Geometry and deformation theory of heterotic systems

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Generalised geometry in interaction,
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Collaborations with

E Svanes, JHEP **1410** (2014) 123; JHEP **1412** (2014) 008

E Svanes and E Hardy, JHEP **1601** (2016) 049

M Larfors and E Svanes,
Adv. Theor. Math. Phys. **19** 837-903 (2015);
in Proc N Hitchin's 70th bday conf (2018) (ArXiV 1709.06974);
JHEP **1611** (2016) 016; Commun. Math. Phys. (2017);

P Candelas and J McOrist,
Commun. Math. Phys. **356** (2017) 567-612; ArXiV 1810.00879;

MA Fiset ArXiV 1809.01138

A Ashmore, R Minasian, C Strickland-Constable and E Svanes,
JHEP 1810 (20)

M Larfors, M Magill E Svanes, hepth 1904.01027

Introduction

General context: interested in effective field theory derived from (minimally) supersymmetric compactifications of heterotic string theories and the CFTs related to these.

What are the mathematical structures encountered?

We are all familiar with the very beautiful structures in mathematics and physics that arise in string theories and corresponding CFTs!

Introduction

Heterotic string theories are 10-dimensional supersymmetric gauge theories: they come with a vector bundle V with connection A which has structure group contained in $E_8 \times E_8$.

This is different from type-II or M-theory!

To study the effective field theory of heterotic string compactifications which give a minimally supersymmetric gauge theory on space time we make an ansatz for the 10-dimensional metric:

$$M_D \times Y, \qquad dim(Y) = d = 10 - D$$

where M_D is D-dim is maximally symmetric (space time).

Introduction

We then have following mathematical objects on these theories:

- Riemannian metric g_{mn} on Y
- a "scalar" ϕ (the dilaton)
- Gauge fields A for the gauge group $G \subseteq E_8 \times E_8$.
  So, V is a vector bundle on Y with connection A with structure group G contained in $E_8 \times E_8$.
- 3-form H, “the flux”, defined by

  $$H = \mathrm{d}B + \frac{\alpha'}{4}(\mathcal{CS}[A] - \mathcal{CS}[\Theta]) , \quad \mathcal{CS}[A] = \mathrm{tr}(A \wedge \mathrm{d}A + \frac{2}{3} A \wedge A \wedge A) ,$$

  where Θ is a connection on TY, and $\alpha' \neq 0$ is a constant.

  Note that the B-field is not gauge invariant!

Introduction

We have a quadruple: $[\,Y, (V, A), (TY, \Theta), H\,]$

Mathematically: supersymmetry constrains the geometry of this quadruple. Together with the requirement that we also satisfy the EOM we obtain a

heterotic system on $\;[\,(Y, \eta), (V, A), (TY, \Theta), H\,]$

Goal in physics: construct the quantum effective field theory and this is largely determined by the geometry and (quantum) moduli of these compactifications. For example: we want to find the massless spectrum, the (effective) 4 or 3 dimensional Lagrangian, correlation functions, and to study dualities....

Introduction

Heteritic system: $[(Y,\eta),(V,A),(TY,\Theta),H]$ where:

- (Y,η) is a manifold which admits an integrable $\mathcal{G}$ structure

  Eg, we $D=4$ leads to the Hull-Strominger system: $\mathcal{G}=SU(3)$, (Y,Ω,ω) is complex conformally balanced

- (V,A) and (TY,Θ) have instanton connections.

  For example: in $D=4$, $F\wedge\Omega=0$, $\omega\lrcorner F=0$ and similarly for Θ

- The anomaly cancelation condition is satisfied: $H=T(\eta)$

$$H=\mathrm{d}B+\frac{\alpha'}{4}(\mathcal{CS}[A]-\mathcal{CS}[\Theta])=T(\eta)$$

$T(\eta)$: totally antisymmetric torsion of the $\mathcal{G}$-connection ∇ $(\nabla\eta=0)$.

