Short talks. Book of abstracts

Achenef, Tesfahun (Nazarbayev University, Kazakhstan):
Dispersive estimates for full dispersion KP equations.

Abstract: We derive some dispersive estimates for the linear part of the full dispersion Kadomtsev-Petviashvili (KP) equation introduced by David Lannes to overcome some shortcomings of the classical KP equations. The proof of these estimates combines the stationary phase method with sharp asymptotics on *asymmetric Bessel functions*, which may be of independent interest. This is a joint work with D. Pilod, J-C. Saut and S. Selberg.

Bakas, Odysseas (Basque Center for Applied Mathematics, Spain):
Codimension one directional multipliers.

Abstract: For a vector field $v : \mathbb{R}^2 \to \mathbb{R}^2$, consider the truncated directional maximal operator along v

$$M_{v,\epsilon}(f)(x) := \sup_{0<r\leq\epsilon} \frac{1}{2r} \int_{|t|\leq r} |f(x - tv(x))| dt.$$

Zygmund's conjecture states that if v is Lipschitz then, for suitable $\epsilon > 0$, $M_{v,\epsilon}$ is of weak-type $(2, 2)$. Stein's conjecture is a singular integral variant of that problem stating that if v is Lipschitz then the truncated directional Hilbert transform along v is of weak-type $(2, 2)$.

In 2006, Lacey and Li proved that the directional Hilbert transform along general measurable vector fields satisfies a weak-type $(2, 2)$ bound when acting on functions that are frequency supported in an annulus in $\mathbb{R}^2$, a result that implies Carleson's theorem on the convergence of Fourier series.

In this talk we present a novel formulation of the aforementioned result of Lacey and Li. More specifically, we present weak-type $(2, 2)$ bounds in the single annulus case for singular integral operators along variable $(n - 1)$-dimensional subspaces of $\mathbb{R}^n$ given by the orthogonal complement of a measurable vector field. Connections of our results with appropriate Carleson-type theorems in higher dimensions will also be discussed.

This is joint work with Francesco Di Plinio, Ioannis Parissis, and Luz Roncal.

Barron, Alex (University of Illinois, Urbana-Champaign):
On the Fourier restriction problem for hyperbolic paraboloids.

Abstract: We give a brief overview of the current methods used to study the Fourier restriction problem for hyperbolic paraboloids, with a focus on the geometric obstructions that distinguish the problem from the (more extensively-studied) elliptic case. We then discuss some speculative ideas for making progress on the known bounds.

Buschenhenke, Stefan (Christian-Albrechts-Universität zu Kiel, Germany):
Factorisation in restriction theory.

Abstract: We give an alternative argument to the application of the so-called Maurey- Nikishin-Pisier factorisation in Fourier restriction theory. Based on an induction-on-scales argument, our comparably simple method applies to any compact quadratic surface, in particular compact parts of the paraboloid and the hyperbolic paraboloid. This is achieved by constructing near extremisers with big mass, which itself might be of interest.

Cao, Mingming (ICMAT, Spain):
Rubio de Francia extrapolation and compact operators.

Abstract: We will talk the Rubio de Francia extrapolation for multilinear compact operators. It allows one to extrapolate the compactness of T from just one space to the full range of weighted spaces, whenever an m-linear operator T is bounded on weighted Lebesgue spaces. This result is indeed established in terms of the multilinear Muckenhoupt weights $A_{\vec{p},\vec{r}}$, and the limited range of the L^p scale. To show extrapolation theorems above, we build a weighted Fréchet-Kolmogorov theorem and a weighted interpolation for multilinear compact operators with full range of exponents. The former is used to characterize the compactness. The latter is shown by a weighted interpolation theorem in mixed-norm Lebesgue spaces.

This is joint work with A. Olivo and K. Yabuta.

Cardona Sanchez, Duvan (Ghent University, Belgium):
On the boundedness of oscillating singular integrals on Lie groups of polynomial growth.

Abstract: It was proved by Fefferman [1] and Fefferman and Stein [2], the weak (1,1) boundedness of oscillating singular integrals on $\mathbb{R}^n$, and their boundedness from the Hardy space H^1 into L^1, respectively. The aim of this talk is to discuss the recent extension of these results on Lie groups of polynomial growth. This talk is based on my joint works [3,4] with Michael Ruzhansky on the subject.

References:

1. Fefferman, C. Inequalities for strongly singular integral operators, Acta Math. 24, 9-36, (1970).
2. Fefferman, C., Stein, E. H^p spaces of several variables, Acta Math., 129, 137-193, (1972).
3. Cardona, D. Ruzhansky, M. Boundedness of oscillating singular integrals on Lie groups of polynomial growth, arXiv:2201.12883.
4. Cardona, D. Ruzhansky, M. Weak (1,1) continuity and Lp-theory for oscillating singular integral operators, arXiv:2201.12881.

Chuhee, Cho (Seoul National University, Republic of Korea):
Almost everywhere convergence of sequence of Schrödinger means.

Abstract: We study pointwise convergence of the generalized Schrödinger mean for a sequence which converges to zero. Our results extend the recent work of Dimou and Seeger, and Li, Wang and Yan to higher dimensions. The main novelty of the proof is to showing that the pointwise convergence of the sequential Schrödinger means follows automatically from the classical results on convergence of the fractional Schrödinger operators. This strategy also works for other dispersive operators, which in turn provides alternative proof of the previous results on the sequential convergence of various operators. This talk is based on a joint work with Hyerim Ko, Youngwoo Koh and Sanghyuk Lee.

De Carli, Laura (Florida International University):
Remarks on piecewise scaling of frames and Riesz bases.

Abstract: Given a frame (or a Riesz basis) $F = \{f_j\}_{j\in I}$ in a space of square integrable functions H, we consider the set φF obtained by multiplying the f_j by piecewise constant functions φ_j. We find conditions that ensure that φF is still a frame (or a Riesz basis) in some special and significant case. When F is finite, we present necessary and sufficient conditions that ensure that φF is a Parseval frame in H. The results presented in this talk are part of joint papers with J. Edward, P. Casazza and T. Tran.

De León-Contreras, Marta (Norwegian University of Science and Technology (NTNU), Norway):
Characterization of discrete Hölder spaces via semigroups associated to the discrete Laplacian.

