II Banach spaces and Banach lattices workshop

Book of abstracts

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Contents

Mini-courses

$C(K)$-spaces associated with almost disjoint families

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Given an almost disjoint family of infinite sets of natural numbers, there is a standard way to associate a compact Hausdorff space with it, and hence a $C(K)$-space. This construction is nearly 100 years old; it is due to Alexandroff and Urysohn, and the resulting topological space is known under many different names in the literature, including AU-compactum, Mròwka space and Ψ-space. In the 1970s, Johnson and Lindenstrauss made the seminal application of this family of $C(K)$-spaces in Banach space theory. They have subsequently been used in a number of important counterexamples, most recently the spectacular construction of Plebanek and Salguero-Alarcón of a $C(K)$-space containing a complemented subspace which is not isomorphic to a $C(K)$-space. I intend to begin from first principles, introducing this class of spaces, before describing my own interest in them:

- In joint work with Koszmider (Adv. Math. 2021), we constructed a $C(K)$-space of this kind with "few operators", thereby removing the Continuum Hypothesis from an earlier result of Koszmider.
- In ongoing work with Arnott, we study whether the associated Calkin algebra has a unique algebra norm.

Free Banach lattices, and their lattice isomorphisms

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We say that X is the free Banach lattice on the Banach space E if every contraction $T : E \to Z$ (Z is a Banach lattice) extends to a contractive lattice homomorphism $\widetilde{T} : X \to Z$; we denote this X by $FBL[E]$. One can modify this definition by imposing the condition that the Banach lattices involved satisfy a certain property; in this talk, I will focus on lattices which are p-convex with constant 1, in which case the free object is denoted by $FBL^{(p)}[E]$. We shall begin by describing a functional representation of a free Banach lattice. After recalling some properties of a free lattices, I shall focus on lattice homomorphisms between then. It turns out that such lattice homomorphisms give rise to (non-linear, in general) maps between the underlying Banach spaces. This permits us to determine under what conditions $FBL^{(p)}[E]$ may, or may not, be lattice isomorphic (or isometric) to $FBL^{(p)}[F]$. This is a joint work with M.Taylor, P.Tradacete, and V.Troitsky.

Plenary lectures

Isometries of combinatorial Banach spaces

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A combinatorial Banach space is induced by a family of finite subsets of some index set. Generalizing previous results, we prove that linear isometries between combinatorial spaces are induced by a signed permutation of its canonical basis, even in the nonseparable context. Moreover, we show that different spreading families induce nonisometric combinatorial spaces. This is a joint work with Claribet Piña.

Understanding the spectrum of an operator via the geometry of the underlying space

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The classical theorems of Perron and Frobenius about matrices with nonnegative entries, as well as their generalisations to infinite-dimensional Banach lattices, illustrate that there is a strong connection between the spectral properties of a positive operator and the geometric properties of the underlying space and its positive cone. We present an overview of some of the classical results from this realm, and then discuss a number of recent insights into how the geometry of a Banach space E affects the spectral properties of operators acting on E.

Stopping-time spaces and their operators

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Stopping-time Banach spaces is a collective term for the class of spaces of eventually null integrable processes that are defined in terms of the behaviour of the stopping times with respect to some fixed filtration. From the point of view of Banach space theory, these spaces in many regards resemble the classical spaces such as L_1 or $C(\Delta)$, but unlike these, they do have unconditional bases. In the talk, we shall explain the link between the canonical bases in the stopping-time spaces and the possibility of factorising the identity operator thereon. Since we work exclusively with the dyadic-tree filtration, this setup enables us to work with tree-indexed bases rather than directly with stochastic processes. En route to the factorisation results, we have developed general criteria that allow one to deduce the uniqueness of the maximal ideal in the algebra of operators on a Banach space. These criteria are applicable to many classical Banach spaces such as (mixed-norm) L_p-spaces, BMO, SL^∞ and others. Joint work with R. Lechner from Linz.

Relations between the linear and the lattice structure of Banach lattices

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Understanding the relation between the linear and lattice structures of Banach lattices has been the driving force behind a large part of research in the theory of Banach Lattices. For instance, a well-known classical result states that a Banach lattice contains a subspace isomorphic to c_0 if and only if it contains a sublattice isomorphic to c_0. The problem we address in this talk is to characterize those Banach lattices which share this property with c_0. Improving results of H.P. Lotz, H.P. Rosenthal and N. Ghoussoub, we will show that a Banach lattice contains a subspace isomorphic to $C[0,1]$ if and only if it contains a sublattice isomorphic to $C[0,1]$. This talk is based on a recent joint work with Antonio Avilés, Abraham Rueda Zoca and Pedro Tradacete.