For example: $D=4$, ∇ is the Bismut connection, the unique metric connection compatible with the $SU(3)$ structure which admits a totally antisymmetric torsion.

Introduction

Today focus on the case $d = 7$, Y has a $\mathcal{G} = G_2$ structure

1) the $d = 6$ case (Hull-Strominger case) can be deduced from this! (XD, M Larfors, E Svanes, 2021);
2) the $d = 8$ case where $\mathcal{G} = Spin(7)$ is also very interesting (in progress ...)

A reason:
Much less is known about 3-dim $N = 1$ supergravity+YM.
The three dimensional space time is AdS_3 or 3-dim Minkowski.

Recall: Compactifications to four dimensions with $N = 1$ supersymmetry give strong constrains on the geometry and on the moduli space. In particular, the moduli space must be complex and Kähler.

Compactifications to three dimensions with $N = 1$ supersymmetry, are not as constrained and we know much less about the geometry of the moduli space.

Key ideas

- The conditions for the quadruple

$$[\,(Y,\varphi),(V,A),(TY,\Theta),H\,]$$

to admit a heterotic structure are equivalent to the existence of a differential $\mathcal{D}$ which satisfies $\check{\mathcal{D}}^2 = 0$ acting on forms with values on a bundle $\mathcal{Q}$ on Y which is topologically

$$\mathcal{Q} = TY \oplus \mathrm{End}(TY) \oplus \mathrm{End}(V)$$

- The infinitesimal moduli of the heterotic structure correspond to classes in

$$H^1_{\check{\mathcal{D}}}(Y,\mathcal{Q})$$

Outline

- Heterotic G_2 systems $[(Y, \varphi), (V, A), (TY, \Theta), H]$

  T Friederich & S Ivanov 2001& 2003; J Gauntlett, N Kim, D Martelli, & D Waldram 2001; J Gauntlett, D Martelli, & D Waldram 2004; P Ivanov & S Ivanov 2005; A Lukas & C Matti 2010;
  J Gray, M Larfors, D Lüst, 2012; XD, M Larfors & E Svanes 2014

- The tangent space of the moduli space of heterotic G_2 systems

  - Moduli space of (Y, φ)

    Gibbons, Page and Pope, D Joyce; S Karigiannis, S Grigorian, C Leung....;
    XD, E Svanes and M Larfors

  - Moduli space of vector bundles (V, A) over (Y, φ)

    Donaldson & Thomas; C Leung & Karigiannis; XD, E Svanes and M Larfors; ...

  - Moduli space of heterotic G_2 systems

    A Clarke, M García Fernández & C Tipler 1607.01219; XD, M Larfors & E Svanes 1607.03473 & 1704.08177 & in progress
    MA Fiset, C Quigley, E Svanes 1710.06865

- Outlook and open problems

The geometry of Y

Supersymmetry requires that Y has an integrable G_2 structure.

A manifold with a G_2 structure is a seven dimensional manifold Y which admits a smooth positive three form φ.

In fact, any 7-dimensional manifold which is spin and orientable (that is its first and second Stiefel-Whitney classes are trivial) admits a G_2 structure.

The three form φ determines a Riemannian metric g_φ and a four form $\psi = *\varphi$.

The geometry of Y

The exterior derivative of φ and ψ can be decomposed into irreducible G_2 representations (structure equations)

$$\begin{aligned} d\varphi &= \tau_0\,\psi + 3\,\tau_1 \wedge \varphi + *\tau_3 \\ 35 &= \quad 1 \quad + \qquad 7 \qquad + \ 27 \\ d\psi &= 4\,\tau_1 \wedge \psi + *\tau_2 \\ 21 &= \qquad 7 \qquad + \ 14 \end{aligned}$$

where $\tau_i \in \Omega^i(Y)$ (the torsion classes) are uniquely determined by the G_2 structure φ on Y.

An integrable G_2 structure satisfies $\quad \tau_2 = 0$.

