Three differential equations, composition C_0-semigroups and Cesàro-like operators

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Abstract: To study the following differential equations

$$\begin{aligned}\frac{\partial u(t,r)}{\partial t} &= (1-r)\frac{\partial u(t,r)}{\partial r} - \frac{1}{p}u(t,r),\\ \frac{\partial v(t,r)}{\partial t} &= r(1-r)\frac{\partial v'(t,r)}{\partial r} + \left(\frac{1}{p} - \gamma r\right)v(t,r),\\ 2\frac{\partial w(t,r)}{\partial t} &= (1-r^2)\frac{\partial w(t,r)}{\partial r} + \left(\gamma - \frac{2}{p} - \gamma r\right)w(t,r),\end{aligned}$$

for $r, t \geq 0$, we introduce three different continuous semigroups $(\phi_t)_{t\geq 0}$, $(\psi_t)_{t\geq 0}$ and $(\varphi_t)_{t\geq 0}$ on the real half-line where

$$\phi_t(r) := e^{-t}r + 1 - e^{-t}, \quad \psi_t(r) := \frac{e^t r}{1 + r(e^t - 1)}, \quad \varphi_t(r) := \frac{(1+e^t)r - 1 + e^t}{(-1+e^t)r + 1 + e^t},$$

for $r, t \geq 0$. These flows induce composition C_0-semigroups, $(T^{\gamma}_{t,p})_{t>0}$, $(S^{\gamma}_{t,p})_{t>0}$ and $(R^{\gamma}_{t,p})_{t>0}$ on the fractional Lebesgue spaces $\mathcal{T}^{(\alpha)}_p(t^\alpha)$, closed subspaces of $L^p(\mathbb{R}^+)$. We describe spectrum sets, point spectrums and resolvent operators of their infinitesimal generators. Three Cesàro-like operators

$$\begin{aligned}\mathcal{C}_{\mu,\nu}f(r) &:= \frac{1}{|r-1|^{\mu+\nu-1}}\int_{\Gamma_{1,r}} |s-1|^{\mu-1}|r-s|^{\nu-1}f(s)ds, \quad r>0,\\ \mathfrak{C}^{\gamma}_{\mu,\nu}f(r) &:= \frac{r^\mu}{|r-1|^{\mu+\nu+\gamma-1}}\int_{\Gamma_{1,r}} \frac{|s-1|^{\mu+\gamma-1}}{s^{\mu+\nu}}|r-s|^{\nu-1}f(s)ds, \quad r>0,\\ \mathcal{C}^{\gamma}_{\mu,\nu}f(r) &:= 2^\nu\frac{|r+1|^{\mu-\gamma}}{|r-1|^{\mu+\nu-1}}\int_{\Gamma_{1,r}} \frac{|s-1|^{\mu-1}}{|s+1|^{\mu+\nu-\gamma}}|r-s|^{\nu-1}f(s)ds, \quad r>0,\end{aligned}$$

where $\mu, \nu, \gamma \in \mathbb{R}$ and $\Gamma_{1,r} := (1, r)$ when $r > 1$ and $\Gamma_{1,r} := (r, 1)$ in the case $0 < r < 1$, are subordinated to these C_0-semigroups which allow to obtain their norms and spectrum sets.

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Keywords: Composition C_0semigroups; Cesàro-like operators; fractional Lebesgue spaces on the real half-line.