
INTERCITY SEMINAR ON ARAKELOV GEOMETRY

(near) Madrid
September 12 - 16, 2022

Book of abstracts



The conference is sponsored by



SCHEDULE

Monday	Tuesday	Wednesday	Thursday	Friday
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9.30 – 10.00 Registration	9.00 – 10.00 Chen	9.00 Bus to Madrid		9.00 – 10.00 Köhler
coffee break			coffee break	
10.30 – 11.30 Soulé	10.30 – 11.30 Künnemann	10.15 – 11.15 Rössler	10.30 – 11.30 Ikoma	10.30 – 11.30 Kühn
		coffee break		
11.30 – 12.30 Czerniawska	11.30 – 12.30 Gubler	11.45 – 12.45 Pazuki	11.30 – 12.30 Luo	11.30 – 12.30 Botero

lunch break	lunch at <i>Café' de Oriente</i> in Madrid	lunch break
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14.30 – 15.30 Pengo	14.30 – 15.30 Sertöz	Visit to the Prado Museum	14.30 – 15.00 Flores	14.30 Bus departure
			15.00 – 15.30 Peralta	
15.30 – 16.30 Dolce	15.30 – 16.30 Maillot		15.30 – 16.00 Sedillot	
			coffee & photo!	
coffee break			16.30 – 17.00 Dutour	
17.00 – 18.00 Viada		20.00 Bus to “La Cristalera”	17.00 – 17.30 Fang	

dinner	barbecue
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TALKS

1. Ana María Botero: Equivariant non-archimedean Arakelov theory of toric varieties

Abstract: In [BGS95] a non-archimedean Arakelov theory based on the Chow groups of all regular proper models over the valuation ring is developed, under the assumption of resolution of singularities. Under this hypothesis, it establishes a dictionary between objects in differential geometry and intersection theory. In this talk, we report on work in progress on an equivariant version of this theory for toric varieties over a DVR. In particular, we define equivariant analogues of closed (p, p) -forms, (p, p) -forms modulo the image of ∂ and $\bar{\partial}$, closed (p, p) -currents and (p, p) -currents modulo the image of ∂ and $\bar{\partial}$. These groups are identified with appropriate direct and inverse limits of equivariant Chow cycle groups and equivariant operational Chow cohomology groups. We relate these groups to groups of piecewise polynomial functions on the polyhedral complexes defining the toric schemes. We also give a combinatorial description of the analogue of the dd^c operator corresponding to the pushforward from the special to the generic fiber, then Poincaré duality, then pullback from the special to the generic fiber. The goal is to give a combinatorial characterization of the equivariant Green currents associated to an invariant cycle, leading to a combinatorial description of the equivariant arithmetic Chow groups.

2. Huayi Chen: Arithmetic Hilbert-Samuel formula over an adelic curve

Abstract: In this talk, I will explain a joint work with Atsushi Moriwaki on an arithmetic Hilbert-Samuel formula in the setting of Arakelov geometry over an adelic curve, which describe the asymptotic behaviour of the arithmetic Hilbert-Samuel function of a relatively ample line bundle by the arithmetic auto-intersection number of the latter.

3. Weronika Czerniawska: Adelic Euler Characteristic for curves

Abstract: I will present a new approach to geometry of one dimensional objects associated to one dimensional global fields in the spirit of Tate's thesis. In particular I will show how to uniformly (for both geometric and arithmetic curves) obtain the Euler characteristic as a single adelic integral. I will discuss the possibilities to extend the approach to the case of dimension two.

4. Paolo Dolce: Numerical equivalence of \mathbb{R} -divisors and Shioda-Tate formula for arithmetic varieties

Abstract: For varieties defined over the ring of integers of a number field, and inspired by the geometric case, Arakelov geometry offers a suitable notion of arithmetic Chow groups and of an arithmetic intersection product. In a joint work with Roberto Gualdi (University of Regensburg), we prove an arithmetic analogue of the classical Shioda-Tate formula, relating the dimension of the first Arakelov-Chow vector space of an arithmetic variety to some of its geometric invariants. In doing so, we also characterize numerically trivial arithmetic divisors, confirming part of a conjecture by Gillet and Soulé.