The geometry of Y

We can write the structure equations when $\tau_2 = 0$ as

$$\mathrm{d}\varphi = i_{T(\varphi)}(\varphi) \qquad \mathrm{d}\psi = i_{T(\varphi)}(\psi)$$

where

$$T(\varphi) = \frac{1}{6}\,\tau_0\,\varphi - \tau_1 \lrcorner \psi - \tau_3 \ .$$

Remarks:

- $T(\varphi) = 0$ means $\tau_i = 0, \forall i$.
  In this case Y has G_2 holonomy ($\mathrm{d}\varphi = 0$ and $\mathrm{d}\psi = 0$)
- $T(\varphi)$ is the torsion of the unique connection ∇ compatible with the integrable G_2 structure ($\nabla\varphi = 0$, $\nabla\psi = 0$) which is totally antisymmetric.
- $\tau_0 = 0 \Rightarrow M_3$, $\qquad \tau_0 \neq 0 \Rightarrow \mathrm{AdS}_3$

.

The geometry of Y

Question: is there a differential $\check{\mathrm{d}}$ with $\check{\mathrm{d}}^2 = 0$ which encodes this geometry?

That is, is there an analogue for (Y, φ) of the Dolbeault differential $\bar{\partial}$ with $\bar{\partial}^2 = 0$ for a complex manifold (X, J)?

The geometry of Y

Canonical G_2 cohomology

Reyes-Carrion, 93;Fernandez-Ugarte, 98

Consider the differential operator $\check{\mathrm{d}}$ defined by

$$\check{\mathrm{d}}_0 = \mathrm{d}\,, \quad \check{\mathrm{d}}_1 = \pi_7 \circ \mathrm{d}\,, \quad \check{\mathrm{d}}_2 = \pi_1 \circ \mathrm{d}\,, \qquad \check{\mathrm{d}}_3 = 0$$

$$\Omega^2(Y) = \Omega^2_7(Y) \oplus \Omega^2_{14}(Y) \qquad \Omega^3(Y) = \Omega^3_1(Y) \oplus \Omega^3_7(Y) \oplus \Omega^3_{27}(Y)$$

That is

$$0 \longrightarrow \Omega^0(Y) \xrightarrow{\mathrm{d}} \Omega^1(Y) \xrightarrow{\check{\mathrm{d}}} \Omega^2_7(Y) \xrightarrow{\check{\mathrm{d}}} \Omega^3_1(Y) \longrightarrow 0$$

Then this is a differential complex, that is

$$\check{\mathrm{d}}^2 = 0 \iff \tau_2 = 0$$

and we have a canonical cohomology $H^*_{\check{\mathrm{d}}}(Y)$ on a manifold Y with an integrable G_2 structure.

Constraints on V

Supersymmetry imposes conditions on the curvature F of the Yang-Mills connection:

$$F \wedge \psi = 0 \ .$$

That is, the connection A on the bundle V is an instanton.

One can construct an operator $\check{\mathrm{d}}_A$ which acts on forms with values on $\mathrm{End}(V)$ where

$$\mathrm{d}_A \alpha = \mathrm{d}\alpha + A \wedge \alpha + (-1)^k \alpha \wedge A \ , \qquad \alpha \in \Omega^k(Y, \mathrm{End}(V))$$

We have

$$\check{\mathrm{d}}_A^2 = 0 \quad \Longleftrightarrow \quad \tau_2 = 0 \quad \text{and} \quad F \wedge \psi = 0$$

This leads to cohomology groups $H^*_{\check{\mathrm{d}}_A}(Y, \mathrm{End}(V))$.

