Titles and Abstracts

Harmonic analysis and von Neumann algebras

Leonard Cadilhac

Semicommutative covering lemmas

This talk will be concerned with adapting covering lemmas to operator-valued functions in order to prove maximal inequalities for families of operators. This type of geometric argument is very natural in the classical setting but was not explored so far in noncommutative measure spaces. First, I will explain how this approach allows to recover the operator-valued Hardy-Littlewood maximal inequality through a Vitali type covering lemma. Then, I will show how the same principles apply to prove an operator-valued Besicovitch lemma and to ergodic theory.

Ismael Cano-Mármol

Schur multipliers on matrix Hardy spaces

Hörmander-Mikhlin-Schur multipliers were studied by Conde-Alonso, González-Perez, Parcet and Tablate via some L_∞-BMO estimates of a certain pair of Calderón-Zygmund operators with diagonal kernels. The introduction of an interpolation result based on a new matrix BMO space yielded adequate estimates for HMS multipliers on Schatten p-classes, as well as many other applications.

In this talk, we approach that piece of work from a perspective based on Hardy spaces by introducing a new matrix Hardy space H_1 . When the underlying Hilbert space is discrete, a simplified version of the interpolation result between matrix spaces follows, and so do some characterizations of matrix H_1 and matrix BMO. On the other hand, we will discuss several results for Calderón-Zygmund operators on H_1 and some applications.

This is joint work with Tao Mei and Éric Ricard.

Max Carter

The type I dichotomy for two-step nilpotent groups

A locally compact group G is called *type I* if every unitary representation of G generates a type I von Neumann algebra. The type I condition is equivalent to the following statements:

(i) the unitary dual $\widehat{G}$ is a standard Borel space when equipped with the Mackey-Borel structure;

(ii) every unitary representation of G *uniquely* decomposes into a direct integral of irreducible unitary representations.

Consequently, if G is not type I, it is effectively impossible to classify the (irreducible) unitary representations of G. The question of which groups are type I has been studied extensively in the literature, especially for Lie/algebraic groups over local fields, where the situation is well understood. One exception to this, however, is that it is still unknown whether unipotent groups over positive characteristic local fields are type I. In this talk, I will discuss recent joint work with Pierre-Emmanuel Caprace, where we produce an algebraic-topological criterion for determining when a two-step nilpotent locally compact group is type I. Using the criterion, on one hand, we prove that many two-step nilpotent unipotent groups over positive characteristic local fields are type I. On the other hand, we exhibit an uncountable class of two-step nilpotent tdlc torsion contraction groups which are not type I, answering an earlier open question of myself.

Soham Chakraborty

Kesten's theorem for measured groupoids

The study of Markov operators on measured groupoids was initiated by Kaimanovich. Many of the known results in the study of such Markov operators on groups extend to measured groupoids, although the techniques used are different when the principal part of the groupoid is nontrivial. We show that a pmp measured groupoid is amenable if and only if every symmetric Markov operator on the groupoid has norm 1, thus extending the classical Kesten criterion to groupoids. This is joint work with Milan Donvil, Felipe Flores and Mario Klisse. .

Milan Donvil

W*-superrigidity for compact quantum groups

A group G is called W*-superrigid if it is completely remembered by its group von Neumann algebra in the following sense: any other group with the same group von Neumann algebra must be isomorphic to G. By now, there are many examples of W*-superrigid groups. Of course, a group von Neumann algebra is in particular a compact quantum group, hence one can ask whether superrigidity also holds within this larger class. Surprisingly, many of the typical W*-superrid groups in the literature fail to remain superrigid as quantum groups. I will present a recent joint work with Stefaan Vaes where we prove certain (cohomological) obstructions to 'quantum W*-superrigidity'. We also build on previous work to find the first examples of discrete groups and (non-cocommutative) compact quantum groups which are quantum.

Guillaume Dumas

Regularity of matrix coefficients of semi-simple Lie groups

In his work introducing strong property (T), Lafforgue noticed that any SO(2)-invariant matrix coefficient of a unitary representation of SO(3) is locally 1/2-Hölder continuous (outside of some singular points). This observation, which played a key role in his proof of strong property (T) for SL(3,R), has since proved to be of central importance in several developments. In particular, it has been used to understand better the behavior of some Fourier multipliers, but also the von Neumann algebras of higher rank lattices and their non commutative Lp spaces. In this talk, I will explain how we can extend the regularity result on SO(3) and completely understand the optimal regularity of coefficients of any (compact or noncompact) semisimple Lie group with finite center.

Adriana Fernández Quero

Non-amenable C*-superrigid groups that are not W*-superrigid

In this talk, I will recall the notions of C*- and W*-superrigidity for groups. I will then present examples of non-amenable groups that can be completely reconstructed from their reduced C*-algebras, but not from their group von Neumann algebras. These groups are constructed as infinite direct sums of amalgamated free product groups and provide the first known examples of non-amenable groups exhibiting this behavior. This is joint work with Juan Felipe Ariza Mejía and Ionut Chifan.

