

# Commutativity and embeddability for univalent self-maps in the unit disk

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**Abstract:** The embeddability problem is the problem to determine whether a given univalent (i.e. injective holomorphic) self-map of the unit disk  $\mathbb{D}$  is contained in a one-parameter semigroup, i.e. in the image of a continuous homomorphism from the semigroup formed by the non-negative reals and endowed with the standard Euclidian topology to the semigroup of all univalent self-maps of  $\mathbb{D}$  with the topology of locally uniform convergence and with the composition as the semigroup operation. For a given univalent self-map, this problem is usually extremely hard, unless the self-map luckily fails to have one of the nice properties known for elements of one-parameter semigroups, such as existence of angular limits everywhere on the unit circle.

For two explicitly given univalent self-maps it is almost straightforward to check whether they commute or not. However, it is still very hard to describe explicitly the centralizer of a univalent self-map, i.e. the set of all univalent self-maps commuting with the given one.

The main results of the talk relate the embeddability of a univalent self-map with the structure of its centralizer. If time permits, we will also discuss some further ideas that may lead to progress in the embeddability problem in certain interesting special cases.