Abstract: Classical Hölder spaces, $C^\alpha(\mathbb{R}^n)$, $\alpha > 0$, $\alpha \notin \mathbb{N}$, are very useful classes of smooth functions in partial differential equations, harmonic analysis and function theory. They are usually defined through pointwise estimates. When $0 < \alpha < 1$ they are defined as the set of (bounded) functions f such that

$$|f(x+z) - f(x)| \leq C|z|^\alpha \;\; x, z \in \mathbb{R}^n. \tag{MDL}$$

In [3,4,5,6], Stein and Taibleson characterized bounded Hölder functions via some integral estimates of the Poisson semigroup, $\{e^{-y\sqrt{-\Delta}}\}_{y>0}$, and of the Gauss semigroup, $\{e^{\tau\Delta}\}_{\tau>0}$. The advantage of these results is that the semigroup descriptions allow to obtain regularity properties in these spaces in a more direct way than using the pointwise expressions. The works of Taibleson and Stein raise the question of analyzing Hölder spaces adapted to different Laplacians and to find their pointwise and semigroup characterizations.

In this talk we shall see how to characterize discrete Hölder and Zygmund spaces through semigroups associated to the discrete Laplacian, for any $\alpha > 0$. This case has a completely different nature from the other cases done previously in the literature (e.g kernels don't have Gaussian estimates). This fact forces to look for new kind of estimates for the heat and Poisson kernels and their derivatives. Those estimates could also be useful to broad the theory to other discrete operators as well as to obtain characterizations of other discrete functional spaces.

The content of this talk is based on a joint work with L. Abadias, [1].

References:

1. L. Abadias and M. De León-Contreras. Discrete Hölder spaces, their characterization via semigroups associated to the discrete Laplacian and kernels estimates. arXiv:2107.06778.

2. I. Bailleul and F. Bernicot. Heat semigroup and singular PDEs, J. Funct. Anal., 270 (2016), pp. 3344-3452.

3. E. M. Stein. Singular integrals and differentiability properties of functions, Princeton Mathematical Series, No. 30, Princeton University Press, Princeton, N.J., 1970.

4. M.H. Taibleson, On the theory of Lipschitz spaces of distributions on Euclidean n-space. I. Principal properties, J. Math. Mech., 13 (1964), pp. 407-479.

5. M.H. Taibleson, On the theory of Lipschitz spaces of distributions on Euclidean n-space. II. Translation invariant operators, duality, and interpolation, J. Math. Mech., 14 (1965), pp. 821-839.

6. M.H. Taibleson, On the theory of Lipschitz spaces of distributions on Euclidean n-space. III. Smoothness and integrability of Fourier tansforms, smoothness of convolution kernels, J. Math. Mech., 15 (1966), pp. 973-981.

Eceizabarrena, Daniel (University of Massachusetts Amherst, United States):
The Schrödinger convergence problem: fractal counterexamples and the Mass Transference Principle.

Abstract: The fractal convergence problem for the Schrödinger equation asks for the smallest Sobolev regularity s such that

$$\lim_{t\to 0} e^{it\Delta} f(x) = f(x) \quad \mathcal{H}^\alpha\text{-almost everywhere}, \qquad \forall f \in H^s, \tag{DE}$$

where $0 \leq \alpha \leq n$ and $\mathcal{H}^\alpha$ is the α-Hausdorff measure.

To construct counterexamples, the usual maximal characterization does not work when $\alpha < n$. Instead, one needs to explicitly:

- Find a datum f and a divergence set $\mathcal{D}(f)$ such that $e^{it\Delta} f$ diverges in $\mathcal{D}(f)$, and

- Check that $\dim_{\mathcal{H}} \mathcal{D}(f) = \alpha$.

Well-known counterexamples like Bourgain's can be adapted to the fractal setting to satisfy the first bullet point. However, computing the Hausdorff dimension of $\mathcal{D}(f)$ is challenging. I will explain how one can use the Mass Transference Principle originated in Diophantine approximation to make this a much easier task, and share the recent improvements on the lower bounds for s in (DE) obtained with Felipe Ponce-Vanegas (BCAM, Bilbao).

Gallegos, Josep M (Universitat Autònoma de Barcelona):
Unique continuation at the boundary for solutions of elliptic PDEs.

Abstract: In 1991 Fang-Hua Lin posed the following question. Let $\Omega \subset \mathbb{R}^d$ be a Lipschitz domain and Σ be an open subset of its boundary $\partial\Omega$. Let u be a harmonic function in Ω, continuous in $\overline{\Omega}$, that vanishes on Σ, and that its normal derivative $\partial_\nu u$ vanishes in a subset of Σ with positive surface measure. Is it true that u must be identically zero?

Last year Xavier Tolsa showed a positive answer to the previous question in the case Ω is a Lipschitz domain with small Lipschitz constant. In this talk, I will explain a recent work where we show that (in the same setting as Tolsa) we can find a family of balls $(B_i)_i$ centered on Σ such that $u|_{B_t \cap \Omega}$ does not change sign and that $K \setminus \cup_i B_i$ has positive Minkowski codimension for any compact $K \subset \Sigma$. We also prove the previous result for solutions of divergence form elliptic PDEs with Lipschitz coefficients, although we will only focus on the harmonic case during the talk. I will also try to motivate why the set of points of Σ where u does not change sign nearby is interesting by comparing it with the usual singular set on the boundary.

Geng, Zhiyuan (Basque Center for Applied Mathematics (BCAM), Spain):
Asymptotic Estimates of the Diffuse Interface for Entire Minimizers in Phase Transitions.

Abstract: We study a class of globally bounded entire solutions of the Allen-Cahn system

$$\Delta u - W_u(u) = 0, \quad u : \mathbb{R}^n \to \mathbb{R}^m,$$

where $n.m \in \mathbb{N}^+$, and $W : \mathbb{R}^m \to \mathbb{R}$ is a nonnegative potential that vanishes only on a finite set $\{W = 0\} = \{a_1, ..., a_N\}$, which represents different phases of a substance. Near the minimal point a_i, the potential $W(u)$ behaves like $|u - a_i|^\alpha$ for some $0 < \alpha < 2$. Such solutions are, over large regions, identically equal to some zeroes of the potential a_i's. We establish the asymptotic measure estimates of the diffuse interface $I_0 := \{x \in \mathbb{R}^n : W(u(x)) > 0\}$ and its boundary ∂I_0. More precisely, we will prove for sufficiently large r,

- for $0 < \alpha < 2$, $\mathcal{L}^n(I_0 \cap B_r) \le c_1 r^{n-1}$, $\mathcal{H}^{n-1}(\partial^* I_0 \cap B_r) \ge c_2 r^{n-1}$.
- for $\alpha = 1$, $\mathcal{H}^{n-1}(\partial^* I_0 \cap B_r) \le c_3 r^{n-1}$.

