Words and twisted sums of C(K)-spaces

Jesús M. F. Castillo

Instituto de Matemáticas Imuex /Universidad de Extremadura

Avenida de Elvas, 06071-Badajoz, Spain (castillo@unex.es).

Workshop on Banach spaces and Banach lattices II 9–13 May 2022, ICMAT Madrid (Spain).

This research was supported in part by project PID2019-103961GB-C21 funded by MCIN and by project IB20038 funded by Junta de Extremadura.



JUNTA DE EXTREMADURA

Consejeria de Economia, Ciencia y Agenda Digital



Words and twisted sums of C(K)-spaces

Jesús M. F. Castillo

The talk contains material from the papers

- J.M.F. Castillo, A. Salguero Alarcón, On twisted sums of c₀(1), arXiv:2204.01704v1
- F. Cabello Sánchez, J.M.F. Castillo, Words and exact sequences of spaces of continuous functions. Not yet ready!

Twisted sums of the form $0 \longrightarrow A \longrightarrow B \longrightarrow C \longrightarrow 0$ in which two of the involved spaces are spaces of continuous functions on some compact spaces (from now on called \mathscr{C} -spaces) are rather complex mathematical objects. The examples in the literature usually respond to one of the following schemas. Pick a continuous function $\varphi: K \to S$ between two compacta:

(1) If ϕ is an injection, the "dual" exact sequence

 $0 \longrightarrow J_{\mathcal{K}} \longrightarrow C(S) \xrightarrow{r} C(\mathcal{K}) \longrightarrow 0$ in which $r = \varphi^{\circ}$ is the natural restriction. Especially interesting examples of this type are the Nakamura-Kakutani sequences generated by almost-disjoint families of subsets of .

(II) If ϕ is a surjection the "dual" exact sequence

 $0 \longrightarrow C(S) \xrightarrow{\phi^{\circ}} C(K) \longrightarrow Q \longrightarrow 0$. This case is quite delicate since we do not have information about the nature of Q.

Pick a continuous function $\varphi: \mathcal{K} \to S$ between two compacta:

(1) If ϕ is an injection, the "dual" exact sequence

 $0 \longrightarrow J_K \longrightarrow C(S) \xrightarrow{r} C(K) \longrightarrow 0$ in which $r = \varphi^{\circ}$ is the natural restriction. Especially interesting examples of this type are the Nakamura-Kakutani sequences generated by almost-disjoint families of subsets of .

(II) If ϕ is a surjection the "dual" exact sequence

 $0 \longrightarrow C(S) \xrightarrow{\phi^{\circ}} C(K) \longrightarrow Q \longrightarrow 0$. This case is quite delicate since we do not have information about the nature of Q.

Against our wishes for a tidy Banach world, it may perfectly happen that two of the spaces A, B, C can be C-spaces and the third one not. One of the extreme examples are the Plebanek-Salguero sequences

$$0 \longrightarrow c_0 \longrightarrow \mathsf{PS}(K) \longrightarrow C(K) \longrightarrow 0$$

(日) (四) (三) (三) (三)

in which PS(K) is not a C-space.