A Further Constraint on $[(Y, \varphi), (V, A)]$
The anomaly cancelation condition

The anomaly cancelation condition: $H = T(\varphi)$

$$\mathrm{d}B + \frac{\alpha'}{4}(\mathcal{CS}[A] - \mathcal{CS}[\Theta]) = \frac{1}{6}\,\tau_0\,\varphi - \tau_1 \lrcorner \psi - \tau_3$$

Moreover: a solution of the supersymmetry conditions, which also satisfies the anomaly cancelation, automatically satisfies the equations of motion iff the connection Θ on TY satisfies

$$R(\Theta) \wedge \psi = 0\ .$$

That is, Θ must be an instanton. (Hull, Ivanov, Martelli and Sparks)

Summary

A heterotic G_2 system is a quadruple

$$[(Y,\varphi),(V,A),(TY,\Theta),H]$$

where:

- (Y,φ) is a manifold with an integrable G_2 structure
- (V,A) and (TY,Θ) have instanton connections.
- The anomaly cancelation condition is satisfied: $H = T(\varphi)$

$$\mathrm{d}B + \frac{\alpha'}{4}(\mathcal{CS}[A] - \mathcal{CS}[\Theta]) = \frac{1}{6}\tau_0\,\varphi - \tau_1 \lrcorner \psi - \tau_3$$

Remark: the BI of the anomaly cancelation condition is

$$\frac{\alpha'}{4}(\mathrm{tr}(F \wedge F) - \mathrm{tr}(R \wedge R))) = \mathrm{d}T(\varphi)$$

Examples of solutions

Not many, more work needs to be done!

G2 holonomy: Joyce; Joyce & Karigiannis; Corti, Haskins, Nordström & Pacini; Braun; etc

Instatons on G2 holonomy manifolds: Wapulski; Sa Earp; Menet, Nordström & Sa Earp; ...

$\tau_0 = 0$: Fernández, Ivanov, Ugarte & Villacampa 2011; Fernández, Ivanov, Ugarte & Vassilev 2015

$\tau_0 \neq 0$: Lotay, Sá Earp, Saavedra 2021; XD & Galdeano 2021

Key ideas

- The heterotic G_2 system $[(Y,\varphi),(V,A),(TY,\Theta),H]$ is equivalent to a the existence of a differential $\mathcal{D}$ such that $\check{\mathcal{D}}^2 = 0$ on forms with values in a bundle $\mathcal{Q}$ on Y which is topologically

$$\mathcal{Q} = TY \oplus \mathrm{End}(TY) \oplus \mathrm{End}(V).$$

- The infinitesimal moduli of the heterotic structure:

$$H^1_{\check{\mathcal{D}}}(Y,\mathcal{Q})$$

The heterotic G_2 system as a differential

Consider the linear operator

$$\mathcal{D} = \begin{pmatrix} \mathrm{d}_\zeta & \mathcal{R} & -\mathcal{F} \\ \mathcal{R} & \mathrm{d}_\Theta & 0 \\ \mathcal{F} & 0 & \mathrm{d}_A \end{pmatrix}$$

acting on forms with values in $\mathcal{Q} = TY \oplus \mathrm{End}(TY) \oplus \mathrm{End}(V)$ with

- $\mathrm{d}_\zeta y = \mathrm{d}y + \zeta \wedge y \,, \qquad y \in \Omega^k(Y, TY)$
  ζ a connection on TY with torsion $-T(\varphi)$
- $\mathrm{d}_A \alpha = \mathrm{d}\alpha + A \wedge \alpha + (-1)^k \alpha \wedge A \,, \quad \alpha \in \Omega^k(Y, \mathrm{End}(V))$, and similar for d_Θ

The heterotic G_2 system as a differential

$$\mathcal{D} = \begin{pmatrix} \mathrm{d}_\zeta & \mathcal{R} & -\mathcal{F} \\ \mathcal{R} & \mathrm{d}_\Theta & 0 \\ \mathcal{F} & 0 & \mathrm{d}_A \end{pmatrix}$$

- $\mathcal{F}$ is a linear map defined by

$$\mathcal{F} : \Omega^k(Y, TY) \oplus \Omega^k(Y, \mathrm{End}(V)) \longrightarrow \Omega^{k+1}(Y, \mathrm{End}(V)) \oplus \Omega^{k+1}(Y, TY)$$

$$\begin{pmatrix} y \\ \alpha \end{pmatrix} \mapsto \begin{pmatrix} \mathcal{F}(y) \\ \mathcal{F}(\alpha) \end{pmatrix} = \begin{pmatrix} (-1)^k \; i_y(F) \\ (-1)^k \, \frac{\alpha'}{4} \, \mathrm{tr}(\alpha \wedge F) \end{pmatrix}$$

- $\mathcal{R}$ is similar.

