Nonlinear continuum theory of three-dimensional curved dislocations

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Abstract

This paper presents a novel continuum dislocation theory rooted in the kinematics of finite plastic deformation, where dislocations are formed without deforming the underlying crystal lattice. We define the resultant Burgers' vector and the corresponding dislocation density tensor in this framework. The crystal's reference configuration is modeled as a non-Riemannian manifold, characterized by a metric tensor representing finite plastic strain and a Cartan torsion tensor associated with dislocation density.

We establish a thermodynamic framework for finite strain continuum dislocation theory based on this kinematics. The theory is applied to an antiplane constrained shear problem, yielding an analytical solution. Subsequently, we extend the theory to crystals containing curved dislocations. In the absence of significant resistance to dislocation motion, we derive a set of governing equations and boundary conditions for the true placement, plastic slips, and loop functions that minimize the crystal's free energy. For non-negligible resistance, the governing equations are derived from a variational equation incorporating dissipation. A simplified theory for small strains is also presented. An asymptotic solution is obtained for the two-dimensional problem of a single crystal beam undergoing single slip and simple shear deformation.