We also prove the regularity and quantitative properties of u near the free boundary ∂I_0.

This problem is motivated by the study of multi-phase solutions in phase transition problems. The set I_0 here plays the role of a diffuse interface that separates the coexisting phases. The proof requires a novel approach of dividing B_r into identical smaller sub-cubes and classifying all the cubes according to how much measure of I_0 they contain.

This is joint work with Nicholas D. Alikakos and Arghir Zarnescu.

Genschaw, Alyssa (Milwaukee School of Engineering, USA):
Hausdorff Dimension of Caloric Measure.

Abstract: Caloric measure is a probability measure supported on the boundary of a domain in $\mathbb{R}^{n+1} = \mathbb{R}^n \times \mathbb{R}$ (space $\times$ time) that is related to the Dirichlet problem for the heat equation in a fundamental way.

Equipped with the parabolic distance, $\mathbb{R}^{n+1}$ has Hausdorff dimension $n+2$. We prove that (even on domains with geometrically very large boundary), the caloric measure is carried by a set of Hausdorff dimension at most $n+2-\beta_n$ for some $\beta_n > 0$. The corresponding theorem for harmonic measure is due to Bourgain (1987), but the proof in that paper contains a gap. Additionally, we prove a caloric analogue of Bourgain's alternative. I will briefly discuss the results, including how we fix the gap in the original proof. This is joint work with Matthew Badger.

Gonçalves, Felipe (Hausdorff Center for Mathematics):
Generalized Collatz Maps with Almost Bounded Orbits.

Abstract: *If dividing by p is a mistake, multiply by q and translate, and so you'll live to iterate.* We show that if we define a Collatz-like map in this form then, under suitable conditions on p and q, almost all orbits of this map attain almost bounded values. This generalizes a recent breakthrough result of Tao for the original Collatz map (i.e., $p = 2$ and $q = 3$). In other words, given an arbitrary growth function $N \mapsto f(N)$ we show that almost every orbit with input N eventually attains a value smaller than $f(N)$.

González-Riquelme, Cristian (ICTP, Italy):
Continuity for maximal operators at the derivative level.

Abstract: Maximal operators are a central object in harmonic analysis, The oscillatory behavior of such objects has been an object of study for many authors over the last decades. However, even in the one dimensional case, there are still interesting questions that remain open. In this talk, we will discuss recent developments and open questions about this topic, particularly about the boundedness and continuity for such operators at the derivative level.

Hagelstein, Paul (Baylor University, USA):
On the finiteness of strong maximal functions associated to functions whose integrals are strongly differentiable.

Abstract: Besicovitch proved that if f is an integrable function on $\mathbb{R}^2$ whose associated strong maximal function $M_S f$ is finite a.e., then the integral of f is strongly differentiable. On the other hand, Papoulis proved the existence of a function in $L^1(\mathbb{R}^2)$ (taking on both positive and negative values) whose integral is strongly differentiable but whose associated strong maximal function is infinite on a set of positive measure. In this talk, we prove that if f is a *nonnegative* measurable function on $\mathbb{R}^n$ whose integral is strongly differentiable and moreover such that $f(1+\log^+ f)^{n-2}$ is integrable, then $M_S f$ is finite a.e. This result is sharp in that, if ϕ is a convex increasing function on $[0,\infty)$ such that $\phi(0) = 0$ and with $\phi(u) = o(u(1+\log^+ u)^{n-2})$ $(u \to \infty)$, then there exists a nonnegative function f on $\mathbb{R}^n$ such that $\phi(f)$ is integrable on $\mathbb{R}^n$ and the integral of f is strongly differentiable, although $M_S f$ is infinite almost everywhere. This work is joint with Giorgi Oniani.

Hoffman, John (University of Missouri, United States):
Parabolic Singular Integral Operators and Quantitative Rectifiability.

Abstract: In this talk, we discuss a recent result which says that parabolic Ahlfors-David regular sets on which sufficiently nice singular integral operators are bounded from L^2 to L^2 are exactly those which are parabolic uniformly rectifiable. This is an analog of a result proved by David and Semmes in 1991 in the elliptic setting. Our proof relies on the corona decomposition - a powerful tool that can be used to link the analytic and geometric characterizations of uniform rectifiability. We will highlight some of the significant

differences between the elliptic and parabolic theories of uniform rectifiability and show how these appear in our proof. This talk is based on joint work with Bortz, Hofmann, García, and Nyström.

Jesurum, Michael (University of Wisconsin-Madison, USA):
Fourier restriction to arbitrary smooth enough curves.

Abstract: We prove Fourier restriction estimates in the (sharp) Drury range for arbitrary compact C^N curves for any $N > d$, using a power of the affine arclength measure as a mitigating factor. In particular, we make no nondegeneracy assumption on the curve.

Jung, Hongki (Indiana University, USA):
A small cap decoupling for the twisted cubic.

Abstract: Small cap decouplings deal with decoupling estimates for caps that are smaller than the canonical size. In 2019, Demeter, Guth and Wang studied small cap decoupling for exponential sums with frequency points supported on the cubic moment curve. In this talk, I will discuss the proof of L^{10} small cap decoupling for general functions, which involves incidence estimates for tubes and planks in $\mathbb{R}^3$.

Kosz, Dariusz (Basque Center for Applied Mathematics, Spain):
Maximal operators on the infinite-dimensional torus.

Abstract: We consider maximal operators $\mathcal{M}^{\mathcal{B}}$ associated with various differentiation bases $\mathcal{B}$ in the infinite-dimensional torus $\mathbb{T}^\omega$. It is known that for the so-called Rubio de Francia basis $\mathcal{R}$ the operator $\mathcal{M}^{\mathcal{R}}$ is unbounded on $L^p(\mathbb{T}^\omega)$ for every $p \in [1, \infty)$. On the other hand, the operator determined by the restricted (dyadic) basis $\mathcal{R}_0$ is of weak type $(1, 1)$, hence bounded on $L^p(\mathbb{T}^\omega)$ for every $p \in (1, \infty)$. Our goal is to understand the interplay between the structure of $\mathcal{B}$ and the behavior of $\mathcal{M}^{\mathcal{B}}$. For this purpose, we look for intermediate bases $\mathcal{R}_0 \subset \widetilde{\mathcal{R}} \subset \mathcal{R}$ which produce operators with more peculiar mapping properties. In particular, for any given $p_0 \in (1, \infty)$ we construct $\widetilde{\mathcal{R}}$ such that $\mathcal{M}^{\widetilde{\mathcal{R}}}$ is bounded on $L^p(\mathbb{T}^\omega)$ if and only if $p \in (p_0, \infty]$.

