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Multiscale Analysis of Effective Properties in Remodelling Composite Media

We present a multiscale analysis of a composite medium comprising two solid phases undergoing a remodelling process described through the evolution of anelastic distortions and contextualised through the use of the Bilby-Kröner-Lee decomposition of the deformation gradient tensor [1] in a multi-scale framework. Our methodology begins with formulating the governing equations for the dynamics of the composite's constituents, based on the balance of linear momentum and the evolution law for inelastic distortions. Then, our mathematical framework leverages the two-scale Asymptotic Homogenisation (AH) technique to upscale the fine-scale anelastic distortions, which affect the constituents' elasticity moduli, to the coarse scale while explicitly accounting for geometrical features [2]. This influence is captured through the AH technique via a tensor field known as the tensor of effective coefficients. A key motivation is to simplify the computational challenges in determining the effective properties of composites undergoing remodelling, which involves managing the transfer of information between scales of the so-called cell and homogenised problems. Thus, our discussion also centres on providing (semi-)analytical expressions for the effective coefficients parameterised by space and time through the tensor of anelastic distortions with a particular focus on the case of multi-layered [3] or uniaxially fibre-reinforced structures [4]. While our work is mathematical in nature, our underlying goal is to explore questions related to biology, as biological systems, such as soft and hard tissues, can change their properties in response to various internal and external factors.

[1] Micunovic M. Thermomechanics of Viscoplasticity - Fundamentals and Applications. Springer, Heidelberg, Germany, 2009.

[2] Ramírez-Torres A, Di Stefano S, Grillo A, Rodríguez-Ramos R, Merodio J, Penta R (2018) An asymptotic homogenization approach to the microstructural evolution of heterogeneous media. International Journal of Non-Linear Mechanics, 106:245–257.

[3] Giammarini A, Ramírez-Torres A, Grillo A (2024) Effective elasto-(visco)plastic coefficients of a bi-phasic composite material with scale-dependent size effects. Mathematical Methods in the Applied Sciences.

[4] Ramírez-Torres A, Roque-Piedra A, Giammarini A, Grillo A, Rodríguez-Ramos R (2024) Analytical expressions for the effective coefficients of fibre-reinforced composite materials under the influence of

inelastic distortions. Journal of Applied Mathematics and Mechanics (ZAMM). Under review