + n - 1) \equiv \Delta x_n(t)$. Through this transformation, we formulate the problem as the following differential-difference equation:

$$c^2 u''(t) - a(V(u(t+1)) - V(u(t))) + acu'(t) = 0. \tag{2}$$

Numerical simulations indicate that when N is sufficiently large, the solution profiles exhibit regions staying close to certain constant states u_1 and u_2, connected by transition layers. It should be noted that these transition layers essentially coincide with the exact solution constructed in [2]. As $N \to \infty$, it is observed that these constant regions extend infinitely, and c asymptotically approaches a constant value. Based on these observations, the goal in this talk is to prove the existence of periodic solutions consisting of alternating transition layers and constant states for a sufficiently large β and period N under the assumption that the OV function V approaches a Heaviside step function $V_0 H(x - l) + V_1$ exponentially as a parameter $\beta \to \infty$.

To construct periodic solutions corresponding to stop-and-go waves on a circuit, we impose a global constraint reflecting the conservation of the total road length. Unlike heteroclinic or homoclinic solutions, constructing periodic solutions directly via perturbation methods is challenging because the associated linearized operator is not invertible. To overcome this difficulty, we introduce a modified differential-difference equation that incorporates the global constraint.

References

[1] M. Bando, K. Hasebe, A. Nakayama, A. Shibata, and Y. Sugiyama. Structure stability of congestion in traffic dynamics. Japan Journal of Industrial and Applied Mathematics, Vol.11 (1994), 203–223.

[2] Y. Sugiyama, and H. Yamada. Simple and exactly solvable model for queue dynamics. Physical Review E, Vol.55, No.6 (1997), 7749.

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On structure of attractors for Lotka-Volterra systems

Piotr Kalita
Uniwersytet Jagielloński

Abstract

The talk will be devoted to the results on the structure of global attractors for the following system of Lotka–Volterra ODEs given by

$$u_i' = u_i \left(b_i + \sum_{j=1}^{n} a_{ij} u_j \right) \quad \text{for} \quad i \in \{1, \dots, n\}.$$

In autonomous case, we revisit the classical situation (see cf. [6]) where the matrix $A = (a_{ij})_{i,j=1}^n$ is Lotka–Volterra stable. For such case, we provide results obtained in [1, 2] and inspired by [5] on the structure of the global attractors, and their relations with the invasion graphs. In the diagonally dominant case we also present some results of the forward attractors for the non-autonomous extension of the model [3, 4].

References

[1] P. Almaraz, P. Kalita, J.A. Langa, F. Soler-Toscano, Structural stability of invasion graphs for Lotka-Volterra systems, Journal of Mathematical Biology vol. 88 (2024), article number 64

[2] M.C. Bortolan, P. Kalita, J.A. Langa, R. de Oliveira Moura, A theoretical and computational study of heteroclinic cycles in Lotka-Volterra systems, Journal of Mathematical Biology vol. 90 (2025), article number 28

[3] J. Garcia-Fuentes, P. Kalita, J.A. Langa, Forwards attractor structures in a planar cooperative non-autonomous Lotka-Volterra system, Qualitative Theory of Dynamical Systems vol. 23 (2024), article number 163

[4] J. Garcia-Fuentes, P. Kalita, J.A. Langa, A. Suarez, Forwards attractors for non-autonomous Lotka-Volterra cooperative systems: a detailed geometrical description, Discrete and Continuous Dynamical Systems - Series B vol. 30 (2025), 189-221

[5] J. Hofbauer, S.J. Schreiber, Permanence via invasion graphs: incorporating community assembly into modern coexistence theory, Journal of Mathematical Biology vol. 85 (2022) article number 54

[6] Y. Takeuchi, Global Dynamical Properties of Lotka-Volterra Systems, World Scientific 1996.

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Heteroclinic and homoclinic orbits for some Mackey–Glass type equations

Tibor Krisztin
Szegedi Tudományegyetem

Abstract

We consider the delay differential equation

$$y'(t) = -ay(t) + bf(y(t-1)) \tag{3}$$

with positive parameters $b > a$ and nonlinear feedback functions of the form $f(\xi) = \frac{\xi^k}{1+\xi^n}$ with $k > 0$ and $n > 0$. The case $k = 1$ is the famous Mackey–Glass equation, the case $k > 1$ appears in population models with the Allee effect, and $k \in (0, 1)$ arises in some economic growth models. For large n, the nonlinear f is close to the discontinuous $g : [0, \infty) \to [0, 1]$ with $g(\xi^k) = \xi^k$ for $\xi \in [0, 1)$ and $g(\xi) = 0$ for $\xi > 1$. Here we consider the case $k > 1$ and n large. There are three equilibria $0 < \xi_{1,n} < \xi_{2,n}$, 0 is stable, $\xi_{1,n}$ and $\xi_{2,n}$ are unstable.

The aim is to understand the structure of the leading unstable set W of $\xi_{1,n}$ and its role in the global dynamics. There is a threshold value $b^* = b^*(a) > a$ such that, for $b < b^*$, W consists of orbits connecting $\xi_{1,n}$ to the stable 0. For $b > b^*$, there is a bistable situation: a subset W^- of W contain connecting orbits from $\xi_{1,n}$ to 0, and another subset W^+ of W connects $\xi_{1,n}$ to a compact invariant set $\mathcal{A}_n$. The set $\mathcal{A}_n$ is disjoint from the equilibria $\xi_{1,n}$, 0, and it can be complicated. In some cases $\mathcal{A}_n$ is a stable periodic orbit.

Analogous results are valid for periodic orbits close to $\xi_{1,n}$.

For the critical parameter value $b = b^*$, there exists a homoclinic solution $h_n : \mathbb{R} \to \mathbb{R}$ with $\lim_{|t|\to\infty} h_n(t) = \xi_{1,n}$. In this case a Shilnikov type chaos is expected in a small neighborhood of h_n.

The results rigorously prove several interesting dynamical properties which were observed earlier only by numerical methods.

This is a joint work with G. Benedek, L.B.N. Pham, M. Polner, and R. Szczelina.

New oscillatory regimes for competing predators with Holling type II response

Phillipo Lappicy
Universidad Complutense de Madrid and ICMAT

Abstract

We present three new results on oscillatory regimes for models of competing predators with Holling type II functional response, which are based on [1, 2, 3], respectively.