Krishnaswamy-Usha, Amudhan (Delft University of Technology, Netherlands):
Local and multilinear noncommutative De Leeuw theorems.

Abstract: Let Γ be a discrete subgroup of a locally compact unimodular group G. Let $m \in C_b(G)$ be the symbol of a bounded Fourier multiplier $T_m : L_p(\widehat{G}) \to L_p(\widehat{G})$, with $1 \leq p < \infty$. Let $T_{m|_\Gamma} : L_p(\widehat{\Gamma}) \to L_p(\widehat{\Gamma})$ be the Fourier multiplier associated to the restriction of m to Γ. De Leeuw's classical restriction theorem states that, when G is abelian, the norm of $T_{m|\Gamma}$ is bounded above by the norm of T_m. Caspers, Parcet, Perrin and Ricard later showed that this statement holds even when G is noncommutative, provided G has small almost Γ-invariant neighbourhoods. In this talk, I will show a local version of these results: namely, that there is a constant $0 \leq c(U) \leq 1$ for every $U \subseteq \Gamma$, such that

$$c(\mathrm{supp}(m|_\Gamma))\|T_{m|_\Gamma} : L_p(\widehat{\Gamma}) \to L_p(\widehat{\Gamma})\| \leq \|T_m : L_p(\widehat{G}) \to L_p(\widehat{G})\|$$

When $c(\Gamma) = 1$, I will also show a multilinear version of De Leeuw's restriction theorem, as well as multilinear versions of his compactification and lattice approximation theorems. These let us construct non-trivial examples of multilinear Fourier multipliers on nonabelian groups.

This function c has explicit lower bounds for real reductive Lie groups, in terms of the maximal dimension of a nilpotent orbit in the adjoint representation. If time permits, I will sketch how this may be computed. This is joint work with Martijn Caspers, Bas Janssens and Lukas Miaskiwskyi.

Langowski, Bartosz (Indiana University Bloomington, United States):
Restriction of exponential sums to hypersurfaces.

Abstract: We prove moment inequalities for exponential sums with respect to singular measures, whose Fourier decay matches those of curved hypersurfaces. Our emphasis is on proving estimates that are sharp with respect to the scale parameter.

Liehr, Lukas (University of Vienna, Austria):
STFT phase retrieval: uniqueness and discretization barriers.

Abstract: Let $V_g f$ be the short-time Fourier transform (STFT) of $f \in L^2(\mathbb{R})$ with respect to the window function $g \in L^2(\mathbb{R})$,

$$V_g f(x, \omega) = \int_{\mathbb{R}} f(t)\overline{g(t-x)} e^{-2\pi i \omega t}\, dt.$$

Further, let $\mathcal{L} = L\mathbb{Z}^2, L \in \mathrm{GL}_2(\mathbb{R})$, be a lattice in the time-frequency plane. Classical results in Gabor analysis state that under suitable density assumptions on $\mathcal{L}$ and mild conditions on g, every $f \in L^2(\mathbb{R})$ is determined by $V_g f(\mathcal{L})$, i.e. $f = h$ whenever $V_g f(\mathcal{L}) = V_g h(\mathcal{L})$. The so-called *STFT phase retrieval problem* concerns the question of whether analogous uniqueness results hold true assuming that only samples of the magnitude of the STFT are available, i.e. whether f is determined (up to trivial ambiguities) by $|V_g f(\mathcal{L})|$. We reveal a stark contrast to the first-mentioned setting: we prove that there exists no window functions g and no lattice $\mathcal{L}$ such that every $f \in L^2(\mathbb{R})$ is determined by $|V_g f(\mathcal{L})|$ [Grohs, P., Liehr, L., *On foundational discretization barriers in STFT phase retrieval*, to appear in: Journal of Fourier Analysis and Applications, 2022]. In particular, uniqueness can be never achieved, no matter how the window function is chosen and no matter how dense the lattice $\mathcal{L}$ is chosen. This complements earlier work where we showed that uniqueness from lattice samples can be achieved under a prior restriction of the signal class to a proper subspace of $L^2(\mathbb{R})$, namely compactly supported functions and functions in Gaussian shift-invariant spaces [Grohs, P., Liehr, L., *Injectivity of Gabor phase retrieval from lattice measurements*, arXiv:2008.07238, 2020].

Lorist, Emiel (University of Helsinki, Finland):
Operator-free sparse domination.

Abstract: Sparse domination is a recent technique, allowing to estimate (in norm, pointwise or dually) many operators in harmonic analysis by simple, positive expressions. This technique has led to a wealth of new results in harmonic analysis over the past decade. In this talk I will discuss a sparse domination principle for an arbitrary family of functions $f(x, Q)$, where $x \in \mathbb{R}^n$ and Q is a cube in $\mathbb{R}^n$. When applied to operators, this result recovers various recent sparse domination results. In contrast to preceding results, our sparse domination principle can also be applied to non-operator objects, which allows one to use sparse techniques in new areas. It for example yields new, generalized Poincaré-Sobolev inequalities and sharp results in the theory of tent spaces. This talk is based on joint work with Andrei Lerner and Sheldy Ombrosi.

Luna Garcia, Jose Luis (McMaster University, Canada):
Coronizations and big pieces in metric spaces.

Abstract: We compare two ways to approximate a set E in a metric measure space X by a family of subsets $\mathcal{A}$ in X; namely coronizations by $\mathcal{A}$ and big pieces of sets in $\mathcal{A}$. Both are motivated by the work of David and Semmes in the context of uniform rectifiability in $\mathbb{R}^n$. Indeed, this work arose as a attempt to understand these notions in the setting of parabolic geometry in $\mathbb{R}^{n+1}$.

We show that, under some structural assumptions on E and sets in $\mathcal{A}$, if E admits a coronization by sets in E then E has big pieces of big pieces of sets in $\mathcal{A}$.

This is joint work with Simon Bortz, John Hoffman, Steve Hofmann, and Kaj Nyström.

Magyar, Akos (University of Georgia, USA):
Multilinear maximal operators associated to simplices.