Geometry of spaces of vector-valued Lipschitz functions

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Universidad de Murcia

In the last years, a big effort has been done in order to analyse the geometric structure of the space of Lipschitz functions $\mathrm{Lip}(M)$ as well as its isometric predual $\mathcal{F}(M)$, obtaining for instance that $\mathrm{Lip}(M)$ has the Daugavet property if, and only if, M is length. As usual in this context, one of the most important tools is the fact that every real-valued Lipschitz function can be extended in a norm preserving way to any bigger domain space (this is classical McShane extension theorem). Since a version of McShane extension theorem is false for Lipschitz functions taking values on arbitrary Banach spaces, it is clear that different approaches are needed in order to study the geometry of spaces of vector-valued Lipschitz functions $\mathrm{Lip}(M, X)$.

In this talk we will present some results about octahedrality and the Daugavet property in the spaces $\mathrm{Lip}(M, X)$, based on results of [1, 2].

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Smoothness in normed spaces

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In the talk we shall report on a joint project with Sheldon Dantas and Petr Hájek concerning the existence of differentiable norms in non-complete normed spaces. It is a well-known fact that the existence of a C^1-smooth norm on a separable Banach space $\mathcal{X}$ implies the separability of the dual $\mathcal{X}^*$. It follows, for example, that no infinite-dimensional closed subspace of ℓ_1 can admit a differentiable norm. However, the situation is radically different if we only consider (not necessarily closed) linear subspaces; for example, there is a linear subspace of ℓ_1 that admits a C^∞-smooth norm. Indeed, one of the first results in this area, due to Hájek in 1995, asserts that every separable Banach space admits a dense subspace with a C^∞-smooth norm. This led us to consider the following question:

Let $\mathcal{X}$ be a Banach space. Does there exist a dense subspace $\mathcal{Y}$ of $\mathcal{X}$ that admits a C^k-smooth norm, for some $k \in \mathbb{N} \cup \{\infty, \omega\}$?

In the talk we shall review the status of the art on the said question and we shall describe our recent results. In particular, we shall see that for separable Banach spaces it is also possible to obtain a dense subspace with an analytic (namely, C^ω-smooth) norm. For non-separable Banach spaces, we will also see that the answer is positive (for $k = \infty$) for every Banach space with a fundamental biorthogonal system. However, no dense subspace of $c_0(\omega_1)$ admits an analytic norm, hence the above question might in general have a negative answer for $k = \omega$. Along the way, we will also mention some open problems and possible directions for further research.

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Free Banach Lattices

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Given a Banach space E, one can associate a Banach lattice FBL[E] with the property that every bounded operator from E to a Banach lattice X extends uniquely to a lattice homomorphism from FBL[E] into X. We will discuss recent progress towards understanding the structure of FBL[E], and its relation to classical topics in Banach space theory.

Renorming AM-spaces

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Independent Scholar

We show that any separable AM-space X can be renormed with an equivalent lattice norm of arbitrarily small distortion so that the only lattice isometry on X is the identity. If in addition X has at most one atom, this renorming can be an AM-norm. To this end, we introduce a class of AM-spaces called "regular" AM-spaces, which can be used to approximate separable AM-spaces, and we explore the geometry of this class.

Contributions

On unbounded norm convergence in Banach lattices

Youssef Azouzi (josefazouzi@gmail.com)
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We present some new results on unbounded norm convergence in Banach lattices; some of them solve open questions asked recently by Kandic, Marabeh and Troitsky.

On uniform Mazur intersection property

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The Mazur Intersection Property (MIP)—every closed bounded convex set is the intersection of closed balls containing it—is an extremely well studied property in Banach space theory. A complete characterisation was obtained by Giles, Gregory & Sims (1978), most well-known criterion stating that the w*-denting points of $B(X^*)$ are norm dense in $S(X^*)$. Chen and Lin (1995), introduced the notion of w*-semidenting points and showed that a Banach space X has the MIP if and only if every $f \in S(X^*)$ is a w*-semidenting point of $B(X^*)$. A much less studied uniform version of the MIP (UMIP or UI) was introduced by Whitfield and Zizler (1987). Characterisations similar to Giles, Gregory & Sims were also obtained, but an analogue of the w*-denting point criterion was missing, which perhaps is a reason for its being less pursued. In this talk, we show that a Banach space X has the UMIP if and only if every $f \in S(X^*)$ is a uniformly w*-semidenting point of $B(X^*)$, thus filling a long felt gap. This is a joint work with Jadav Ganesh & Deepak Gothwal.