First, we introduce a new mechanism that generates periodic orbits, which we call hybrid bifurcations: a classical bifurcation occurring at a bifurcation without parameters. We establish existence and stability criteria for the bifurcating periodic solutions and obtain stable periodic coexistence states far from extinction regimes, where all populations remain bounded away from zero.

Second, we prove global continuation and stability of families of periodic orbits with respect to a parameter using a computer-assisted framework. As a consequence, we solve a conjecture of Butler and Waltman (1981) concerning one-parameter families of periodic orbits connecting two extinction regimes.

Finally, we analyze a candidate Poincaré return map for these systems. Using computer-assisted techniques, we prove the existence of periodic orbits together with associated period-doubling bifurcations, indicating a transition to a chaotic regime.

References

[1] A. López-Nieto, P. Lappicy, N. Vassena, H. Stuke, J.-Y. Dai. Hybrid bifurcations: periodicity from eliminating a line of equilibria. *Mathematische Annalen*, Vol. **391**, pg. 6373–6399, (2025).

[2] K. E. M. Church, J.-Y. Dai, O. Hénot, P. Lappicy, N. Vassena. Global continuation of stable periodic orbits in systems of competing predators. To appear in *SIAM Journal on Applied Dynamical Systems* (2026).

[3] L. Alonso Mozo, O. Hénot, P. Lappicy. A Practical Guide to Rigorously Locate Periodic Orbits in Discrete Dynamics. Submitted; see: arxiv:2510.06373 (2026).

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An Epidemic Enigma: Challenges in Modeling the 1978 Influenza Epidemic in a British Boarding School

Michael Li
University of Alberta

Abstract

I will revisit a classical modeling example of an influenza epidemic in a boarding school in the UK, which was first described in the British Medical Journal (BMJ, March 4, 1978) [1]. The application of an SIR model to describe the epidemic has appeared in well-known textbooks, as well as numerous lecture notes and presentations on the internet. It is shown that the number of infected $I(t)$ can fit the data very well, allowing estimation of the two model parameters and the basic reproduction number. What the modeling examples failed to check is that the final size, which is the total number or percentage of infected people during the epidemic, predicted by the calibrated models (close to 730 students or 96%) hugely over-estimated the final size given in the BMJ paper (512 students or 67%). How to reconcile this huge discrepancy in model predictions and data? I will provide some explanations, outlining several challenges in mathematical modeling of general epidemics, and update recent solutions. For the influenza epidemic described in the BMJ, there is still no model that has satisfactorily described both the time series data and the final size, and this well-known problem remains as an epidemic enigma for modellers.

References

[1] Anonymous. Influenza in a boarding school, British Med. J. March 4, 1:587 (1978).

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Continuity of the Lyapunov exponent and measure entropy for stochastic differential equations

Zhenxin Liu
Dalian University of Technology

Abstract

For non-autonomous linear stochastic differential equations (SDE), we establish that the top Lyapunov exponent is continuous if the coefficients “almost" uniformly converge. For autonomous SDEs, assuming the existence of invariant measures and the convergence of coefficients and their derivatives in pointwise sense, we get the continuity of all Lyapunov exponents. Furthermore, we talk about the Lipschitz and Hölder continuity of Lyapunov exponents. For autonomous SDEs, we establish a relationship between the measure entropy of the stochastic flow and the rate of volume growth of stable submanifolds under iteration. By combining the results of Kifer and Yomdin, we obtain that for such systems with C^∞ coefficients, under suitable integrability conditions, the measure entropy is upper semicontinuous with respect to the coefficients. This talk is based on joint works with Lixin Zhang.

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Nonautonomous differential equations in the presence of bounded noise

Iacopo P. Longo
University of Exeter

Abstract

Nonautonomous systems subject to noise arise naturally in many applied contexts. Particularly relevant are those featuring time-dependent parameters and uncertainties, which may drive the emergence of tipping points. In this talk, we focus on the case of bounded noise, where the dynamics can be captured topologically via deterministic set-valued dynamical systems: initial conditions are evolved under all admissible noise realisations, abstracting from probabilistic details. However, as these systems operate on the space of nonempty compact subsets—a space lacking Banach structure—they pose substantial analytical and numerical challenges. In particular, bifurcation analysis of attractors remains a significant obstacle. To address this, we derive a higher-dimensional single-valued determinisitic dynamical system which characterizes the evolution of the boundary of invariant sets of certain differential inclusions.

This is a joint work with Konstantinos Kourliouros and Martin Rasmussen.

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Stability results for functional Volterra–Stieltjes integral equations and applications

Jaqueline G. Mesquita
Universidade Estadual de Campinas

Abstract

We investigate stability for functional Volterra–Stieltjes integral equations, a highly general class of models encompassing systems with memory, discontinuities, and non-smooth dynamics. Using Lyapunov functionals in a nonsmooth and nonclassical setting, we establish uniform stability results under different sets of assumptions, thereby extending and unifying several classical asymptotic results. The abstract theory is illustrated through concrete applications, including a nonlinear RC circuit with memory effects and the classical Mackey–Glass model of respiratory dynamics. In addition, we show how the results naturally extend to impulsive functional Volterra–Stieltjes equations, yielding new attractivity criteria for systems subject to instantaneous perturbations. This work has the financial support of FAPESP.

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Time-Series Data can Probably Characterize its own Dynamics

Konstantin Mischaikow
Rutgers University

Abstract

The identification of dynamics from time series data is a problem of rapidly growing interest. I will discuss a novel method that combines Conley theory and Gaussian Process surrogate modeling with uncertainty quantification to characterize local and global dynamics along with lower bounds on the confidence that this characterization of the dynamics is correct. Furthermore, numerical experiments indicate that it is possible to identify nontrivial dynamics with high confidence with surprisingly small data sets.

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Homogenization of the heat equation in a time-oscillatory moving thin domain

Jean Carlos Nakasato
Universidade Federal do ABC

Abstract

We study the asymptotic behavior of solutions of the heat equation in a time oscillatory thin domain. We start by determining uniform estimates of solutions in the perturbed domain and then find the effective problem. This is a joint work with Tatsu-Hiko Miura (University of Hirosaki).

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Tracking nonautonomous attractors in singularly perturbed systems of ODEs with dependence on the fast time

Rafael Obaya
University of Valladolid

Abstract

Results on the behaviour of the fast motion in slow-fast systems of ODEs with dependence on the fast time are given in terms of tracking of nonautonomous attractors. Under quite general assumptions, including the uniform ultimate boundedness of the solutions of the layer problems, inflated pullback attractors are considered. The problem of the limit of the solutions of the slow-fast systems at each fixed positive value of the slow time is also analyzed.