Abstract: We prove $l^{p_1}\times\ldots\times l^{p_k}\to l^r$ bounds for multilinear maximal operators associated to averages over all isometric copies of a given non-degenerate k-simplex, both in the continuous and the discrete settings. This provides a natural extension of $l^p\to l^p$-bounds for Stein's spherical maximal operator as well as its discrete analogue, which also serve as a key ingredient of our proofs. Joint work with B. Cook and N. Lyall.

Meroño, Cristóbal J. (Universidad Politécnica de Madrid, Espaã):
The Born approximation in the Calderón problem.

Abstract: Uniqueness and reconstruction in the three-dimensional Calderón inverse conductivity problem can be reduced to the study of the inverse boundary problem for Schrödinger operators $-\Delta+q$.

In this talk we will introduce the Born approximation of q in the ball, which amounts to studying the linearization of the inverse problem. We first analyze this approximation for real and radial potentials in any dimension. We show that this approximation is well-defined and obtain a closed formula that involves the spectrum of the Dirichlet-to-Neumann map associated to $-\Delta+q$. We then turn to general real and essentially bounded potentials in three dimensions and introduce the notion of averaged Born approximation, which captures the invariance properties of the exact inverse problem. We obtain explicit formulas for the averaged Born approximation in terms of the matrix elements of the Dirichlet to Neumann map in the basis spherical harmonics. Motivated by these formulas we also study the high-energy behaviour of the matrix elements of the Dirichlet to Neumann map.

Monti, Matteo (Politecnico di Torino, Italy):
Boundedness of harmonic Bergman projectors on homogeneous trees.

Abstract: Bergman spaces are realized on the hyperbolic disk as holomorphic functions that are square-integrable w.r.t. weighted versions of the hyperbolic measure. Through the analogy between hyperbolic disk and homogeneous trees, Bergman spaces of harmonic functions have been introduced on homogeneous trees by J. Cohen, F. Colonna, M. Picardello and D. Singman. In collaboration with F. De Mari and M. Vallarino we show that such Bergman spaces are reproducing kernel Hilbert spaces and we provide an explicit formula for the kernels. Furthermore, we state boundedness results for the extension of the Bergman projector to weighted L^p-spaces for $p>1$ and we present a counterexample for $p=1$.

Negro, Giuseppe (Instituto Superior Técnico, Portugal):
Extremizers for Fourier adjoint restriction estimates to cones.

Abstract: In 2004, Foschi found the best constant, and the extremizing functions, for the Fourier adjoint restriction estimate to the light cone in $\mathbb{R}^{1+d}$ with $d=3$. He also formulated a conjecture, concerning the extremizers to this estimate in all spatial dimensions $d\geq 2$. We disprove such conjecture via the conformal compactification of the Minkowski space-time given by the Penrose transform.

Niedorf, Lars (Kiel University, Germany):
L^p-spectral multiplier theorems with sharp p-specific regularity bounds.

Abstract: Due to a celebrated theorem of Christ, and Mauceri and Meda, for a left-invariant sub-Laplacian on a stratified Lie group G, an operator of the form $F(L)$ is of weak type $(1,1)$ and bounded on $L^p(G)$ for $1 < p < \infty$ if the spectral multiplier F satisfies a scale-invariant smoothness condition of order $s > Q/2$, where Q is the homogeneous dimension of G. In the early nineties, Müller and Stein, and Hebisch discovered that if G is a Heisenberg (-type) group, this threshold can even be pushed down to $s > d/2$, with d being the topological dimension of G, a result which has since then been extended to various other settings. In this talk, I present two of such L^p-spectral multiplier theorems in the settings of Métivier groups and Grushin operators on $\mathbb{R}^d$, requiring only $s > d(1/p - 1/2)$ as regularity condition for L^p-boundedness. Both proofs rely on restriction type estimates where the multiplier is additionally truncated along the spectrum of a sub-Laplacian on the second layer of the underlying space.

Nieraeth, Zoe (BCAM, Spain):
Some notable equivalences and open problems for maximal operators.

Abstract: Defining the bisublinear Hardy-Littlewood maximal operators by

$$M(f,g) := \sup_{Q\subseteq\mathbb{R}^n} \left(\frac{1}{|Q|}\int_Q |f|\,\mathrm{d}x\right)\left(\frac{1}{|Q|}\int_Q |g|\,\mathrm{d}x\right)\chi_Q(x),$$

where the supremum is taken over all cubes in $\mathbb{R}^n$, one can prove the inequality

$$\int_{\mathbb{R}^n} (Mf)|g|\,\mathrm{d}x + \int_{\mathbb{R}^n} |f| Mg\,\mathrm{d}x \lesssim_n \int_{\mathbb{R}^n} M(f,g)\,\mathrm{d}x.$$

through sparse domination. Interestingly enough, it is not known if the reverse inequality $\gtrsim$ also holds. If true, this would immediately imply the known sharp two-weight bounds for the sparse form as well as some new ones. In this talk I will give some insights into this problem. Moreover, I will discuss the equivalence

$$M(|f|^r)^{\frac{1}{r}} \eqsim \sum_{k=0}^{\infty} \frac{M^{k+1}f}{(r')^k},$$

which is essentially an equivalence between two methods of creating weights that are invariant under M. It is used in various places throughout the literature to obtain self-improvement results, but never explicitly written down in this sharp form.

Nowak, Adam (Polish Academy of Sciences, Poland):
Sharp heat kernel estimates on the sphere, on compact rank-one symmetric spaces, and for Jacobi expansions.

Abstract: We find sharp two-sided global estimates for the heat kernel associated with a Euclidean sphere of arbitrary dimension. Curiously enough, this seems to be a new result even for the ordinary sphere of dimension 2.

More generally, we also obtain analogous heat kernel estimates on all compact rank-one Riemannian symmetric spaces, that is on the real, complex and quaternionic projective spaces and on the exceptional Cayley's projective plane over octonions.

Moreover, we prove sharp two-sided bounds for the heat kernel associated with expansions in Jacobi polynomials, which can be interpreted as further substantial generalization of the above results.

The genuinely sharp estimates we get are far more precise than only qualitatively sharp (i.e. where constants in exponential factors in the lower and upper bound are different) heat kernel bounds commonly appearing in the literature in a great variety of settings. This is joint work with Peter Sjögren and Tomasz Z. Szarek.

