Book of abstracts

Non-Archimedean analogues of the valuations algebra

Semyon Alesker, Tel Aviv University

The space of translation invariant valuations on convex sets carries a number of structures with non-trivial properties and geometric applications. These include product, convolution, and Fourier type transform. In the talk we describe an analogue of the space of even valuations over local fields and similar structures on it. Product and convolution satisfy a hard Lefschetz type theorem.

Floating bodies and polarity on the unit sphere

Florian Besau , Technische Universität Wien

We investigate a natural analog to Lutwak's p-affine surface area, for $p = -d/(d+2)$, in d-dimensional spherical space. This curvature measure arises as the intrinsic volume derivative of the spherical floating body of a spherically convex body conjugated by polarity in spherical space.
The relation between p-affine surface area and the floating body was established in Euclidean space by Meyer & Werner and our results show, that it naturally extends from the flat geometry to the classical spaces of constant curvature.
Based on joint work with E. Werner.

Unitary equivariant translation invariant tensor valuations

Karoly Boroczky, Renyi Institute, Budapest

Following the work of Semyon Alesker in the scalar valued case and of Thomas Wannerer in the vector valued case, the dimensions of the spaces of continuous translation invariant and unitary equivariant tensor valuations are computed. In addition, a basis in the vector valued case is presented. Joint work with Matyas Domokos and Gil Solanes

Fixed points of Mean Section Operators

Leo Brauner, TU Wien

Several geometric inequalities for Minkowski valuations can be reduced to the determination of their fixed points. In this talk, we prove that for a large class of Minkowski valuations, including the Mean Section Operators, there is a neighborhood of the unit ball where the only fixed points are Euclidean balls. Our approach unifies and extends previous results by Ivaki, Schuster, and Ortega-Moreno. This is joint work with Oscar Ortega-Moreno.

Valuations on spaces of functions

Andrea Colesanti, University of Florence

We will review the main directions of research and the main developments in the theory of valuations on spaces of functions. We will focus on recent results concerning valuations on spaces of convex functions and we will present some natural questions in this theory, which are currently unsolved.

On a problem of Fejes Toth

Susanna Dann, Universidad de los Andes

Complex L_p-Intersection Bodies

Simon Ellmeyer, TU Wien

Interpolating between the classical notions of intersection and polar centroid bodies, (real) L_p-intersection bodies, for $-1 < p - 2$, is introduced, interpolating between polar complex centroid and complex intersection bodies.

On the existence of valuations with prescribed restrictions

Dmitry Faifman, Tel Aviv University

Assume that a collection of valuations is given on a family of subspaces in $\mathbb{R}^n$. Are they the restrictions of a single valuation on $\mathbb{R}^n$? One quickly realizes that compatibility of the given data on intersections is a necessary condition. Is it sufficient? We will discuss several geometric instances of this problem, whence it acquires distinct flavors. In particular, we will deduce a Nash embedding theorem for valuations on manifolds. Based on a joint work with Georg Hofstätter.

Dual volumes, dual Steiner polynomials and more

María A. Hernández Cifre, Universidad de Murcia

Dual (mixed) volumes and dual Steiner polynomials are central notions in the well-known dual Brunn-Minkowski theory. Many interesting questions and generalizations arise around these two concepts, and our aim in this talk is to present some of them: how to characterize dual Steiner polynomials, how they behave, convex bodies that are determined by their dual volumes, or possible extensions to other functionals or close related polynomials. The contrast of these results with the corresponding ones in the classical Brunn-Minkowski setting will be also shown.

Equivariant Valuations on Convex Functions

Georg Hofstätter, Friedrich-Schiller-University, Jena

In the affine geometry of convex bodies, many fundamental constructions like the difference body map and the projection body map can be characterized by continuity, a valuation property, translation-invariance and their equi- or contravariance, respectively, with respect to volume preserving linear maps. In this talk, we transfer this approach to the theory of valuations on convex functions and give a characterization of all continuous and dually epi-translation invariant valuations on finite convex functions with values in the same space, which are SL_n equi- or contravariant, respectively, thereby generalizing theorems by M. Ludwig. This is joint work with J. Knoerr.