References

[1] V. I. Arnold. Geometrical methods in the theory of ordinary differential equations. Springer (1988).

[2] N. Fenichel. Geometric singular perturbation theory for ordinary differential equations. *J. Differential Equations* 31 (1979).

[3] I. P. Longo, R. Obaya, A. M. Sanz. Tracking nonautonomous attractors in singularly perturbed systems of ODEs with dependence on the fast time. *J. Differential Equations* 414 (2025).

[4] A. N. Tikhonov. Systems of differential equations containing a small parameter multiplying the derivative. *Math. Sb.* 31 (1952).

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On the exponential decay in some simple chromatographic models

Marta Pellicer
Universitat de Girona

Abstract

Chromatographic separation plays a central role in many applications of chemical engineering, providing an efficient way to isolate different components of a mixture. In this talk, we focus on two simplified linear models arising from chromatography, both formulated as systems of transport equations. Our interest lies in analyzing the asymptotic behaviour of their solutions.

We prove that, in both settings, the dynamics are governed by exponential decay towards equilibrium. The decay rate is shown to be optimal and is characterized by the dominant eigenvalue of the underlying operator. Despite this common outcome, the mathematical approaches required are substantially different. For the first model, the analysis relies on the Spectral Mapping Theorem, while for the second one the Krein–Rutman Theorem is used.

Beyond the theoretical results, these models also offer explicit quantitative information. In particular, the optimal decay rates can be computed explicitly, and, in some situations, the associated asymptotic profiles can be completely determined.

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Time-delayed opinion dynamics with leader-follower interactions

Cristina Pignotti
Università degli Studi dell'Aquila

Abstract

We study a time-delayed variant of the Hegselmann–Krause opinion formation model featuring a small group of leaders and a large group of non-leaders. In this model, leaders influence all agents but only interact among themselves. At the same time, non-leaders update their opinions via interactions with their peers and the leaders, with time delays accounting for communication and decision-making lags. We prove the exponential convergence to consensus of the particle system, without imposing smallness assumptions on the delay parameters. Furthermore, we analyze the mean-field limit in two regimes: (i) with a fixed number of leaders and an infinite number of non-leaders, and (ii) with both populations tending to infinity, obtaining existence, uniqueness, and exponential decay estimates for the corresponding macroscopic models.

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From invariant manifolds to fiber bundles: Numerical dynamics of integrodifference equations

Christian Pötzsche
Universität Klagenfurt

Abstract

Integrodifference equations are popular tools in theoretical ecology to model the spatial dispersal of populations evolving with nonoverlapping generations. Their behavior is often illustrated using numerical simulations based on various discretization methods.

In this talk, we discuss the persistence and behavior of their invariant manifolds under spatial discretization and establish convergence preserving the order of the numerical method. Our approach ranges from classical stable and unstable manifolds of autonomous equations to the full hierarchy of invariant fiber bundles for nonautonomous problems. Moreover, various ambient state spaces are discussed.

References

[1] F. Lutscher. *Integrodifference Equations in Spatial Ecology*, Interdisciplinary Applied Mathematics 49. Springer, Cham, 2019.

[2] C. Pötzsche. Numerical dynamics of integrodifference equations: Periodic solutions and invariant manifolds in $C^\alpha(\Omega)$. *Numer. Math.* 154:249–281, 2023.

[3] C. Pötzsche. Numerical dynamics of integrodifference equations: Hierarchies of invariant bundles in $L^p(\Omega)$. *Numer. Funct. Anal. Optimization* 44(7):653–686, 2023.

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Intrinsic vs. Extrinsic Noise in Transient Dynamics: A Quasi-Stationary Approach

Weiwei Qi
Academy of Mathematics and Systems Science

Abstract

Transient dynamics–long-lasting but ultimately finite-time behaviors–are ubiquitous in complex stochastic systems. Understanding their underlying mechanisms remains a fundamental challenge. In this talk, we investigate randomly perturbed processes arising in chemical reaction networks and population dynamics, where extinction is inevitable but preceded by long-lived persistence. Using quasi-stationary distributions (QSDs), we characterize these transient dynamics and analyze their asymptotics in the vanishing-noise regime. We show that intrinsic and extrinsic noise can induce fundamentally different transient dynamics, leading to distinct persistence and extinction patterns. We conclude with a discussion of broader implications and open problems.

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A new class of generalized ordinary differential equations

Ana M. Sanz
Universidad de Valladolid

Abstract

In this talk, we present a new class of generalized ordinary differential equations. Our motivation arises from the fact that solutions of limiting equations associated with certain Carathéodory ODEs may lose the property of absolute continuity, while remaining continuous and of bounded variation on compact intervals. Several approaches can be employed to address these irregular limiting equations. We propose a novel method, based on the introduction of a class of generalized ODEs defined by parametric b-measures, aimed at preserving the aforementioned properties of solutions to the equations in the hull, as well as ensuring the compactness of the hull. This framework allows us to apply techniques from nonautonomous dynamical systems to the study of precompact families of Carathéodory ODEs.

This is joint work with Sylvia Novo and Rafael Obaya.

Principal eigenvalue of local – non-local equations

Silvia Sastre Gómez
Universidad de Sevilla

Abstract

We consider an eigenvalue problem for a local – non-local equation involving the Laplace operator and a non-local operator with a regular kernel. A comprehensive analysis of the principal eigenvalue is carried out. We study also several monotonicity properties of the principal eigenvalue with respect to the domain and the kernel. In particular, we are interested in the principal eigenvalue of the equation with the Laplace operator term multiplied by epsilon. Finally, we analyze the convergence of this principal eigenvalue as epsilon goes to zero and as epsilon goes to infinity.

Joint work with: Willian Cintra (UnB) and Antonio Suárez-Fernández (US).

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Chaos in generic analytic convex billiards

Tere M.-Seara
Universitat Politècnica de Catalunya

Abstract

The object of this talk is to show the existence of Chaos in analytic convex billiards. It is well known that, for a convex billiard table, the corresponding billiard map is a twist map on the cylinder. To obtain the existence of chaos in such a map, we find hyperbolic periodic orbits whose stable and unstable manifolds intersect transversally, giving rise to the existence of horseshoes and, therefore, to hyperbolic invariant sets whose dynamics is conjugated to the Bernoulli shift. This is the classical approach to prove chaos; nevertheless, the abundance of transverse homoclinic orbits has been only proved for generic smooth billiard tables. The novelty of our work is that, going back to some ideas of Zehnder of 1972, we prove that the set of analytic billiards such that "given any rational rotation number, there is a unique periodic orbit in the associated Aubry-Mather set, whose stable and unstable manifolds intersect and whose intersections are all transversal" form a Baire generic set with the usual analytic topology.

