Workshop on Geometrical aspects of material modelling
Instituto de Ciencias Matemáticas
21 − 23 August 2024, Madrid, España

Non-Holonomic Micromorphic Placements and their Induced Geometry and Static Equations

Mewen Crespo*[1], Guy Casale[2] and Loïc Le Marrec[3]

[123] Univ Rennes, CNRS, IRMAR-UMR 6625,
263 avenue du Général Leclerc, CS 74205, 35042 Rennes, France

[1] mewen.crespo@univ-rennes.fr https://mewen-crespo.fr
[2] guy.casale@univ-rennes.fr https://perso.univ-rennes1.fr/guy.casale
[3] loic.lemarrec@univ-rennes.fr https://researchgate.net/profile/Loic-Le-Marrec

Keywords: *Generalized continua, Fibre bundles, Connections, Pseudo-metrics, Principle of virtual work*

The kinematics of a micro-structured material is geometrically modelled, as in [1, 2, 3], through the framework of fibre bundle geometry. The material continuum is a fibre bundle $\mathcal{B} \to \mathbb{B}$ where $\mathbb{B}$ is compact and orientable. It is commonly agreed that connections with curvature and torsion can describe defect densities in micro-structured materials. The first goal of this work is to introduce a method to derive the connection from the kinematics in an intrinsic way and derive the generic form of a frame invariant energy in such a model. Then, in the remaining time, it will be shown how the principle of virtual work can be used to derive static equilibrium equations for this model.

The material bundle $\mathcal{B}$ is placed in the Euclidean fibre bundle $\mathcal{E} \equiv \mathrm{T}\mathbb{E} \to \mathbb{E}$ using a punctual placement map $\varphi : \mathcal{B} \to \mathcal{E}$. A first-order placement map $\mathbf{F} : \mathrm{T}\mathcal{B} \to \mathrm{T}\mathcal{E}$ generalizing $\mathrm{T}\varphi$ is then introduced. This placement has a macroscopic, a microscopic and a mixed part as in [4]. A macroscopic metric on $\mathbb{E}$, a connection on $\mathcal{E}$ and a solder form on $\mathcal{E}$ are canonically prescribed from the Euclidean structure on $\mathcal{E}$ (see [5, 6, 7]). Using those, a generalisation of the notion of metric is prescribed on $\mathcal{E}$. This later takes the form of a pseudo-metric, defined as the only one satisfying a certain compatibility condition with the connection, solder form and macroscopic metric. Finally, using $\mathbf{F}$, a metric on $\mathbb{B}$, a connection on $\mathcal{B}$, a solder form on $\mathcal{B}$ and a pseudo-metric on $\mathcal{B}$ are inferred.

A generalisation of the group of Euclidean displacements is proposed, and the orbit of all possible first-order placement maps $\mathbf{F}$ under this group are computed. Invariance of the energy under this group, analogous to the notion of frame indifference, is then shown to be equivalent to the expressibility of the energy as a functional of the induced material connection, material solder form and material microscopic metric (canonically obtained from the material pseudo-metric). In the micro-linear holonomic case, it is proven that the data of these tensors is equivalent to the data of the pseudo-metric on $\mathcal{B}$.

It is then shown how the kinematic of the proposed model includes the kinematics of some well known models such as Eringen microcontinua [4], Cosserat media [2] and Timoschenko or Euler-Bernoulli beams. Following the methodology of P. Germain [8, 9], it is shown how the principle of virtual work can be successfully used to derive the static equilibrium equations of this model. It is then shown how the static of the proposed model includes the static of some well known models.

The authors would like to emphasise their desire to establish a thorough comparison of this model with other models of generalised continua, especially those allowing for the emergence of non-holonomy and defects. They therefore encourage every participant of this workshop to come and exchange with them regarding this matter.

References

[1] É. Cartan, *Comptes Rendus Ac. Sc. Paris* **174**, 593–595 (1922).
[2] E. Cosserat et al., *Théorie des corps déformables*, A. Hermann et fils (1909).
[3] V. H. Nguyen et al., *Mathematics and Mechanics of Solids* (2021).
[4] A. C. Eringen, *Microcontinuum Field Theories*, Springer (1998).
[5] O. Gonzalez et al., *A First Course in Countinuum Mechanics*, Cambridge Univ. Press (2008).
[6] M. Epstein, *The Geometrical Language of Continuum Mechanics*, Springer (2010).
[7] J. Marsden et al., *Mathematical Foundations of Elasticity*, Dover (1994).
[8] P. Germain, *La méthode des puissances virtuelles en mécanique des milieux continus*, Journal de Mécanique (1973)
[9] P. Germain, *The method of virtual power in continuum mechanics. Part 2: Microstructure*, SIAM Journal on Applied Mathematics (1973)