The heterotic G_2 system as a differential

Theorem

Let Y be a manifold with a G_2 structure φ, V a bundle on Y with connection A, and TY the tangent bundle of Y with connection Θ. Let ζ be the connection one-form on TY defined earlier. Consider the exterior covariant derivative $\mathcal{D}$ defined above. Then

$$\check{\mathcal{D}}^2 = 0 \iff \big([Y, \varphi], [V, A], [TY, \Theta], H\big) \text{ is a heterotic system}$$

where

$$\check{\mathcal{D}}_0 = \mathcal{D} \ , \quad \check{\mathcal{D}}_1 = \pi_7 \circ \mathcal{D} \ , \quad \check{\mathcal{D}}_2 = \pi_1 \circ \mathcal{D} \ .$$

The heterotic G_2 system as a differential

Comments on the proof: Consider

$$\mathcal{D}^2 = \begin{pmatrix} \mathrm{d}_\zeta^2 + \mathcal{R}^2 - \mathcal{F}^2 & \mathrm{d}_\zeta \mathcal{R} + \mathcal{R}\mathrm{d}_\Theta & -(\mathrm{d}_\zeta \mathcal{F} + \mathcal{F}\mathrm{d}_A) \\ \mathcal{R}\mathrm{d}_\zeta + \mathrm{d}_\Theta \mathcal{R} & \mathcal{R}^2 + \mathrm{d}_\Theta^2 & -\mathcal{R}\mathcal{F} \\ \mathcal{F}\mathrm{d}_\zeta + \mathrm{d}_A \mathcal{F} & \mathcal{F}\mathcal{R} & -\mathcal{F}^2 + \mathrm{d}_A^2 \end{pmatrix}$$

It is not too hard to see that $\check{\mathcal{D}}^2 = 0$ is satisfied for the heterotic G_2 system as long as the Bianchi identity of the anomaly is satisfied

The converse however is more involved: in particular the vanishing of the (1,1) entry implies the Bianchi identity of the anomaly.

Key ideas

- The heterotic G_2 system $[(Y,\varphi),(V,A),(TY,\Theta),H]$ is equivalent to a the existence of a differential $\mathcal{D}$ such that $\check{\mathcal{D}}^2 = 0$ on forms with values in a bundle $\mathcal{Q}$ on Y which is topologically
$$\mathcal{Q} = TY \oplus \mathrm{End}(TY) \oplus \mathrm{End}(V).$$
- The infinitesimal moduli of the heterotic structure are given by
$$H^1_{\check{\mathcal{D}}}(Y,\mathcal{Q}) \ .$$

Infinitesimal deformations of (Y, φ)

Let (Y, φ) be a manifold with an integrable G_2 structure and consider infinitesimal deformations $(Y, \varphi(t))$ with $\varphi(0) = \varphi$.
Idea: study integrable G_2 structures in terms of $M_t \in \Omega^1(Y, TY)$

$$\partial_t \varphi = i_{M_t}(\varphi)$$

$$\partial_t \psi = i_{M_t}(\psi)$$

Remark: The **14** part of the antisymmetric part of M_t drops out.

First order variations of the metric in terms of M_t:

$$\partial_t g_{\varphi\, ab} = 2\, M_{t\,(ab)}\ , \qquad \partial_t \sqrt{\det g_\varphi} = (\mathrm{tr}\, M_t)\, \sqrt{\det g_\varphi}\ .$$

Remark: The **7** part of the antisymmetric part of M_t leaves the metric invariant.