The result is in collaboration with I. Baldomá (U.Politecnica de Catalunya) A. Florio (U. Paris Dauphine) and M. Leguil (E. Polytechnique Paris).

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An example of Lyapunov stability (*In memoriam* of Hildebrando M. Rodrigues)

J. Solà-Morales
Universitat Politècnica de Catalunya

Abstract

In this communication I will present our last joint result with Hildebrando M. Rodrigues, who unexpectedly and sadly died on 2023. The result was published in [1] and is a construction of a $\mathcal{C}^1$ (in the Fréchet sense) map T in a infinite-dimensional separable Hilbert space for which the origin is an asymptotically stable fixed point for the discrete dynamical system T^n, but such that the derivative of T at the origin has spectral radius larger than unity, and this means that the origin is linearly unstable.

The possible existence or not of an example of this kind was an open question for some time. The construction is based on a classical example in Operator Theory due to Kakutani. Certainly, the nonlinear map T does not satisfy the Rutman-Daleckii-Krein sufficient condition $T(x) = DT(0) + O(\|x\|^p)$ with $p > 1$ as $x \to 0$.

I will present a short history of this old problem, including references to the previous work [2] and the subsequent paper [3].

References

[1] Hildebrando M. Rodrigues, J. Solà-Morales. An example on Lyapunov stability and linearization. J. Differential Equations 269 (2020)1349–1359.

[2] T. Gallay, B. Texier, K. Zembrun. On nonlinear stabilization of linearly unstable maps. J. Nonlinear Sci. 27(5) (2017) 1641–1666.

[3] Á. Garab, M. Pituk, C. Pötzsche. Linearized stability in the context of an example by Rodrigues and Solà-Morales. J. Differential Equations 269 (2020), 9838–9845

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A panoramic view of exponential attractors

Stefanie Sonner
Radboud Universiteit

Abstract

We provide a unifying framework for the construction of exponential attractors for infinite dimensional dynamical systems that allows us to generalize, improve and compare existing methods that are commonly used to construct exponential attractors. For autonomous deterministic systems we formulate necessary and sufficient conditions for the existence of discrete exponential attractors in terms of a covering condition for iterates of the absorbing set under the time evolution of the semigroup. The parameters in the covering condition determine the estimate for the fractal dimension of the exponential attractor and the exponential rate of attraction.

We then verify the covering condition for existing approaches to construct exponential attractors where the most general setting concerns quasi-stable semigroups in complete metric spaces. Generalizing previous notions and methods used in the literature on exponential attractors then allows us to compare widely used approaches. To conclude, we mention generalizations of the constructions for non-autonomous and random dynamical systems. This is joint work with R. Czaja (University of Silesia in Katowice).

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On the connectedness of solutions of the three-dimensional Navier-Stokes equations with damping

José Valero
Universidad Miguel Hernández de Elche

Abstract

We study the connectedness and compactness of the attainability set of solutions to the three-dimensional Navier-Stokes equations with damping (known as the Kneser property), given by

$$\begin{cases} u_t - \mu\Delta u + (u\cdot\nabla)u + \nabla p + \alpha|u|^{\beta-1}u = f,\ (x,t)\in\Omega\times(\tau,T), \\ \operatorname{div} u = 0, \\ u|_{\partial\Omega} = 0, \\ u(\tau) = u_\tau, \end{cases}$$

where $\alpha,\mu>0$ and $\beta\geq 1$ are constants, and Ω is a bounded open subset of $\mathbb{R}^3$ with smooth boundary $\partial\Omega$.

First, for $\beta\in[1,3)$, we establish the Kneser property of weak solutions with respect to the weak topology of the phase space $H = cl_{(L^2(\Omega))^3}\mathcal{V}$, where $\mathcal{V}=\{u\in(C_0^\infty(\Omega))^3 : \operatorname{div} u = 0\}$.

Second, for $\beta = 3$, $4\alpha\mu<1$, we obtain the Kneser property of weak solutions with respect to the strong topology of the phase space H and the connectedeness of the global attractor as well.

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Effective models in thin domains with rough boundaries: the role of competing scales

Manuel Villanueva-Pesqueira
Universidad Pontificia Comillas

Abstract

In this talk I will discuss some homogenization results for partial differential equations posed in thin domains with rough boundaries. The main focus will be on how the effective model depends on the relation between two characteristic small scales: the thickness of the domain and the frequency of the boundary oscillations. Different balances between these parameters lead to different limit equations and, at the same time, call for different analytical strategies.

The underlying question is how the combined effect of domain degeneration and boundary roughness is transmitted to the macroscopic description of the problem. According to the scaling regime, the asymptotic behavior may reflect predominantly the dimension reduction, the oscillatory geometry, or a nontrivial interaction of both. As a consequence, the effective equations may display markedly different structures, showing the sensitivity of the limit problem to the geometry of the thin rough domain.

I will describe several representative regimes and the techniques used in their analysis, including rescaling methods, compactness tools, and homogenization procedures adapted to singular and highly oscillatory geometries. The resulting models illustrate how variations in the competing scales lead to genuinely different effective descriptions, highlighting the richness of thin rough domains as a framework for asymptotic analysis.

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A direct approach to simplicity of solution manifolds

Hans-Otto Walther
Justus-Liebig-Universität Gießen

Abstract

Differential equations with state-dependent delays define a semiflow of continuously differentiable solution operators in general only on the associated *solution manifold* $X \subset C^1([-h,0],\mathbb{R}^n)$. For systems with discrete state-dependent delays we construct a diffeomorphism on a neighbourhood of X which takes X to an open subset of the subspace given by $\phi'(0) = 0$. This is in line with earlier work on the nature of solution manifolds. The present approach, however, is new and dismisses all hypotheses beyond smoothness which have been instrumental so far. Compared to a recent case study it is more direct in the sense that theory of *algebraic delay systems* is avoided.

References

[1] H.O. Walther. A direct approach to simplicity of solution manifolds. https://arxiv.org/abs/2601.00642, 21 p. (2026).