Adrián González-Pérez

Schatten class bounds for matricial Riesz transforms

Dimension-free estimates for classical Riesz transforms are a central theme in harmonic analysis and admit several noncommutative counterparts arising from semigroups, group von Neumann algebras, and Markov dilations. Results in this direction have been previously obtained by Junge, Mei and Parcet and by Arhancet and Krigler. In this talk, we discuss analogous operators in the setting of Schur multipliers acting on Schatten classes. Given a map $\beta\colon I \to H$ taking values in a Hilbert space H, one can associate the matricial Riesz transform

$$\mathcal{R}(A) = \Big[a_{j,k} \otimes \frac{\beta(j) - \beta(k)}{\|\beta(j) - \beta(k)\|}\Big]_{j,k\in I}.$$

These operators naturally arise from conditionally negative kernels and the corresponding semigroups of Schur multipliers. We present recent joint work with Parcet, Pérez, and Ricard establishing the boundedness of these transforms on Schatten classes S_p, $1 < p < \infty$, through a simple direct argument. We will also discuss several open problems related to the endpoint behaviour. In particular, it remains open whether these transforms satisfy dimension-free BMO estimates and are of weak type $(1, 1)$.

Guixiang Hong

A noncommutative analogue of Ionescu-Wainger's multiplier theorem

By establishing acceptable L_p bounds for Fourier multipliers with symbols localized to major arcs, the Ionescu-Wainger multiplier theorem has become an indispensable tool in discrete Fourier analysis. Motivated by a program on noncommutative discrete harmonic analysis, in particular a possible complete resolution of the problem on a quantum analogue of Bourgain's ergodic theorems along arithmetic subsets, we have established recently a matrix-valued (or more generally a von Neumann algebra-valued) Ionescu-Wainger multiplier theorem. This is based on a joint work in progress with Eric Ricard.

David Jekel

The gradient flow of entropy in free probability

A classical result of Jordan, Kinderlehrer, and Otto shows that the heat evolution with respect to a measure (e.g. the standard heat flow in the case Lebesgue measure) is the gradient flow of entropy on the Wasserstein space (the space of probability measures with the L^2-Wasserstein distance coming from optimal transportation). The analogous statement for free probability theory is well-understood for the free entropy of a single self-adjoint element, but extremely difficult for the multivariable setting, and connects to the long-standing open problem relating two different notions of free entropy χ and χ^* defined by Voiculescu. I will describe a new version of free entropy and Wasserstein distance which does satisfy the evolution variational inequality, a metric formulation of gradient flow. This is defined by expanding the class of test functions from the original space of traces of non-commutative polynomials to ensure that it is closed both under partial suprema/infima and the free heat semigroup. I also describe why at least some expansion of the original space of test functions seems to be necessary.

Marius Junge

Noncommutative function spaces to the rescue

Starting from the work of Minyu Zhao, we will study complete version of classical heat kernel estimates-based on noncommutative functions space (lots of them). We will indicate the connection to concentration inequalities and the role of amenability.

Christian Le Merdy

Absolute dilations of Schur and Fourier multipliers

We consider the notion of absolute dilation for a nomal, unital, completely positive and trace preserving map $T\colon M \to M$ acting on a von Neumann algebra equipped with a nsf trace. We first present the links between this notion and that of isometric dilation in non-commutative L^p-spaces on the one hand, and with that of factorisable operators on the other. Next, we focus more specifically on the absolutely dilatable Schur multipliers, both in the discrete and in the measurable cases, and on the absolutely dilatable Fourier multipliers.

Tao Mei

The Λ_p property of well-separated branches of hyperbolic groups

Let F_n be the free group with free generators g_i. In a previous joint work with Eric Ricard, we showed that the branches B_i of F_n consisting of reduced words starting with g_i has the so-called Λ_p property. This means that, for functions f_i supported on B_i, the noncommutative L_p norms of $\sum_i f_i$ and $\sum_i \varepsilon_i f_i$ are equivalent for arbitrary choices of $\varepsilon_i = \pm$. In this talk, I will try to explain that such Λ_p-property extends to well-separated branches of hyperbolic groups.

Basile Morando

Factoriality of von Neumann algebras of totally disconnected groups

To any locally compact group G, one can associate a von Neumann algebra L(G), generated by the left regular representation of G. This algebra reflects the decomposition properties of the representation: L(G) is a factor — i.e., has trivial center — if and only if the regular representation does not split as a direct sum of two disjoint subrepresentations. In the discrete case, Murray and von Neumann showed that L(G) is a factor if and only if all non-trivial conjugacy classes are infinite. By contrast, for non-discrete groups, determining factoriality becomes more subtle.