Karigiannis 2003

Infinitesimal deformations of (Y, φ)

Deformations preserving the integrability of the G_2 structure are given by

$$i_{\check{\mathrm{d}}_\zeta M_t}(\psi) = 0 \ ,$$

Diffeomorphisms: $\qquad \mathcal{L}_V \psi = i_{\check{\mathrm{d}}_\zeta V}(\psi)$

The dimension of the space of infinitesimal deformations of integrable G_2 structures, $\mathcal{TM}_0$, is in general is not finite.

An exception (of course!): Y has G_2 holonomy
In this case

$$\mathcal{TM}_0 = H^1_{\check{\mathrm{d}}_\zeta}(Y, TY)$$

precisely matches the deRham cohomology $H^3(Y)$.

Deformations of $[(Y, \varphi), (V, A)]$

Consider now deformations of $[(Y, \varphi), (V, A)]$

Want:

- deformations M of the integrable G_2 structure φ on Y which preserve the integrability of the G_2 structure together with
- deformations $\alpha \in \Omega^1(Y, \mathrm{End}(V))$ of the instanton connection A such that simultaneous deformations of φ and A preserve the instanton condition $F \wedge \psi = 0$ on V.

Deformations of $[\,(Y,\varphi),(V,A)\,]$

Varying the instanton equation $F\wedge\psi=0$ we find

$$\check{\mathrm{d}}_A(\alpha_t)=-\check{\mathcal{F}}(M_t)=-\pi_7(\mathcal{F}(M_t))\ .$$

where

$$\begin{array}{cccc} \mathcal{F}: & \Omega^k(Y,TY) & \longrightarrow & \Omega^{k+1}(Y,\mathrm{End}(V)) \\ & M & \mapsto & \mathcal{F}(M)=(-1)^k\, i_M(F)\ . \end{array}$$

$\check{\mathcal{F}}$ maps M_t into a two form with values in $\mathrm{End}(V)$ which is exact in $\check{\mathrm{d}}_A$-cohomology.

Deformations of $[\,(Y,\varphi),(V,A)\,]$

Then we have so far

$$\check{\mathrm{d}}_A(\alpha_t) = -\check{\mathcal{F}}(M_t) \quad \text{and} \quad i_{\check{\mathrm{d}}_\varsigma M_t}(\psi) = 0$$

which gives

$$\mathcal{T}\mathcal{M}_1 = H^1_{\check{\mathrm{d}}_A}(Y, \mathrm{End}(V)) \oplus \ker \check{\mathcal{F}}\ , \quad \ker \check{\mathcal{F}} \subseteq \mathcal{T}\mathcal{M}_0$$

Again: there is no reason why the dimension should be finite (except in the case where Y has G_2 holonomy)

Deformations of the heterotic G_2 system

Moduli

Consider the action of $\mathcal{D}$ on one forms $\mathcal{Z}$ with values in

$$\mathcal{Q} = TY \oplus \mathrm{End}(TY) \oplus \mathrm{End}(V)$$

$$\mathcal{D}\mathcal{Z} = \begin{pmatrix} \mathrm{d}_\zeta\, y + \mathcal{R}(\kappa) - \mathcal{F}(\alpha) \\ \mathrm{d}_\Theta \kappa + \mathcal{R}(y) \\ \mathrm{d}_A \alpha + \mathcal{F}(y) \end{pmatrix}, \quad \mathcal{Z} = \begin{pmatrix} y \\ \kappa \\ \alpha \end{pmatrix}$$

Equations for moduli: $\check{\mathcal{D}}\mathcal{Z} = 0$

Deformations of the heterotic G_2 system

In particular,

$$\check{\mathrm{d}}_\zeta\, y_t + \check{\mathcal{R}}(\kappa_t) - \check{\mathcal{F}}(\alpha_t) = 0$$

turns out to be the same equation as

$$i_{\check{\mathrm{d}}_\zeta y_t}(\psi) = 0\ .$$

by the anomaly cancelation condition, if we identify the degrees of freedom corresponding to the antisymmetric part of y_t, with the (covariant) variations $\mathcal{B}_t$ of the B field

$$\partial_t H = \mathrm{d}\mathcal{B}_t + \frac{\alpha'}{2}\left(\mathrm{tr}(\alpha_t \wedge F) - \mathrm{tr}(\kappa_t \wedge R(\Theta))\right)\ .$$