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Synchronization in Markov random networks

Shirou Wang
Jilin University & University of Alberta

Abstract

Many complex biological and physical networks are naturally subject to both random influences, i.e., extrinsic randomness, from their surrounding environment, and uncertainties, i.e., intrinsic noise, from their individuals. Among many interesting network dynamics, of particular importance is the synchronization property which is closely related to the network reliability especially in cellular bio-networks. It has been speculated that whereas extrinsic randomness may cause noise-induced synchronization, intrinsic noises can drive synchronized individuals apart. This talk presents an appropriate framework of (discrete-state and discrete time) Markov random networks to incorporate both extrinsic randomness and intrinsic noise into the rigorous study of such synchronization and desynchronization scenario. Convergence in distribution is established via a Lyapunov spectrum analysis. A probabilistic characterizations of the alternating pattern between synchronization and desynchronization behaviors is given if the deterministic random network (i.e. absent any intrinsic noise) is fully synchronized.

This talk is based on several joint works with Arno Berger, Wen Huang, Hong Qian, Felix X.-F. Ye, and Yingfei Yi.

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Exponentially-long time stability and its application on N-vortex systems

Lu Xu
Jilin University

Abstract

I will talk about the establishment of exponentially-long-time stability for multi-scale, nearly integrable Hamiltonian systems. We focus on two specific scenarios: (I) the integrable component at each scale satisfies (α, K) complete non-resonance; and (II) resonance occurs exclusively in the highest scale of the integrable part. I will show the Nekhoroshev estimates under these two cases and also mention the characteristics and the differences between multi-scale systems and classical systems. This topic is inspired by celestial mechanics and N-vortex systems. As a consequence, I will illustrate our results through explaining the stability of the singularity point of the $(1+1)$ restricted vortex systems.

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Quasi-Stationary Distributions

Yingfei Yi
University of Alberta & Jilin University

Abstract

Quasi-stationary distributions (QSDs) are those almost invariant to a diffusion process over exponentially long time. Representing important transient stochastic dynamics, they arise frequently in applications especially in chemical reactions and population systems admitting extinction states. This talk will present some rigorous results on the existence, uniqueness, concentration, and convergence of QSDs along with their connections to the spectrum of the Fokker-Planck operator.

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Spatial temporal dynamics of a diffusive Nicholson blowfly equation with two shifting parameters

Xingfu Zou
University of Western Ontario

Abstract

In this talk, I will report some recent results contained in [1] on the spreading speed and asymptotic propagation properties for a revised diffusive Nicholson equation with two shifting parameters. By the method of iterative strategies of travelling wave maps, we give some *a priori*estimates on nontrivial solutions which play a key role in the proof of global asymptotic behaviour. These estimates in combination with proper test functions allow us to establish the spatial-temporal propagation dynamics of this revised Nicholson equation. These results enable us to develop a unified method for exploring the spreading speeds and asymptotic propagation phenomena for a class of non-translation invariant delay reaction-diffusion equations in $\mathbb{R}^N$.

References

[1] T. Yi, Y. Zhang and X. Zou, Spatial-temporal dynamics of Nicholson blowfly equation with two shifting parameters, SIAM J Math. Anal. 57(3)(2025), 2880-2910.

Contributed Communications

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The Bianchi IX Attractor in Modified Gravity

Ester Beatriz
Universidade de São Paulo

Abstract

In this talk, I introduce models describing generic spatial singularities (e.g., the Big Bang) in modified gravity theories. These perturb Bianchi models from general relativity (GR) via a parameter $v \in (0, 1)$, recovering GR at $v = 1/2$. I overview the invariant sets (Bianchi types) and their dynamics, focusing on the type IX attractor for $v \in (1/2, 1)$, proved in [1]. This establishes an analogue of Ringström's attractor theorem in GR [3, 4] for supercritical theories.

References

[1] E. Beatriz; E. Bonotto; P. Lappicy. The Bianchi IX Attractor in Modified Gravity. Submitted. `https://arxiv.org/abs/2603.13104`.

[2] J. Hell; P. Lappicy; C. Uglla. Bifurcations and Chaos in Hořava-Lifshitz Cosmology. Adv. Theor. Math. Phys. 26, 2095 p. (2022).

[3] H. Ringström. Curvature blow up in Bianchi VIII and IX vacuum spacetimes. 17, 713 p. (2000).

[4] H. Ringström. The Bianchi IX attractor. Ann. Henri Poincaré. 2, 405–500 p. (2001).

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Splitting and Merging of Stagnation Points for the 2D Navier-Stokes Equations

Isidro Benaroya
ICMAT

Abstract

The two-dimensional Navier-Stokes equations model the behavior of viscous fluids whose motion is restricted to a surface. Their solution, starting from a given initial condition, is the velocity field of the particles making up the fluid, assumed here to be incompressible. A bifurcation happens when its stagnation points, i.e. the zeroes of the velocity field, change their number or type.

In this talk, I will discuss a new result proving the existence of solutions to the two-dimensional Navier-Stokes equations, both in $\mathbb{R}^2$ and $\mathbb{T}^2$, with an arbitrary number of stagnation points that merge or split along prescribed trajectories, up to a small deformation. The proof combines ideas from the geometric theory of dynamical systems and recent techniques on the approximation of parabolic partial differential equations. This is joint work with A. Enciso and D. Peralta-Salas, as the speaker's PhD supervisors.

References

[1] I. Benaroya, A. Enciso and D. Peralta-Salas, Splitting and Merging of Stagnation Points of Solutions to the 2D Navier-Stokes Equations, preprint, arXiv:2510.00784 (2025).

E-compact convergence and interpolation spaces

Simone M. Bruschi
George Mason University

Abstract

In [2] Carvalho and Piskarev presented a complete set of abstract results for proving continuity of attractors using E- compact convergence. By working with domain perturbation we developed an alternative approach that allows these results to be applied.

This work presents a definition of extension operators and convergence in spaces with negative exponents and we will prove that the notion of E-convergence is the same as the notion of convergence stablished in an abstract way in [16].

References

[1] J. Arrieta, G. Aragao, and S. M. Bruschi Continuity of attractors of parabolic equations with nonlinear boundary conditions and rapidly varying boundaries. The case of a Lipschitz deformation Journal of Differential Equations, 429, p. 460-502, (2025).

[2] A. N. Carvalho, and S. Piskarev. A general approximation scheme for attractors of abstract parabolic problems. Numer Funct Anal Optim, 27, p.785-829, (2006).