Oliveira, Lucas (Universidade Federal do Rio Grande do Sul, Brasil):
On the connection between the Hardy and L^2-Cowling–Price Gaussian space-frequency spaces in the sub-critical case.

Abstract: We will discuss a relation between the classes of Schwartz functions present in Cowling-Price's and Hardy's uncertainty principle: they are almost-equivalent. We also discuss on an endpoint conjecture version of our main result, exhibiting particular classes where it holds, and present applications of our main theorem, making connections with the recent machinery of Escauriaza-Kenig-Ponce-Vega and with bounds on pointwise Gaussian decay of solutions to harmonic oscillators, settling a conjecture of Vemuri in all but a discrete set of times. Joint work with João Pedro Ramos (ETH) and Aleksei Kulikov (NTNU).

Olivo, Andrea (ICTP, Italy)
Fourier decay of self-similar measures on $\mathbb{C}$.

Abstract: R. Kaufman and M. Tsujii proved that the Fourier transform of self-similar measures on the real line has a power decay outside of a sparse set of frequencies. We present a version of this result for homogeneous self-similar complex measures, with quantitative estimates, and derive several applications. This is a joint work with Carolina Mosquera and Pablo Shmerkin.

Olsen, Jan-Fredrik (Lund University, Sweden):
Nazarov-type uncertainty principles in finite dimensions.

Abstract: In his thesis from 1992, F. Nazarov established that there exist constants C and D so that for all $f \in L^2(\mathbb{R})$ and all measurable subsets $\mathcal{L}, \mathcal{R} \subset \mathbb{R}$, it holds that

$$\int_{\mathbb{R}\setminus\mathcal{L}} |f(x)|^2 \mathrm{d}x + \int_{\mathbb{R}\setminus\mathcal{R}} |\hat{f}(\xi)|^2 \mathrm{d}\xi \geq C\mathrm{e}^{-D|\mathcal{L}||\mathcal{R}|}\|f\|_2^2.$$

This inequality improved previous results by Amrein and Berthier.

In this joint work with Shahaf Nitzan, we discuss recent progress on obtaining a version of this result for finite sequences.

Ombrosi, Sheldy Javier (Universidad Nacional del Sur, Argentina & ICTP, Italy):
Theory of weights in k-trees.

Abstract: In this talk we present weighted endpoint estimates for the Hardy-Littlewood maximal function on the infinite rooted k-ary tree. Namely, the following Fefferman-Stein estimate

$$w\left(\{x \in T \,:\, Mf(x) > \lambda\}\right) \leq c_s \frac{1}{\lambda} \int_T |f(x)| M(w^s)(x)^{\frac{1}{s}} dx \qquad s > 1$$

is settled and moreover it is shown it is sharp, in the sense that it does not hold in general if $s = 1$. This result is a generalization of the unweighted case ($w \equiv 1$) independently obtained by Naor-Tao and Cowling-Meda-Setti.

We will also present more general sufficient conditions for the strong estimates in the case $p > 1$.

This talk is based in joint works with Israel Rivera-Ríos and Martín Safe.

Poggi, Bruno (Universitat Autonoma de Barcelona, Spain):
Applications of the Poisson problem toward the regularity problem with rough coefficients.

Abstract: For $p > 1$ and p' its Hölder conjugate, we consider the Dirichlet problem with $L^{p'}$ data for the Poisson equation $-\mathrm{div}(A\nabla u) = -\mathrm{div}\ F$ in a bounded domain $\Omega \subset \mathbb{R}^{n+1}$, $n \geq 3$, with $n-$Ahlfors-David regular boundary and with interior Corkscrews, where F is a continuous vector function in Ω verifying

a natural Carleson-type condition. Assuming the solvability of the homogeneous Dirichlet problem with $L^{p'}$ data on Ω, we obtain an estimate for the $L^{p'}$ norm of a modified non-tangential maximal function of the solution to the Poisson equation. This result, with the natural condition on F, is new even for the Laplacian, and on the ball. We apply our result to solve the regularity problem with L^p data for L on uniformly rectifiable domains, assuming the matrix A verifies the Dahlberg-Kenig-Pipher conditions. This is joint work with Mihalis Mourgoglou and Xavier Tolsa.

Puliatti, Carmelo (University of the Basque Country, Spain):
L^2-boundedness of gradients of single layer potentials for elliptic operators with coefficients of Dini mean oscillation-type.

Abstract: We consider a uniformly elliptic operator L_A in divergence form associated with an $(n+1)\times(n+1)$-matrix A with real, bounded, and possibly non-symmetric coefficients. If a proper L^1-mean oscillation of the coefficients of A satisfies suitable Dini-type assumptions, we prove the following: if μ is a compactly supported Radon measure in $\mathbb{R}^{n+1}$, $n \geq 2$, and $T_\mu f(x) = \int \nabla_x \Gamma_A(x,y) f(y)\, d\mu(y)$ denotes the gradient of the single layer potential associated with L_A, then

$$1 + \|T_\mu\|_{L^2(\mu)\to L^2(\mu)} \approx 1 + \|\mathcal{R}_\mu\|_{L^2(\mu)\to L^2(\mu)},$$

where $\mathcal{R}_\mu$ indicates the n-dimensional Riesz transform. This makes possible to obtain direct generalization of some deep geometric results, initially obtained for $\mathcal{R}_\mu$, which were recently extended to T_μ under a Hölder continuity assumption on the coefficients of the matrix A.

This is a joint work with Alejandro Molero, Mihalis Mourgoglou, and Xavier Tolsa.

Rios, Cristian (University of Calgary, Canada):
On the regularity theory for infinitely degenerate elliptic equations.

Abstract: We consider elliptic equations in divergence form $L = -\mathrm{div} A(x)\nabla$ where the ellipticity parameters are allowed to vanish with infinite order. Under certain structural assumptions we develop an abstract theory of boundedness and continuity of solutions under the conditions of existence of a Poincaré's inequality and an appropriate Orlicz-Sobolev inequality. We also show particular examples where this theory applies. For the proofs, we extend the classical De Giorgi-Moser approach to non-doubling geometries.

This is work done in collaboration with Luda Korobenko, Eric Sawyer, and Ruipeng Shen.

Rivera-Ríos, Israel P. (University of Málaga, Spain):
Two weight estimates for iterated commutators.

Abstract: In this talk we shall present some recent results related to two weight estimates for Coifman, Rochberg and Weiss iterated commutators.

Rottensteiner, David (Ghent University, Belgium):
Homogeneous Function Spaces Related to Rockland Operators.