Deformations of the heterotic G_2 system

Then

$$\mathcal{TM} = H^1_{\check{\mathcal{D}}}(Y, \mathcal{Q}) \ .$$

The $\check{\mathcal{D}}$-exact forms correspond to diffeomorphisms of Y and gauge transformations.

$\mathcal{TM}$ is finite dimensional.

Main result

heterotic structure on $[(Y,\eta),(V,A),(TY,\Theta),H]$	$\longleftrightarrow$	special structure $\check{D}$ on $\mathcal{Q}$
infinitesimal moduli of the heterotic structure	$\longleftrightarrow$	$H^1_{\check{\mathcal{D}}}(Y,\mathcal{Q})$

Outlook and open problems

- Examples??? Need more to play with.
- And who can compute the cohomologies?
- Better understanding of the structure of moduli space of heterotic systems $[Y, V, TY, H]$: What is the mathematical structure?
- Global questions: metric? geometrical structure? singularities of the moduli space? higher order deformations and obstructions?

Outlook and open problems

- We have derived the same results from a superpotential

$$W = \frac{1}{2} \int_Y e^{-2\phi} \left(H + h\varphi - \frac{1}{2} * \mathrm{d}\varphi \right) \wedge \psi$$

XD, M Larfors, M Magill, E Svanes (1904.01027):
Het system $\Longleftrightarrow$ $W = 0$ & $\delta W = 0$; also showed that $h = \tau_0/3$

Equations for infinitesimal moduli $\Longleftrightarrow$ $\delta^2 W = 0$.

XD, M Larfors, E Svanes, M Magill, in progress

- $SU(3)$ case (Hull-Strominger):
  - Used the superpotential to built an analogue of the Maurer-Cartan equation which leads to an L_3-algebra

    A. Ashmore, XD, R. Minasian, C. Strickland-Constable, E. Svanes 1806.08367
    See also M. García Fernández, C. Rubio, C. Shahbazi, C. Tipler, 1803.01873, 1807.10329.
  - Metrics and the Universal Bundle

    P. Candelas, XD, J McOrist 1605.05256;P. Candelas, XD, J McOrist, R. Sisca 1810.00879

Outlook and open problems

- CFT and σ-model perspective

  Here we consider $(1,0)$ superconformal NLσM with a target space Y which admits some $\mathcal{G}$ structure and a bundle with connection on Y: this gives a model with extra symmetries (Howe Papdopoulos symmetries)

  Howe and Papadopoulos 9202036...XD, MA Fiset 1809.01138
  & in progress with M Galdeano and E Marchetto

  Moduli: marginal deformations of the σ-model

  I Melnikov & E Sharpe arXiv:1110.1886....MA Fiset, C Quigley, E Svanes 1710.06865

Outlook and open problems

- Quantum corrections? We have world sheet instanton corrections and NS5branes. For example

$$\mathrm{d}H = \frac{\alpha'}{4}(\mathrm{tr}F^2 - \mathrm{tr}R^2) + \Sigma$$

  What are the non perturbative corrections the moduli space? What are the generalisations of the Donaldson-Thomas invariants?

- Concept of mirror symmetry? Dualities?

$$\Gamma[\,(Y,\varphi),(V,A),(TY,\Theta),H\,] = \Gamma[\,(Y',\varphi'),(V',A'),(TY',\Theta'),H'\,]$$

- Relation with Type-II, M-theory and F-theory?

Thank you!