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A brush problem. Homogenization involving thin domains and PDEs in graphs.

Joaquín Domínguez-de-Tena
ICMAT and Universidad Complutense de Madrid

Abstract

In this talk, we will discuss some results that study the behavior of solutions to PDEs when a domain (or part of it) is thin, meaning it has a dimension negligibly small compared to the rest. We will address the case where this combines with periodic or non-periodic structures, giving rise to homogenization problems. The goal will be to introduce these types of problems without diving so much into the details, aiming to motivate the study of the recent project [1]. Finally, we will present this work, where we study a non-periodic brush-type domain, explaining our approach as well as the main difficulties we ran into.

References

[1] J. M. Arrieta and J. Domínguez-de-Tena, *A brush problem. Homogenization involving thin domains and PDEs in graphs*, preprint arXiv:2601.18430 [math.AP], 2026.

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One-phase free boundary problems: From calibrations to viscosity properties

Juan Carlos Felipe-Navarro
Universidad Complutense de Madrid

Abstract

The study of one-phase free boundary problems of Bernoulli type is a classical topic in the Calculus of Variations and PDEs. While the theory of calibrations and fields of extremals is a well-known method to prove minimality for standard elliptic functionals, it has rarely been applied to free boundary problems.

This talk will be devoted to presenting a novel calibration approach adapted to the p-Laplacian one-phase functional

$$\mathcal{E}(w) = \int_{\Omega \cap \{w>0\}} \left(\frac{1}{p-1} |\nabla w(x)|^p + Q(x)^p \right) dx,$$

acting on nonnegative functions, where Ω is a regular domain, $p \in (1, +\infty)$, and Q is a nonnegative weight.

First, I will briefly recap the one-phase free boundary problem as the first variation of the functional and the classical theory of calibrations. Next, I will explain how to extend this theory to the one-phase setting. Finally, I will show that this approach allows us to recover viscosity properties of minimizers (a fundamental step for free boundary regularity).

This is a joint work with Iñigo U. Erneta (Rutgers University)

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Existence of a Non-Autonomous Simplex in the May–Leonard System

Juan García Fuentes
Universidad Pablo de Olavide

Abstract

The May–Leonard model is a particular case of the classical Lotka–Volterra framework, in which a cyclic rock–paper–scissors-type dynamics naturally arises [1]. This model is of particular interest because, in the autonomous setting, it exhibits a heteroclinic cycle connecting one-dimensional equilibrium points [2]. Moreover, this cycle can act as an attractor for the system's trajectories under suitable conditions.

In the autonomous case, the construction of this cycle can be achieved through a geometric structure known as a simplex, which is an invariant, unordered set that attracts the system's solutions, as shown by Stephen Baigent in [3, 4].

From an applied perspective, it is natural to consider ecological models with time-dependent parameters, leading to the study of non-autonomous systems [5]. In this talk, we analyze the following non-autonomous May–Leonard model:

$$\begin{cases} u_1'(t) = u_1(t)(\gamma_1(t) - u_1(t) - \alpha(t)u_2(t) - \beta(t)u_3(t)), \\ u_2'(t) = u_2(t)(\gamma_2(t) - u_2(t) - \beta(t)u_1(t) - \alpha(t)u_3(t)), \\ u_3'(t) = u_3(t)(\gamma_3(t) - u_3(t) - \alpha(t)u_1(t) - \beta(t)u_2(t)). \end{cases}$$

Our main goal is to prove the existence of a non-autonomous simplex, which enables the construction of a non-autonomous heteroclinic cycle of one-dimensional solutions—a phenomenon that, to the best of our knowledge, has not yet been described in the literature on non-autonomous dynamical systems.

References

[1] R.M. May and W.J. Leonard, Nonlinear aspects of competition between three species, SIAM Journal on Applied Mathematics, v.29, p.243-253 (1975).

[2] M.C. Bortolan and P.Kalita and J.A. Langa and R.O. Moura, A theoretical and computational study of heteroclinic cycles in Lotka–Volterra systems, Journal of Mathematical Biology, v.90, n.28 (2025).

[3] Miercynski, J. and Baigent, S., Existence of carrying simplex for a retrotone map, Journal of Difference Equations and Application", v.30, p.287-319"(2024).

[4] Baigent, S., Convex geometry of the carrying simplex for the May-Leonard Map, Discrete and Continuous Dynamical Systems Series B, v.24, p.287-319"(2019). volume="24"

[5] J. García-Fuentes and J.A. Langa and P. Kalita, Forwards attractor structures in a planar cooperative non-autonomous Lotka-Volterra system, Qualitative theory of Dynamical Systems, v.23 , p.163 (2024).

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Formal Power Series for NHIMs under Singular and Functional Perturbations

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Abstract

Normally hyperbolic invariant manifolds (NHIMs) play a central role in dynamical systems by organizing nearby motion and providing reduced descriptions of the dynamics. In this talk, I will discuss formal power series expansions for families of solutions emerging from an unperturbed NHIM of an ODE when the perturbation may be singular or functional, including fast-slow terms, delays, and state-dependent effects.

The main result gives an order-by-order procedure to compute both the asymptotic perturbed manifold and its internal dynamics under standard spectral gap assumptions and mild regularity conditions on the perturbation. A notable feature is that the construction still applies when the perturbed problem is no longer an evolution equation, so the natural object becomes a family of approximate solutions rather than an invariant set of initial conditions. I will conclude with examples motivated by delay equations and relativistic charged-particle models.

This is a joint work with R. de la Llave and J. Yang.

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Existence and Regularity of Stable Resonant Spectral Submanifolds and Conjugacy Maps

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Abstract

We prove the existence of $C^{r,1-\varepsilon}$-regular families of stable invariant manifolds and linear conjugacies for analytic maps near a hyperbolic fixed point in the presence of outer resonances. The regularity exponent r depends on the minimal resonant index, while the Hölder exponent can be chosen arbitrarily close to 1, i.e., $1-\varepsilon$ for any $\varepsilon > 0$. This result implies the Hartman conjecture for stable analytical systems.
Our approach introduces a new class of functional expansions based on logarithmic polynomials, which enables the invariance equation to be solved explicitly to arbitrary order. The existence results are obtained via a fixed-point argument in a suitably constructed Banach space. We further present several examples that both illustrate the theorem and demonstrate the necessity of its assumptions.