Abstract: We present a theory of homogeneous Besov-Triebel-Lizorkin spaces on Lie groups with gradable Lie algebra, i.e., those which permit a homogeneous translation-invariant hypoelliptic differential operator, a so-called Rockland operator. These necessarily nilpotent Lie groups include all stratified groups with negative (sub)Laplacians like $\mathbb{R}^n$ or the Heisenberg group.

We provide several characterizations for the full range of spaces $\dot{B}^{\sigma}_{p,q}(N)$ and $\dot{F}^{\sigma}_{p,q}(N)$, which we use to identify very well-known types of function spaces among them: the homogeneous Sobolev spaces $\dot{L}^p_\sigma(N)$, $1 < p < \infty$, introduced by Folland on stratified groups and extended to graded groups by Fischer and Ruzhansky; Folland and Stein's Hardy spaces $H^p(N)$, $0 < p \leq 1$; Folland's Lipschitz spaces $\dot{\Lambda}^\sigma(N)$ and Folland and Stein's BMO(N), which encompass the duals of the Hardy spaces $H^p(N)$.

Employing novel results on the theory of matrix coefficients, we prove the existence of atomic wavelet frames, generated by discrete group dilations and translations, whose dual frames are so-called molecules. This allows us to study the boundedness of a wide range of singular integral operators on any of the aforementioned spaces. Our approach avoids any dependence on finite speed of propagation, which is commonly used for operators of order 2 like sub-Laplacians but which is not available for general Rockland operators.

This is based on a collaboration with Guorong Hu, Jordy van Velthoven and Michael Ruzhansky.

Ryu, Jaehyeon (Korea Institute for Advanced Study, South Korea):
Bochner-Riesz mean for the Hermite expansion.

Abstract: We study the L^p boundedness of the Bochner-Riesz mean for the Hermite expansion. Considering the problem in a local setting, we extend the previously known range of p and the summability index δ. Especially, in two dimensions, we obtain the optimal range of p, δ in which the boundedness holds. To prove this result, we exploit the fact that the induced phase function associated to the kernel satisfies the Carleson-Sjölin condition and the elliptic condition. This talk is based on the recent joint work with Sanghyuk Lee.

Saari, Olli (University of Bonn, Germany):
A right inverse of the divergence in time-dependent domains.

Abstract: Consider an open and connected set in the $(1+n)$-dimensional space time. I discuss a construction of a right inverse of the (spatial) divergence that respects vanishing boundary values at the lateral boundary. The domain is assumed to be Hölder regular both in time and space variables. Depending on the actual regularity, a priori estimates in Sobolev spaces of positive and negative order can be proved. In particular, the construction exhibits a non-trivial amount of regularity in time variable, which is of interest from the point of view of fluid mechanics. This is based on joint work with S. Schwarzacher.

Sengupta, Banhirup (Universitat Autònoma de Barcelona, España):
Rotational bounds for homeomorphisms with integrable distortion and Hölder continuous inverse.

Abstract: In this talk I will explain a recent work, in collaboration with Albert Clop and Lauri Hitruhin, where we have obtained sharp spiraling bounds for homeomorphisms with L^p-integrable distortion having Hölder continuous inverse. Our result certainly improves the bounds obtained by Hitruhin for homeomorphisms with integrable distortion without a priori assumption of Hölder continuous inverse. As an application, we estimate the spiraling rate of Euler flows for small times.

Sjögren, Peter (University of Gothenburg, Sweden):
Functional calculus in a general Gaussian setting.

Abstract: A general Ornstein-Uhlenbeck operator $\mathcal{L}$ in $\mathbb{R}^n$ has drift given by a real matrix B whose eigenvalues have negative real parts. We prove that if m is a function of Laplace transform type defined

in the right half-plane, then $m(\mathcal{L})$ is of weak type $(1,1)$ with respect to the invariant measure in $\mathbb{R}^n$. The strong type (p,p), $1 < p < \infty$, of $m(\mathcal{L})$ was already known. After discussing the definition of $m(\mathcal{L})$, we sketch how this operator is split in the proof of our result, which involves many estimates of the relevant integral kernels.

This is joint work with V. Casarino and P. Ciatti.

Slamić, Ivana (University of Rijeka, Croatia):
Dual integrability condition for unitary representations of locally compact groups.

Abstract: Dual integrable representations were first introduced and studied for locally compact abelian groups. A unitary representation $\Pi : G \to \mathcal{U}(\mathbb{H})$ of an LCA group G on a separable Hibert space $\mathbb{H}$ is called dual integrable if there exists a function $[\cdot,\cdot] : \mathbb{H} \times \mathbb{H} \to L^1(\widehat{G})$, called bracket, such that

$$\langle \varphi, \Pi(g)\psi \rangle = \int_{\widehat{G}} [\varphi, \psi](\xi) e_{-g}(\xi) d\xi$$

for all $g \in G$ and all $\varphi, \psi \in \mathbb{H}$. In recent years the concept has been studied in the setting of non-abelian discrete groups and compact groups. Its importance relies on the possibility of defining an operative bracket map, in terms of which one is able to characterize various properties of orbits and invariant subspaces. We introduce and study the concept in the general setting of locally compact groups. This is joint work with Hrvoje Šikić.

Sousa, Mateus (BCAM, Spain):
Recent developments in Fourier interpolation theory.

Abstract: In this talk we will discuss about some recent developments involving the now called Fourier interpolation formulas, and how they are related to certain kinds of uncertainty principles and the theory of sphere packing.

Srivastava, Rajula (University of Wisconsin at Madison, USA):
On the Korányi Spherical Maximal Function on Heisenberg Groups.

Abstract: We discuss sharp (up to end points) $L^p \to L^q$ estimates for the local maximal operator associated with dilates of the Korányi sphere. This is a surface of codimension one in the Heisenberg group, compatible with the non-isotropic dilation structure on the group. The Lebesgue space estimates imply new bounds on sparse domination for the corresponding global maximal operator. Unlike the Euclidean version, the Korányi sphere has points of vanishing curvature at the north and south poles, which makes the associated maximal operators interesting objects of study. We shall see that despite the non-Euclidean setting, the theory of Fourier Integral Operators can be applied to establish our estimates.

Szarek, Tomasz Z. (BCAM, Spain):
Pointwise ergodic theorems on nilpotent groups.

