Main talks and minicourses. Book of abstracts

Beltran, David (Universitat de València, Spain):
Maximal cubic modulations of the Hilbert transform along the parabola.

Abstract: Pierce and Yung introduced in 2015 a family of operators that is a hybrid between polynomial Carleson operators and Radon transforms: maximally modulated singular integrals along paraboloids. They established their $L^p(\mathbb{R}^n)$-boundedness for all $1 < p < \infty$ and $n \geq 3$, assuming that the modulations are neither linear nor multiplies of $|t|^2$. In this talk, we will discuss the challenges of the planar case and present positive results for a model case: maximal cubic modulations of the Hilbert transform along the parabola. Our approach is inspired by techniques used in proving L^p-bounds for certain geometric maximal functions; in particular, it relies on local smoothing estimates for a class of oscillatory integral operators.

This is joint work with Shaoming Guo and Jonathan Hickman.

Cadilhac, Léonard (Sorbonne Université, France):
Noncommutative maximal inequality on amenable groups.

Abstract: This talk will discuss generalizations of the Hardy-Littlewood maximal inequality to certain groups, meaning that the functions considered will no longer be defined on $\mathbb{R}^d$ but on amenable groups. I will start by presenting the motivations for this work, coming from ergodic theory and noncommutative analysis. Then, after an introduction of the objects at play, the talk will mainly focus on the geometric aspect of the problem and how martingale techniques can be used to import Calderón-Zygmund theory in this irregular setting.

Caro, Pedro (Basque Center for Applied Mathematics, Spain):
An inverse problem for data-driven prediction in quantum mechanics.

Abstract: Data-driven prediction in quantum mechanics consists in providing an approximative description of the motion of any particles at any given time, from data that have been previously collected for a certain number of particles under the influence of the same Hamiltonian. The difficulty of this problem comes from the ignorance of the exact Hamiltonian ruling the dynamic. In order to address this problem, Alberto Ruiz and I have formulated an inverse problem consisting in determining the Hamiltonian of a quantum system from the knowledge of the state at some fixed finite time for each initial state. We focus on the simplest case where the Hamiltonian is given by $-\Delta + V$, where the electric potential V is non-compactly supported. During the talk I will present several uniqueness results for time-dependent potentials $V = V(\mathrm{t}, \mathrm{x})$ and stationary potentials $V = V(\mathrm{x})$, and the difference between them. Roughly speaking, these results are uniqueness theorems, that explain why the Hamiltonians ruling the dynamics of all quantum particles are determined by the corresponding initial and final states of all these particles. As a consequence, one expects to be able to solve the data-driven prediction problem in quantum mechanics.

The theorems I will discuss are the results of collaborations with Alberto Ruiz; and Manuel Cañizares, Ioannis Parissis and Athanasios Zacharopoulos.

Greenfeld, Rachel (Northwestern University, USA):
Integer distance sets.

Abstract: A set in the Euclidean plane is called an integer distance set if the distance between any pair of its points is an integer. All so-far-known integer distance sets have all but up to four of their points on

a single line or circle; and it had long been suspected, going back to Erdős, that any integer distance set must be of this special form. In joint work with Marina Iliopoulou and Sarah Peluse we developed a new approach to the problem, which enabled us to make the first progress towards confirming this suspicion. In the talk, I will discuss the study of integer distance sets, its connections with other problems, and our new developments.

Hickman, Jonathan (University of Edinburgh, United Kingdom):
Geometric maximal functions.

Abstract: This minicourse explores the theory of geometric maximal functions. For us, a prototypical example is Stein's *spherical maximal function*, a singular variant of the classical Hardy–Littlewood maximal function, formed by taking maximal averages over concentric spheres. In recent years the theory of geometric maximal functions has flourished: many new techniques and results have become available, and the theory itself has found new and important applications.

The minicourse, which serves as a gentle introduction accessible to students and nonexperts, studies geometric, combinatorial and analytic methods used in the modern theory of geometric maximal functions. To do this, we examine some recent results, such as multiparameter and higher codimensional variants of Stein's spherical maximal theorem, and also revisit some classical theory from new perspectives.

Iliopoulou, Marina (University of Athens, Greece):
Improved bounds on oscillatory integral operators under weak assumptions on the phase function.

Abstract: We will discuss operators of two types: (a) Hörmander-type operators and (b) variable coefficient Schrödinger propagators. In each case the corresponding waves evolve inside tubes, whose central curves arise from the phase function. While in case (a) these are Kakeya sets of curves, in case (b) they are Nikodym sets of curves - and it is known that this causes essential differences in the L^p-theory of the two cases. In the talk, we will present appropriate, quite weak conditions on the phase function (distinct for each case), which prohibit the corresponding tubes to cluster in little space. This leads to improved Kakeya and Nikodym estimates for the corresponding sets of tubes, which in turn yield improved L^p-bounds for the corresponding operators. This is joint work with M. Chen, S. Gan, S. Guo, J. Hickman and J. Wright.

Maggioni, Mauro (Johns Hopkins University, USA):
Some estimation problems in high dimensions.