5. **Walter Gubler: Monge-Ampère measures for toric metrics on abelian varieties**

Abstract: This is joint work with Stefan Stadloeder. Toric metrics on a line bundle of an abelian variety A are the invariant metrics under the natural torus action coming from Raynaud's uniformization theory. We compute here the associated Monge-Ampère measures for the restriction to any closed subvariety of A . These techniques were useful for Yamaki's reduction step to nowhere degenerate abelian varieties in the proof of the geometric Bogomolov conjecture.

6. **Hideaki Ikoma: Differentiability of the arithmetic volume function for pairs**

Abstract: I will explain that the arithmetic volume function defined on the space of pairs is one-sided differentiable at a big pair, and that its derivative is given by the arithmetic restricted positive intersection number. In particular, we will see that the arithmetic volume function is one-sided differentiable along the directions given by singular functions.

7. **Kai Köhler: Asymptotics of equivariant torsion forms**

Abstract: The talk describes an extension of Puchol's results about asymptotics of torsion forms to the equivariant case.

8. **Ulf Kühn: A survey on multiple q -zeta values**

Abstract: This is a survey on conjectures and recent results for the algebra of multiple q -zeta.

9. **Klaus Künnemann: On the non-archimedean Monge-Ampère equation in mixed characteristic**

Abstract: We report on joint work with Yanbo Fang and Walter Gubler. Let X be a smooth projective variety over a complete discretely valued field of mixed characteristic. We solve non-archimedean Monge-Ampère equations on X assuming resolution and embedded resolution of singularities and subadditivity for $+$ -test ideals on models of X . We follow the variational approach of Boucksom, Favre, and Jonsson proving the continuity of the plurisubharmonic envelope of a continuous metric on an ample line bundle on X . We replace the use of multiplier ideals in equicharacteristic zero by the use of $+$ -test ideals introduced in mixed characteristic by Hacon, Lamarche, and Schwede.

10. **Wenbin Luo: A relative bigness inequality and equidistribution theorem over function fields**

Abstract: For any line bundle written as a subtraction of two ample line bundles, Siu's inequality gives a criterion on its bigness. We generalize this inequality to a relative case. The arithmetic meaning behind the inequality leads to its application on algebraic dynamic systems, which is the equidistribution theorem of generic and small net of subvarieties over a function field.

11. **Vincent Maillot: Exotic relations for arithmetic characteristic classes of motivic fibrations with non-abelian coefficients.**

Abstract: Building on a web of theorems and conjectures published long time ago (joint work with D. Rossler) I will use a few new ingredients to compute arithmetic numbers on some interesting Shimura varieties (some results being conditional). Time permitting I will explain connections with the theory of Galois modules.

12. Fabien Pazuki: Northcott numbers and applications

Abstract: A set of algebraic numbers with bounded degree and bounded height is a finite set, by Northcott’s theorem. The set of roots of unity is of height zero, but is infinite. What about other sets of algebraic numbers? When is a set of bounded height still infinite? A way to approach this question is through the Northcott number of these sets. We will study some of their properties, discuss links to Julia Robinson’s work on undecidability, and explain other applications towards height controls in Bertini statements. The talk is based on joint work with Technau and Widmer.

13. Riccardo Pengo: Standard conjectures for arithmetic Grassmannians, and a new description of their Arakelov Chow ring

Abstract: The validity of the Hodge-Riemann relations and the hard Lefschetz theorem for the Chow ring of a smooth and projective variety defined over the complex numbers are two of the biggest open problems in algebraic geometry, part of the celebrated standard conjectures. Gillet & Soulé conjectured that some analogous properties should hold for the arithmetic Chow ring of an arithmetic variety.

In this talk, based on joint work in progress with Paolo Dolce and Roberto Gualdi, I will explain our attempts to prove the strong version of these conjectures for arithmetic Grassmannians (generalizing previous work of Künnemann and Kresch & Tamvakis), which rely on a new description of the Arakelov Chow ring of these arithmetic varieties.

14. Damian Rössler: A conjectural relative height pairing (partially joint with Tamás Szamuely)

Abstract: In a recent article, Tamas Szamuely and the speaker defined a geometric height pairing for varieties defined over the function field of a smooth variety defined over an algebraically closed field. This pairing has values in the second l -adic cohomology group of a model of the function field and conjecturally the pairing should arise from a “motivic” pairing with values in the rational Picard group of the model. This leads to some speculation on the possible existence of a similar pairing for varieties defined over function fields of arithmetic varieties, which would have values in an Arakelov Picard group. I will discuss some special cases of this conjecture and I will explain what analytic problems arise when one tries to make it more precise. This is very much work in progress.