Abstract: Let $(X, \mathcal{B}(X), \mu)$ denotes a σ-finite measure space. Given any family of invertible measure-preserving transformations $T_1, \ldots, T_d \colon X \to X$, $d \geq 1$, a measurable function $f \in L^p(X)$, $p \geq 1$, polynomials $P_1, \ldots, P_d$, and an integer $N \geq 1$, we define the polynomial ergodic averages

$$A_N(f)(x) = \frac{1}{|[-N,N] \cap \mathbb{Z}|} \sum_{n \in [-N,N] \cap \mathbb{Z}} f(T_1^{P_1(n)} \cdots T_d^{P_d(n)} x), \qquad x \in X.$$

A fundamental problem in ergodic theory is to establish convergence in norm and pointwise almost everywhere for $A_N(f)$ as $N \to \infty$ for functions $f \in L^p(X)$, $1 \le p \le \infty$.

The famous Furstenberg–Bergelson–Leibman conjecture says that the averages $A_N(f)$ converge for μ-almost every $x \in X$ as $N \to \infty$ provided that the transformations $T_1, \dots, T_d$ generate a nilpotent group. The purpose of this talk is to discuss recent progress towards solving this conjecture. We focus on the case of nilpotent groups of step 2 in which this conjecture was recently solved. This is joint work with A.D. Ionescu, Á. Magyar and M. Mirek.

Tapiola, Olli (Universitat Autònoma de Barcelona, Catalonia, Spain):
Two-sided local John condition implies Harnack chains and boundary Poincaré inequalities.

Abstract: Inspired by recent work of Mourgoglou and Tolsa, we prove the following two results. First, we show that a 2-sided corkscrew domain in $\mathbb{R}^{n+1}$ satisfying the so-called local John condition, with codimension 1 Ahlfors-David regular boundary, is a chord-arc domain. Second, we show that the boundary of a 2-sided chord-arc domain supports a Heinonen–Koskela type weak 1-Poincaré inequality. Our proofs utilize significant advances in e.g. harmonic measure, uniform rectifiability and metric Poincaré theories. This is a joint work with Xavier Tolsa.

Van Velthoven, Jordy Timo (Delft University of Technology):
Density conditions with stabilizers for lattice orbits of Bergman kernels.

Abstract: Let π_α be a holomorphic discrete series of a connected semi-simple Lie group G, acting on a weighted Bergman space $A^2_\alpha(\Omega)$ on a bounded symmetric domain Ω, of formal dimension $d_{\pi_\alpha} > 0$.

The talk concerns density conditions on a lattice $\Gamma \le G$ for the Bergman kernels $(\pi_\alpha(\gamma)k_z^{(\alpha)})_{\gamma\in\Gamma}$ to be a frame or Riesz sequence in $A^2_\alpha(\Omega)$. The condition $\mathrm{vol}(G/\Gamma)d_{\pi_\alpha} \le |\Gamma_z|^{-1}$ (resp. $\mathrm{vol}(G/\Gamma)d_{\pi_\alpha} \ge |\Gamma_z|^{-1}$) is shown to be necessary for $(\pi_\alpha(\gamma)k_z^{(\alpha)})_{\gamma\in\Gamma}$ to be a frame (resp. Riesz sequence).

These estimates improve on general density theorems for restricted discrete series through the dependence on the stabilizers Γ_z of $z \in \Omega$.

The talk is based on joint work with M.Caspers.

Vardakis, Dimitris (Michigan State University, United States):
Free boundary problems via Sakai's theorem.

Abstract: A Schwarz function on an open domain Ω is a holomorphic function satisfying $S(\zeta) = \overline{\zeta}$ on a part, Γ, of Ω's boundary. Sakai in 1991 gave a complete characterization of the boundary of a domain admitting a Schwarz function. In fact, if Ω is simply connected and $\Gamma = \partial\Omega \cap D(\zeta, r)$, then Γ has to be regular real analytic. We'll try to describe Γ over a simply connected domain Ω in three different scenarios when the boundary condition is slightly relaxed: When $f_1(\zeta) = \overline{\zeta} f_2(\zeta)$ on Γ, with f_1, f_2 holomorphic and continuous up to the boundary, when $S(\zeta) = \Phi(\zeta, \overline{\zeta})$ on Γ, with Φ a holomorphic function of two variables, and when $\mathcal{U}/\mathcal{V}$ equals certain real analytic function on Γ with $\mathcal{U}, \mathcal{V}$ positive and harmonic on Ω and vanishing on Γ. It turns out the boundary piece Γ can be, respectively, anything from C^∞ to just C^1, regular except for a zero-measure set, or regular except finitely many points.

Weigt, Julian (Aalto University, Finland):
Endpoint regularity bounds of maximal functions in any dimensions.

Abstract: We prove the endpoint Sobolev bound

$$\|\nabla Mf\|_{L^1(\mathbb{R}^n)} \le C_n \|\nabla f\|_{L^1(\mathbb{R}^n)}$$

for various maximal functions Mf in any dimension $n \geq 1$.

The key arguments of the proofs are of geometric nature. For example new variants of the isoperimetric inequality and of the Vitali covering lemma are proven and used. All proofs are mostly elementary up to applications of classical results like the relative isoperimetric inequality and the coarea formula and approximation schemes.

Some of the arguments only work for cubes and not for balls. Thus, for the uncentered Hardy-Littlewood maximal operator we can only prove the above endpoint Sobolev bound in the case of characteristic functions. However, we are able to prove it for general functions for example for the maximal operator that averages over uncentered cubes with any orientation instead of balls. The methods also enable a proof of the corresponding bound for the fractional centered and uncentered Hardy-Littlewood maximal functions.

Yang, Tongou (University of British Columbia):
Restricted projections along C^2 curves on the sphere.

Abstract: Given a C^2 curve $\gamma(\theta)$ lying on the sphere $\mathbb{S}^2$ and a Borel set $A \subseteq \mathbb{R}^3$. Consider the projections $P_\theta(A)$ of A into straight lines in the directions $\gamma(\theta)$. We prove that if γ satisfies the following non-degeneracy condition: $\det(\gamma, \gamma', \gamma'')(\theta) \neq 0$ for any θ, then for almost every θ, the Hausdorff dimension of $P_\theta(A)$ is equal to $\text{mín}\{1, \dim_H(A)\}$. This solves a conjecture of Fässler and Orponen. One key feature of our argument is a result of Marcus-Tardos in topological graph theory.

This is a joint work with Malabika Pramanik, Orit Raz and Josh Zahl.