In this talk, we address this question for totally disconnected groups. We present a sufficient criterion for factoriality, apply it to certain groups acting on (boundaries of) trees, and discuss the consequences of these results for the type of the group and its representations.

Hugues Moyart

Interpolation of noncommutative Hardy spaces

The theory of Hardy spaces plays a central role in harmonic analysis. The interpolation properties of classical analytic Hardy spaces have been investigated by many authors since the seminal work of P. W. Jones, whose main result can be formulated in terms of K-closedness. An operator-valued extension of Jones' theorem was later obtained by G. Pisier and Q. Xu. In this talk, we present recent advances that resulted in a (noncommutative) martingale counterpart of these results. The approach sheds light on the relationship between the properties of noncommutative martingale Hardy spaces and those of noncommutative martingale transforms.

Eduardo Tablate

Pseudodifferential operators over matrices and group algebras

Classical pseudodifferential operators generalize Fourier multipliers and play a central role in the analysis of linear PDEs. In $\mathbb{R}^n$, given a symbol $a\colon \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{C}$, the pseudodifferential operator T_a is defined by

$$T_a f(x) = \int_{\mathbb{R}^n} a(x,\xi)\, \widehat{f}(\xi)\, e^{2\pi i x\cdot\xi}\, d\xi,$$

Fourier multipliers correspond to the special case $a(x,\xi) = m(\xi)$. The L_p-theory of these operators was initiated by Kohn, Nirenberg, and Hörmander, and lies methodologically in Calderón–Zygmund techniques. In noncommutative settings these objects have been studied in quantum Euclidean spaces, but not in the general context of group von Neumann algebras. In this talk we introduce pseudodifferential operators over group von Neumann algebras $\mathcal{L}(\mathrm{G})$. We prove that these operators are naturally connected with suitable generalizations of Schur multipliers, and we establish transference principles analogous to the Fourier/Schur correspondence. As first applications, we obtain L_2 and L_p boundedness results for natural classes of symbols in both group and matrix algebra settings. This is joint work with J. Conde, A. González-Pérez and J. Parcet.

Simeng Wang

On ball Fourier multipliers on free groups

We prove that the ball Fourier multipliers on free groups, defined with respect to the natural word length, are uniformly bounded on the noncommutative analogue of Rubio de Francia's mixed radial–angular spaces $L_p(L_2)$ for $2/3 < p < 3$. This range is sharp, as unboundedness for other values of p was established by Bożejko and Fendler. To this end, we relate the ball multipliers to radial Hardy–Littlewood maximal functions on weighted L_p spaces, which can be viewed as noncommutative analogues of restricted Kakeya maximal functions. As a key ingredient, we explicitly compute the noncommutative convolution kernel of the free radial Fourier multipliers via the Poisson boundary of the free group. The talk is based on joint work with A. González-Pérez, B. Lyu, J. Parcet, J. Pérez-García and T. Zhao.

Runlian Xia

Hilbert transforms on graph products of finite von Neumann algebras

In this talk, we will discuss Hilbert transforms and their boundedness on graph products of finite von Neumann algebras, which generalize both Voiculescu's free products and tensor products of von Neumann algebras. The free Hilbert transform on free products of von Neumann algebras was studied by Mei and Ricard. However, in the setting of graph products, the situation is more delicate, as reduced operators may admit several equivalent reduced expressions.

To address this issue, we introduce a natural analogue of the free Hilbert transform in the graph product setting. The boundedness relies on two main ingredients: a generalized Cotlar identity and a Haagerup-type inequality. We show that when the block length of reduced operators exceeds a constant depending on the graph, a Cotlar identity holds.

This is based on a joint work with Xiaoqi Lu.

Safoura Zadeh

Pointwise convergence for random ergodic averages in non-commutative L^p-spaces

Let M be a semifinite von Neumann algebra and T a positive contraction on both $L^1(M)$ and $L^\infty(M)$. In this talk, we consider ergodic averages along a random sparse subsequence determined by independent Bernoulli variables $(X_n)_{n\geq 1}$ with $\mathbb{P}(X_n = 1) = n^{-\alpha}$, and set $W_N = \sum_{n=1}^N \mathbb{E}[X_n]$. We see that, almost surely, the averages $\frac{1}{W_N}\sum_{n=1}^N X_n\, T^n(x)$ converge bilaterally almost uniformly to the ergodic projection for all $1 < p < \infty$. This extends a theorem of Bourgain to the non-commutative setting, using Junge–Xu ergodic theorem and some concentration inequalities. The talk is based on a joint work with Christian Le Merdy.

Hao Zhang

Boundedness of Riesz transforms on weighted hypercubes

We establish the dimension-free boundedness of Riesz transforms on weighted hypercubes. The proof is based on the boundedness of Riesz transforms associated with the nontracial baby Fock model. Our results also extend to q-Araki-Woods algebras.