Valuations, distributions, and Monge-Ampère operators

Jonas Knoerr, TU Wien

In recent years, valuations on functions arose as a natural generalization of valuations on sets, and due to their intimate relation with convex bodies, valuations on convex functions have been the focus of intense research. I will talk about a construction that allows us to interpret dually epi-translation invariant valuations on convex functions as distributions with special properties, and we will discuss how this interpretation can in turn be used to obtain strong characterization results of certain subclasses of valuations. In particular, we will focus an the information encoded in the Fourier-Laplace transform of these distributions and their relation to certain equivariant Monge-Ampère operators.

From harmonic analysis of translation-invariant valuations to geometric inequalities

Jan Kotrbatý, Goethe University Frankfurt

The Alesker-Bernig-Schuster theorem describes how the space of translation-invariant continuous valuations decomposes under the orthogonal group. In a joint work with Thomas Wannerer, we construct an explicit set of the corresponding highest weight vectors and characterize important algebraic operations on valuations (Alesker-Poincaré pairing, Fourier transform, Lefschetz operator) in terms of their action on these vectors. Our results imply the Hodge-Riemann relations for valuations and consequently new linear inequalities between mixed volumes of convex bodies.

Canal Classes of Convex Bodies and Related Inequalities.

Nico Lombardi, TU Wien

We will first recall the classical notion of canal class of a convex body in relation to the problem of finding linear refinements of Brunn-Minkowski type inequalities, and further geometric and matrix inequalities.

We will also discuss and present possible interconnections between canal classes and Cheeger sets in relation to some of these geometric inequalities.

This is an ongoing project with Christian Richter and Eugenia Saorín Gómez.

Integral Geometry on Convex Functions

Fabian Mussnig, TU Wien

We present integral geometric results for valuations on convex functions. Our starting point are recently introduced functional Cauchy–Kubota formulas, which generalize their classical counterparts for convex bodies and which can be obtained from a Hadwiger-type theorem. We will then explain how mixed Monge–Ampère measures can be used to establish improved versions of these formulas. If time permits, we will also discuss functional versions of the additive kinematic formulas that can be obtained by using a combination of all of the above. Based on joint work with Andrea Colesanti and Monika Ludwig as well as Daniel Hug and Jacopo Ulivelli.

Iterations of Minkowski Valuations

Oscar Ortega Moreno , TU Wien

In this talk, we show that for any sufficiently regular even Minkowski valuation Φ which is homogeneous and intertwines rigid motions, and for any convex body K in a smooth neighborhood of the unit ball, there exists a sequence of positive numbers $(\gamma_m)_{m=1}^{\infty}$ such that $(\gamma_m \Phi^m K)_{m=1}^{\infty}$ converges to the unit ball with respect to the Hausdorff metric.

Reduced convex bodies in spaces of constant curvature and Pál‘s isominwidth inequality

Ádám Sagmeister, Eötvös Loránd University

We call a convex body K reduced, if for any different convex body contained in K has a smaller minimal width. Reduced bodies are extremizers to some inequalities in convex geometry, and they also give a different perspective to the broadly studied family of bodies of constant width. There are multiple recent studies about reduced bodies in Minkowski spaces and spherical reduced bodies, and we also present a hyperbolic approach after introducing an extended version of Leichtweiss‘ width function. We prove the hyperbolic version of Pál‘s inequality, stating that the regular triangle has the smallest area among convex bodies of minimal width, and we will also discuss a stability version of the theorem. The talk is based on joint works with Károly J. Böröczky, András Csépai and Ansgar Freyer.

Continuous Translation-Invariant Curvature Measures

Jakob Schuhmacher, Friedrich-Schiller-University Jena

We introduce the concept of continuous translation-invariant curvature measures on a real finite dimensional vector space, which generalizes the notion of smooth translation-invariant curvature measures due to Bernig and Fu. The set of these curvature measures forms a vector space, denoted by Curv. We will see that Curv admits a grading analogous to the McMullen decomposition for valuations and we equip Curv with a Banach space topology. Making use of an embedding into a space of vector-valued valuations, we will give explicit descriptions of certain special classes of curvature measures. This is joint work in progress with T. Wannerer.