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Symmetric periodic solutions in a nonlinear Volterra equation with asymmetric negative feedback

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Abstract

We study the nonlinear convolution equation

$$u(t) = \frac{1}{b-a} \int_{t-b}^{t-a} f\big(u(s)\big)\, ds, \tag{4}$$

where the parameters a, b satisfy $0 < a < b$. The nonlinearity f is assumed to be continuous, strictly decreasing, with $f(0) = 0$. Such equations arise as phenomenological models of population growth and have recently attracted renewed interest in connection with Gurtin–MacCamy-type age-structured models. We establish the existence of even, cosine-like slowly oscillating $(a+b)$-periodic solutions under minimal assumptions, without assuming the commonly used oddness of f or the restriction $b \le 2a$.

Our result is optimal in the spirit of the classical theorem of Hadeler and Tomiuk, [1], asserting that the linear instability of the trivial equilibrium implies the existence of a family of slowly oscillating periodic solutions.

Periodic solutions are obtained as fixed points of a nonlinear compact operator. By constructing an invariant cone of periodic functions and applying the Browder fixed point theorem, we prove the existence of nontrivial symmetric periodic solutions previously observed in numerical simulations.

This is a joint work with Sergei Trofimchuk and Karel Hasík.

References

[1] K.P. Hadeler and J. Tomiuk, Periodic solutions of difference–differential equations, *Arch. Rational Mech. Anal.* **65** (1977), 82–95.

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Results on joint observability-identifiability for autonomous ODE systems

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Abstract

The analysis of identifiability, observability and joint observability-identifiability is of paramount interest when modeling real-world phenomena. These properties determine which unknown quantities of a system can be uniquely inferred from available data. Most existing results focus on rational ODE systems, or on rational approximations of non-rational ones, whereas the study of general nonlinearities remains less developed. Moreover, even within the rational setting, some gaps persist when determining whether a system satisfies these properties. In this talk, we review some results for both rational and non-rational systems and extend them to general autonomous ODE systems, addressing the aforementioned issues and illustrating them through some examples [1].

This is a joint work with Benjamin Ivorra, Ángel M. Ramos and Alain Rapaport.

References

[1] A.B. Kubik, A. Rapaport, B. Ivorra, & Á.M. Ramos. About identifiability and observability for a class of dynamical systems. Automatica, 188:112955 (2026).

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Low regularity perturbations of linear non-autonomous parabolic problems

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Abstract

We present general techniques for low regularity perturbations for evolution operators of linear non-autonomous parabolic problems, obtaining results to determine in which spaces of a scale of Banach spaces the problem has a solution and how it regularizes. This abstract results can be applied to specific cases in scales of Bessel potential spaces and Lebesgue spaces, addressing perturbations defined by interior and boundary potentials, or transport terms, where the spaces in which there is existence of solution and regularization can be explicitly determined.

This is a joint on-going project with Aníbal Rodríguez-Bernal (UCM-ICMAT).

References

[1] A. Rodríguez-Bernal. *Perturbation of analytic semigroups in scales of Banach spaces and applications to linear parabolic equations with low regularity data.* SEMA Journal, 53, pp. 3-54 (2011).

[2] J. W. Cholewa and A. Rodríguez-Bernal. *Linear higher order parabolic problems in locally uniform Lebesgue's spaces.* Journal of Mathematical Analysis and Applications, 449, pp. 1-45 (2017).

[3] J. W. Cholewa and A. Rodríguez-Bernal. *On linear Schrödinger parabolic problems in Morrey spaces.* Journal of Mathematical Analysis and Applications, 545 (2025).

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Differential Systems under Uncertainty for the Study of Epidemic Propagation

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Abstract

Epidemics have had a profound impact on human history, causing not only significant loss of life but also severe social and economic consequences. Many infectious diseases persist today, while new ones continue to emerge, making it essential to develop mathematical models that describe their evolution and support effective control strategies.

One of the foundational frameworks in mathematical epidemiology is the classical SIR model, introduced by Kermack and McKendrick in 1927. This compartmental model divides the population into susceptible (S), infected (I), and recovered (R) individuals, and describes their interaction through a nonlinear system of ordinary differential equations. Despite its importance, the classical SIR model relies on restrictive assumptions, such as constant population size and permanent immunity after infection.

To obtain a more realistic description, we extend the classical SIR model by incorporating vital dynamics (birth and natural death rates) and reinfection. This leads to a modified nonlinear differential system that accounts for population changes and the possibility that recovered individuals may lose immunity.

Motivated by observed variability in real epidemics, we further introduce uncertainty into the transmission process by considering a random perturbation of the transmission coefficient. This results in a random differential system that better captures environmental and social fluctuations affecting disease spread.

In this work, we analyze the resulting model within the framework of random dynamical systems, using the theory of pullback attractors to study its long-term behavior. In particular, we derive conditions under which the disease dies out or becomes endemic. These results provide insight into the qualitative dynamics of the system and are interpreted from a biological perspective. Finally, numerical simulations are presented to illustrate the theoretical findings.

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Fractional Diffusion in the Bessel Potential Scale: Smoothing and Low-Regularity Perturbations

Pablo Molina-Benito
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Abstract

This work aims to study fractional powers, A^μ, with $0 < \mu < 1$, of uniformly elliptic operators of order $2m$ with real and constant coefficients. Our goal is to describe the semigroup generated by $-A^\mu$ in the Bessel potential scale of spaces. In other words, we want to solve the evolution problem $u_t + A^\mu u = 0$ with $u(0) = u_0$ in all the Bessel potential spaces, $H^s_p(\mathbb{R}^N)$, with $1 < p < \infty$ and $s \in \mathbb{R}$ and show that the associated fractional semigroup acts consistently on this scale. We also obtain smoothing estimates that quantify the regularizing effect of the fractional diffusion equation for positive times.

Additionally, we discuss perturbative results in order to add lower-order terms to the equation, including divergence-form operators with low-regularity coefficients. Thus, we will be able to solve the perturbed problem and obtain the corresponding smoothing estimates under certain conditions that depend on the indices of the scale.

This project is being developed in collaboration with Aníbal Rodríguez-Bernal (UCM-ICMAT).

References

[1] H. Amann, *Linear and Quasilinear Parabolic Problems, Vol. I: Abstract Linear Theory*, vol. 1, Springer, 1995.

[2] J. W. Cholewa and A. Rodriguez-Bernal, *Sharp estimates for homogeneous semigroups in homogeneous spaces. Applications to PDEs and fractional diffusion in* $\mathbb{R}^N$, Comm. Contemp. Math. **24**(01) (2022), 2050070.