Abstract: I will discuss a couple of recent results in the area of high-dimensional estimation problems.

First, we consider systems of interacting agents or particles, which are commonly used in models throughout the sciences, and can exhibit complex, emergent large-scale dynamics, even when driven by simple interaction laws. We consider the following inference problem: given only observations of trajectories of the agents in the system, can we learn the unknown laws of interactions? We cast this as an inverse problem, discuss when this problem is well-posed, construct estimators for the interaction kernels with provably good statistical and computational properties, even in the nonparametric estimation regime when only minimal information is provided about the form of such interaction laws. We also demonstrate numerically that the estimated systems can accurately reproduce the emergent behaviors of the original systems, even when the observations are so short that no emergent behavior was witnessed in the training data.

In the second part of the talk, I will discuss instances of estimation of a function f in high dimensions from data in the form of input-output data pairs $\{(x_i, y_i)\}_{i=1}^n$, when there is special structure in the problem that allows one to avoid the curse of dimensionality, either in the form of a low-dimensional manifold on which the input data lies, or in the form of a special compositionality structure of the function itself.

Mourgoglou, Mihalis (University of the Basque Country (UPV/EHU) & Ikerbasque, Spain):
Solvability of the Neumann problem for elliptic PDE's in chord-arc domains with very big pieces of good superdomains.

Abstract: One of the central open problems in the theory of elliptic partial differential equations is the L^p-solvability of the Neumann problem for the Laplacian in chord-arc domains, for some $p > 1$. In this talk, I will present a recent result obtained in collaboration with Xavier Tolsa that advances our understanding of this question. Specifically, we consider a bounded chord-arc domain $\Omega \subset \mathbb{R}^{n+1}$ and $\mathcal{L} = -\mathrm{div} A\nabla$ an elliptic operator in Ω associated with a matrix A having Dini mean oscillation coefficients. If $1 < p \leq 2$, we show that if the regularity problem for $\mathcal{L}$ is solvable in L^q for some $q > p$ in Ω, $\partial\Omega$ supports a weak p-Poincaré inequality, and Ω has very big pieces of superdomains for which the Neumann problem for $\mathcal{L}$ is solvable uniformly in L^q, then the Neumann problem for $\mathcal{L}$ is solvable in L^p in Ω. This result provides a new mechanism to transfer Neumann solvability from larger domains to a given domain under structural and geometric assumptions.

Orponen, Tuomas (University of Jyväskylä, Finland):
Radial and restricted projections.

Abstract: The mini course will cover two results from fractal geometry, in particular *projection theory*, that were useful in the resolution for the *sticky Kakeya conjecture* by H. Wang and J. Zahl.

The first result concerns the *radial projections* $\pi_x(y) = (x - y)/|x - y|$ in the plane. It states that if $E, F \subset \mathbb{R}^2$ are compact disjoint 1-dimensional sets, and E is not contained on any line, then

$$\sup_{x \in E} \dim_{\mathrm{H}} \pi_x(F) = 1.$$

The second result concerns orthogonal projections to sufficiently "curved" 1-parameter families of lines in $\mathbb{R}^3$. I will mostly discuss the projection family $\rho_\theta(x, y, z) := (x, y, z) \cdot (\cos\theta, \sin\theta, 1)$, $\theta \in [0, \pi)$. The result is that if $B \subset \mathbb{R}^3$ is a compact 1-dimensional set, then

$$\dim_{\mathrm{H}} \rho_\theta(B) = 1 \quad \text{for a.e. } \theta \in [0, \pi).$$

Lecture notes: `https://sites.google.com/view/tuomaths/teaching`

Shmerkin, Pablo (University of British Columbia, Canada):
Fourier decay of multiplicative convolutions.

Abstract: The sum-product philosophy suggests that multiplicative convolutions of measures of positive dimension should enjoy good Fourier decay properties. Bourgain proved such a statement in 2010. I will discuss joint work with T. Orponen and N. de Saxcé, in which we prove a sharp version of Bourgain's result, as well as other quantitative improvements, using recent progress in fractal sum-product theory. A corollary of our results is that if $\dim_{\mathsf{H}}(A) > 1/k$, then the k-fold self-product of A generates $\mathbb{R}$ as an additive group (and this threshold is sharp).

Steinerberger, Stefan (University of Washington, USA):
Some Open Problems in Fourier Analysis.

Abstract: I will survey a number of open problems in Fourier Analysis and adjacent areas that are maybe less known or have been forgotten. New ideas are needed!

Zhong, Xiao (Sun Yat-sen University, China):
Variational problems with gradient constraints.

Abstract: I will talk about three different types of variational problems with gradient constraints. They arise from elastic-plastic torsion, hypersurfaces in the Lorentz-Minkowski spaces with given mean curvature and dimer models.

Hong Wang (New York University, Courant Institute of Mathematical Science):
Union of tubes and Kakeya sets.

Abstract: We discuss techniques (Córdoba argument, projection theory) to prove volume bounds on union of tubes in $\mathbb{R}^2$ and $\mathbb{R}^3$, with applications to the Kakeya problem. This is joint work with Josh Zahl.