Heterotic $SU(3)$ system

XD & E Svanes: 1402.1725 and 1409.3347; XD, E Svanes & E Hardy: 1506.08724;
L Anderson, J Gray and E Sharpe 1402.1532; M García Fernández, R Rubio & C Tipler 1503.07562

This is the Hull-Strominger system and one obtains a YM-SUGRA theory on 4-dim Minkowski.

The heterotic $SU(3)$ system

$$[(Y, \omega, \Omega), (V, A), (TY, \Theta), H]$$

is equivalent to a the existence of a differential $\mathcal{D}$ such that $\overline{\mathcal{D}}^2 = 0$ on forms with values in a bundle $\mathcal{Q}$ on Y which is topologically

$$\mathcal{Q} = \mathcal{T}^* Y \oplus \mathrm{End}(TY) \oplus \mathrm{End}(V) \oplus \mathcal{T} Y.$$

That is: $\mathcal{Q}$ is a holomorphic bundle over Y.

Heterotic $SU(3)$ system

The infinitesimal moduli space of the heterotic structure corresponds to the deformations of the holomorphic structure defined by $\bar{D}$ on the extension bundle $\mathcal{Q}$

$$\mathcal{TM} = H^1_{\bar{D}}(Y, \mathcal{Q}) \cong \left[H^1_{\bar{\partial}}(Y, T^*Y) \Big/ \mathrm{Im}(\mathcal{H}_0) \right] \oplus \ker(\mathcal{H})$$

- Deformations of the holomorphic structure on E

$$\ker(\mathcal{H}) \subseteq H^{(0,1)}_{\bar{\partial}_E}(Y, E) \ ,$$

- Moduli of the (complexified) hermitian structure

$$\mathcal{M}_{HS} = \left[H^1_{\bar{\partial}}(Y, T^*Y) \Big/ \mathrm{Im}(\mathcal{H}_0) \right]$$

The elements are $\quad \mathcal{Z}_t = \mathcal{B}_t + i\partial_t\omega$

$$\text{where} \qquad \mathcal{B}_t = \partial_t B + \frac{\alpha'}{4} \left(\mathrm{tr}(\alpha_t \wedge A) - \mathrm{tr}(\kappa_t \wedge \Theta) \right)$$

Heterotic $SU(3)$ system: Kähler metric on moduli

P Candelas, XD & J McOrist: 1605.05256; J McOrist 1606.0521

Consider a basis for the deformations of the fields in the heterotic structure $(Y, [V, A], [TY, \Theta], H)$ obtained by the action of $\mathcal{D}$

$$\chi_\alpha, \quad \mathcal{D}_\xi \omega\ , \quad \mathcal{D}_\xi\,\mathcal{A}$$

The Kähler metric on $\mathcal{M}$ is the natural inner product of these forms.

We find

$$K = -\log\left(i\int_Y \Omega \wedge \overline{\Omega}\right) - \log\left(\frac{4}{3}\int_Y \omega^3\right)$$

where ω is the α' corrected hermitian form on Y.

$$ds^2 = G_{\xi\bar{\eta}}\, dy^{\xi}\, dy^{\bar{\eta}} + G^0_{\alpha\beta}\, dz^{\alpha}\, dz^{\bar{\beta}}$$

$$G^0_{\alpha\bar{\beta}} = \frac{1}{\int_Y \Omega \wedge \overline{\Omega}} \int_Y \chi_\alpha \wedge \bar{\chi}_{\bar{\beta}}$$

$$G_{\xi\bar{\eta}} = \frac{1}{V} \int_Y \mathcal{D}_\xi\, \omega \wedge *\mathcal{D}_{\bar{\eta}}\, \omega + \frac{i\alpha'}{8V} \int_Y \mathrm{Tr}\left(\mathcal{D}_\xi \mathcal{A}\, \mathcal{D}_{\bar{\eta}} \mathcal{A}^\dagger - \mathcal{D}_\xi\, \vartheta\, \mathcal{D}_{\bar{\eta}} \vartheta^\dagger\right)$$