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Cosserat media in dynamics

Our aim is to develop a general approach for the dynamics of material bodies of dimension d represented by a submanifold N of dimension (d + 1) of the space-time M, defined by an injection i: N ⟶ M: ξ ⟼ X. It can be declined for d = 0 (material particles and rigid bodies, represented by a curve), d = 1 (arch if solid, flow in a pipe or jet if fluid, represented by a surface), d = 2 (plate or shell if solid, sheet of fluid, represented by a volume), d = 3 (bulky bodies, represented by a submanifold of dimension 4).

We call torsor a skew-symmetric bilinear form on the vector space of R-valued affine maps on the affine tangent space to M at X. The components of this affine tensor, valued in the tangent space to N at X = i(ξ), are obtained by decomposition in an affine frame (composed of a basis and an origin of the tangent space to M) and a basis of the tangent space to N.

According to É. Cartan, we define affine connections of which the coefficients describe the infinitesimal motion of an affine moving frame on M. For applications to classical mechanics, we consider the Galilean connections associated to the G-structure where G is Galilei group, a Lie group of dimension 10.

After some insights into the cases d = 0 and d = 4, we treat more deeply the case d = 1. We discuss the physical meaning of the torsor components. We claim that the covariant affine divergence of the torsor field vanishes and we deduce 10 conservation laws.