[3] J. W. Cholewa and A. Rodriguez-Bernal, *Linear higher order parabolic problems in locally uniform Lebesgue's spaces*, J. Math. Anal. Appl. **449** (2017), 1–45.

[4] A. Rodríguez-Bernal, *Perturbation of analytic semigroups in scales of Banach spaces and applications to linear parabolic equations with low regularity data*, Sema J. **53** (2011), 3–54.

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The spectrum of a linear operator associated to a thin domain problem

Estefani M. Moreira
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Abstract

In this presentation, we analyze the spectrum of a linear operator that is related to a thin domain problem. We consider a parabolic problem defined in a thin domain that collapses to an interval with two different scales. We then describe the asymptotic behavior the eigenvalues of the linear operator associated to the limit problem, which is the coupled PDE-ODE system.

Joint work with Leonardo Pires (UEPG) and Marcone Correa Pereira (USP).

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Effective Stability of Near-Rectilinear Halo Orbits in the Earth-Moon System

Luke Peterson
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Abstract

Near-rectilinear halo orbits (NRHOs) around Earth-Moon L_2 in the Circular Restricted 3-Body Problem (CR3BP) exhibit a complex dynamical landscape, featuring a band of normally elliptic orbits embedded within regions of strong instability. This coexistence of stable and unstable dynamics, amplified by the numerical sensitivity associated with close lunar passages, makes the long-term behavior of trajectories near NRHOs a delicate and intrinsically nonlinear problem. Understanding the effective stability of these elliptic orbits is therefore a critical challenge, lying at the intersection of local normal form theory and global instability mechanisms.

To quantify finite-time confinement, we formulate a rigorous framework for effective stability using discrete Poincaré maps. By employing jet transport to compute high-order Taylor expansions, we construct explicit polynomial normal forms. We derive discrete Nekhoroshev-type estimates by identifying the optimal normalization order, which balances the asymptotic convergence of the map's analyticity domain against the cumulative penalty of low-order small divisors.

Applying this framework to the Earth-Moon system, we map the resulting geometric limits directly into physical spatial coordinates. Crucially, we demonstrate that for practical mission lifetimes (e.g., 10–50 years), the required stability is vastly shorter than the characteristic Nekhoroshev accumulation time. Consequently, the effective stability region is not constrained by time-dependent exponential drift but is instead governed entirely by the maximum analytical domain of the optimized normal form. These derived spatial envelopes establish explicit geometric boundaries for the intrinsic local stability of elliptic NRHOs, providing a rigorous mathematical characterization of their nonlinear confinement within the CR3BP.

Joint work with Joan Gimeno.

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Topological properties of rough fluid equilibria

Javier Peñafiel-Tomás
ICMAT

Abstract

It has been shown that fluid equilibria are often chaotic and that they may contain structures as complex as desired. However, there are topological obstructions that prevent steady states of the Euler equations from generating all the dynamics available to a general divergence-free field. In this talk we will show that these obstructions are not present in weak solutions, which can be constructed to be *weakly* topologically equivalent to any prescribed divergence-free field. This is proved using a novel implementation of convex integration in which the topology is preserved throughout the process. These ideas also have applications in a conjecture concerning the symmetry of plasma equilibria.

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Transcritical and Pitchfork Bifurcations in Scalar Nonautonomous Difference Equations

David Rackl
Universität Klagenfurt

Abstract

We study transcritical and pitchfork bifurcations in scalar nonautonomous difference equations. In the nonautonomous setting, these bifurcations are no longer described by isolated equilibria, but by bounded solutions and their stability properties.

Using explicitly solvable model equations as prototypes, we show that classical bifurcation scenarios persist via comparison with these model systems under suitable spectral and nonlinear balance conditions. Our approach is local in nature and applies when the dichotomy spectrum reduces to a single point, which allows it to depend smoothly on the parameter under additional summability assumptions.

Finally, we discuss the structural limitations of this approach, particularly when hyperbolicity is lost over a parameter interval, and indicate how it fits into the broader landscape of nonautonomous bifurcation theory.

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Gradual smoothing: strong hypercontractivity and logarithmic Sobolev inequalities

Jorge Ruiz-Cases
Universidad Autónoma de Madrid and ICMAT

Abstract

In this talk we will study the possibility of a gradual improvement as time progresses of the regularity of solutions to evolution problems of parabolic type driven by Lévy operators, not necessarily translation invariant. The model operator for this behaviour is $\log(I-\Delta)$. In the course of our analysis, we study the equivalence between general smoothing effects and a family of logarithmic Sobolev inequalities. This equivalence allows us to identify a new type of regularization, strong hypercontractivity, characterized by the existence of a time at which solutions belong to every L^p space with p finite. It can also be used to prove logarithmic Sobolev inequalities in a context not previously seen in the literature. We then show that any purely nonlocal Lévy operator whose kernel is comparable to that of $\log(I-\Delta)$ satisfies exactly the same improvement in integrability as $\log(I-\Delta)$ does. Furthermore, in the translation-invariant case, we also prove that solutions get bounded eventually and start improving in differentiability right after. Finally, we show that this behaviour only appears if the kernel defining the operator behaves as $|x-y|^{-N}$ for small interactions (0^+-*order operators*), and this is precisely the threshold: any radial perturbation makes it disappear.

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Relations between different attractors related to impulsive dynamical systems

José Manuel Uzal
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Abstract

Impulsive dynamical systems provide a natural framework for modeling processes in which a continuous evolution is intermittently interrupted by sudden, instantaneous changes in state. Such systems arise in a wide range of applications, including mechanics, biology, and control theory. In this talk, we present the basic concepts and mathematical formulation of impulsive dynamical systems, emphasizing the interplay between their continuous flow and discrete jump dynamics.

We then investigate the existence and qualitative properties of global attractors for these systems, which describe their long-term behavior. In particular, we establish conditions under which an impulsive dynamical system admits a global attractor and analyze its structure. Furthermore, we examine the relationship between three different types of attractors: the global attractor of the impulsive system, the attractor associated with the underlying continuous semiflow, and the attractor corresponding to the induced discrete dynamics generated by the impulses.

The results reveal that the asymptotic behavior is closely linked across these three perspectives. In particular, we show that key features of the long-term dynamics are transferred between the continuous and discrete components, providing a deeper understanding of how impulsive effects influence the global behavior of the system.

This is based on a joint work with Everaldo M. Bonotto (ICMC-USP).