Isoperimetric Inequalities for Minkowski and Asplund Endomorphisms

Franz Schuster, Vienna University of Technology

We present new isoperimetric inequalities for monotone Minkowski endomorphisms of convex bodies, each one stronger than the classical Urysohn inequality. Among this large family of new inequalities, the only affine invariant one - the Blaschke-Santalo inequality - turns out to be the strongest one. A further extension of these inequalities to merely weakly monotone Minkowski endomorphisms is proven to be impossible which, in turn, uncovers an unexpected phenomenon. Moreover, for Asplund endomorphisms of log-concave functions, which are functional analogues of Minkowski endomorphisms, a family of analytic inequalities is presented which generalizes the functional Blaschke–Santalo inequality. This is joint work with Georg Hofstaetter.

First variation of Wulff-shapes of convex functions

Jacopo Ulivelli, La Sapienza, University of Rome

We present a comprehensive approach to obtaining the first variation of the weighted volume of the epigraph of convex functions. The perturbations we consider generalize the classical notion of Wulff-shape. The method we employ derives functional results from the geometric counterparts and allows us to recover as special cases results on log-concave functions and valuations on super-coercive functions, tying up different paths that have been followed in recent years.

Extremal affine surface area in a functional setting

Elisabeth Werner, Case Western Reserve University

We introduce extremal affine surface areas in a functional setting. We show their main properties, in particular we estimate the size of these quantities.

This parallels results in the setting of convex bodies.

Based on joint work with Stephanie Egler.

On complemented Brunn-Minkowski type inequalities

Jesús Yepes Nicolás, Universidad de Murcia

A classical result shown independently by Borell, and Brascamp & Lieb, yields that any absolutely continuous measure μ on $\mathbb{R}^n$ associated to a p-concave density (for $p \in [-1/n, +\infty]$) is q-concave, with $1/q = n+1/p$. In other words, it satisfies a q-Brunn-Minkowski inequality, namely,

$$\mu\big((1-\lambda)A + \lambda B\big) \geq \big((1-\lambda)\mu(A)^q + \lambda\mu(B)^q\big)^{1/q}$$

for all measurable sets $A, B \subset \mathbb{R}^n$ with $\mu(A)\mu(B) > 0$ such that $(1-\lambda)A+\lambda B$ is also measurable, and all $\lambda \in (0,1)$.

Following the duality of concave and convex functions, it is natural to wonder about a q-complemented Brunn-Minkowski inequality, i.e., whether

$$\mu\big(\mathbb{R}^n \setminus \big((1-\lambda)A + \lambda B\big)\big) \leq \big((1-\lambda)\mu\big(\mathbb{R}^n \setminus A\big)^q + \lambda\mu\big(\mathbb{R}^n \setminus B\big)^q\big)^{1/q},$$

provided that $\mu\big(\mathbb{R}^n \setminus A\big), \mu\big(\mathbb{R}^n \setminus B\big) < +\infty$.

When (μ is finite and) $q = 1$ both conditions above are trivially equivalent, but this equivalence is no longer true in general for other values of q. However, Milman and Rotem in 2014 showed that under certain assumptions of concavity and homogeneity for the density of μ such an inequality holds. In particular, the restriction of the Lebesgue measure $\mathrm{vol}(\cdot)$ to a convex cone C (which is its support) satisfies the latter inequality for $q = 1/n$ and any $A, B \subset C$ with $\mathrm{vol}\big(C \setminus A\big), \mathrm{vol}\big(C \setminus B\big) < +\infty$. This case was later studied also by Schneider in 2018, who gave a different proof and characterized its equality case when A and B are convex.

In this talk we will discuss about different functional and geometric forms of complemented Brunn-Minkowski type inequalities for certain absolutely continuous measures μ on $\mathbb{R}^n$.

This is about joint work in progress with A. Zvavitch.

Topics in the Minkowski problem for non-compact convex surface

Ning Zhang, Huazhong University of Science and Technology

In this talk, we will present a solution of the Minkowski problem for non-compact convex surface with asymptotic conditions based on the coconvex methods.