15. Emre Sertöz: Local non-Archimedean height pairing via tropical biextensions

Abstract: We show how the local non-Archimedean Néron–Tate pairing on a local curve resembles the formula for the Archimedean height of a biextension. This motivates our definition of a tropical biextension which can be defined on a tropical curve and a tropical divisor. We show that this definition can be used to compute local Néron–Tate heights using the reduction graphs of curves. Joint work with Robin de Jong and Spencer Bloch.

16. Christophe Soulé: On abc and the absence of Siegel zeros

Abstract: Granville and Stark have shown that the validity of the uniform abc conjecture would imply the absence of Siegel zero for the L-function of imaginary quadratic characters. I will explain that the same conclusion holds under a weaker assumption.

17. Evelina Viada: The distribution of small points in subvarieties

Abstract: In this talk I intend to present an older result with F. Amoroso where we described how small points distribute in subvarieties of tori, giving in particular sharp lower bounds for the essential minimum of the variety.

In a recent work, A. Galateau and C. Martinez use a method similar to our, for the torsion of a subvariety V of an abelian variety. They obtain a remarkable bound for the number of components of the closure of the torsion of V and other sharp results.

SHORT TALKS

1. Mathieu Dutour: Loop groups and Pro-Hermitian vector bundles

Abstract: The link between Lie groups and Lie algebras is an explicit one. In order to go from the former to the latter, one takes the tangent space at identity, while Chevalley groups are interesting for the converse. This last construction can be adapted for some Kac–Moody algebras, as shown by Garland in the case of (central extensions of) loop algebras. The resulting group is called a loop group.

In this talk, we will see how elements of these loop groups yield interesting Pro-Hermitian vector bundles, a notion studied by Bost and Charles in relation to theta invariants. They are ultimately expected to be useful to study loop Eisenstein series for number fields.

2. Yanbo Fang: Non-Archimedean critical Fubini-Study metric

Abstract: Over a non-Archimedean place, the (local) height of a projective variety X embedded in \mathbb{P}^n by ample line bundle L equipped with a F-S metric is given by the (local) height of its Chow form. Under the natural action by $SL(n+1)$ on $H^0(X, L)$, a F-S metric h on L is called (Chow-)critical if it minimises this height among other F-S metrics in the same orbit. Using the n.-A. Kempf-Ness criteria, we characterise the criticality of h in terms of the Monge-Ampère polytope (a polymatroid) of (X, L, h) . This can be seen as a n.-A. analogue of results over the complex place in the mid 90s.

3. Marco Flores: A cohomological approach to formal Fourier-Jacobi series

Abstract: Siegel modular forms admit various expansions, one of the most important being the Fourier–Jacobi expansion. Algebraically, these expansions take the form of a series whose coefficients are Jacobi forms satisfying a certain symmetry condition. One then poses the following modularity question: does every formal series of that shape come from a Siegel modular form? Bruinier and Raum answered the question affirmatively, over the complex numbers, in 2014. In this talk I will consider this question over the ring of integers, and reformulate it as a matter of cohomological vanishing. I will present a weaker version of the desired cohomological vanishing, and a result highlighting how special the case of genus $g = 2$ potentially is.

4. Gari Yamel Peralta Alvarez: Toric varieties and singular metrics

Abstract: Toric varieties come with a dictionary which translates algebraic-geometric properties into combinatorics of convex objects. Burgos, Philippon and Sombra expanded this dictionary to the Arakelov setting. In particular, they give a formula for the local height of a toric variety with respect to an arithmetic divisor with torus invariant properties. In this talk, we present an extension of this result for toric divisors with singular toric Green’s functions. This extension is based on the theory of adelic line bundles by Yuan and Zhang.

5. Antoine Sedillot: Diophantine problems over adelic curves

Abstract: Recently, Chen and Moriwaki introduced the framework of adelic curves, which provides a flexible language for doing Arakelov geometry in a wide range of situations. In this talk, after introducing the necessary tools, we will address the generalisation of classical diophantine questions in this framework. In particular, we will present some of the expected consequences of such generalisations.