Horizontal and uo convergence on vector lattices

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University of Alberta

In this talk we will consider a new mode of convergence on vector lattices. It turns out that lateral completeness is equivalent to completeness with respect to this convergence. We also revisit the unbounded order convergence, provide its description independent of the order convergence and discuss the fact that universal completion of a vector lattice is the completion with respect of the uo convergence

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Words and twisted sums of spaces of continuous functions

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Twisted sums of the form $0 \longrightarrow A \longrightarrow B \longrightarrow C \longrightarrow 0$ in which two of the involved spaces are spaces of continuous functions on some compact spaces (from now on called $\mathcal{C}$-spaces) are rather complex mathematical objects. The examples in the literature usually respond to one of the following schemas. Pick a continuous function $\varphi : K \to S$ between two compacta:

(I) If φ is an injection, the "dual" exact sequence

$$0 \longrightarrow J_K \longrightarrow C(S) \xrightarrow{r} C(K) \longrightarrow 0$$

in which $r = \varphi^\circ$ is the natural restriction. Especially interesting examples of this type are the Nakamura-Kakutani sequences generated by almost-disjoint families of subsets of $\mathbb{N}$.

(II) If φ is a surjection the "dual" exact sequence

$$0 \longrightarrow C(S) \xrightarrow{\varphi^\circ} C(K) \longrightarrow Q \longrightarrow 0\,.$$

This case is quite delicate since we do not have information about the nature of Q.

Against our wishes for a tidy Banach world, it may perfectly happen that two of the spaces A, B, C can be $\mathcal{C}$-spaces and the third one not. One of the extreme examples are the Plebanek-Salguero sequences $0 \longrightarrow c_0 \longrightarrow \mathsf{PS}(K) \longrightarrow C(K) \longrightarrow 0$ in which $\mathsf{PS}(K)$ is not a $\mathcal{C}$-space.

We will show that every exact sequence of type (I) has an ultrapower that splits which, in particular, implies that every twisted sum of c_0 and a $\mathcal{C}$-space admits an ultrapower that is a $\mathcal{C}$-space. Yes, including $\mathsf{PS}(K)$.

We will also will pay a closer look at the spaces generated by almost disjoint families, some of which are $\mathcal{C}$-spaces like the Nakamura-Kakutani sequences; and some are not, like the spaces $\mathsf{PS}(K)$ that we will connect with (much) older constructions of Heinrich, Henson and Moore.

This research has been supported in part by project IB20038 funded by Junta de Extremadura and project PID2019-103961GB funded by MINCIN.

Generalized Mazur maps in the noncommutative setting

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The classical Mazur map is a uniform homeomorphism between unit spheres of L_p spaces, and the version for noncommutative L_p spaces has the same property (see, e.g. the work of Ricard). Two types of generalized Mazur maps were used by Odell and Schlumprecht to prove that the unit sphere of a Banach space X with an unconditional basis is uniformly homeomorphic to the unit sphere of a Hilbert space if and only if X does not contain ℓ_∞^n's uniformly. We study noncommutative versions of both of these types of generalized Mazur maps, yielding uniform homeomorphisms between the unit spheres of unitarily invariant operator ideals. The techniques used involve matrix analysis and quantum information theory.

Transfinite almost square Banach spaces

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A Banach space X is said to be *almost square* if for every $\varepsilon > 0$ and every finite-dimensional subspace $Y \subset X$ there exists x in the unit sphere of X satisfying

$$(1-\varepsilon)\max\{\|y\|, |\lambda|\} \leq \|y + \lambda x\| \leq (1+\varepsilon)\max\{\|y\|, |\lambda|\}$$

holds for all $y \in Y$ and $\lambda \in \mathbb{R}$. For example c_0, the Gurariĭ space $\mathbb{G}$ and M-embedded spaces are almost square. In this talk we introduce a more rigid version of almost squareness and a transfinite generalization to arbitrary cardinalities. Moreover, we discuss about both isometric and isomorphic properties of these spaces. In particular renorming problems, duality, direct sums, tensor products and ultra-powers are discussed.

Subspaces of C(K) with unique Hahn-Banach extensions

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In 1960 Phelps introduced property U to refer to a subspace of a Banach space with unique norm-preserving extensions through the Hahn–Banach Theorem. In this presentation, we study Banach spaces that admit what we called a U-embedding into a $C(K)$ space, that is, those Banach spaces X that can be isometrically embedded into a $C(K)$ space in such a way that any functional on its isometric image has a unique Hahn-Banach extension. We provide results whether X is separable or X is a $C(K)$ space.

Free complex Banach lattices

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ICMAT-CSIC

The construction of the free Banach lattice generated by a real Banach space is adjusted to the complex setting. It is shown that for every complex Banach space E there is a complex Banach lattice $FBL_{\mathbb{C}}[E]$ containing an isometric copy of E and satisfying the corresponding universal property; that is, any (complex) operator $T : E \to X_{\mathbb{C}}$, may be extended to a unique lattice homomorphism $\hat{T} : FBL_{\mathbb{C}}[E] \to X_{\mathbb{C}}$ with $\|T\| = \|\hat{T}\|$. We shall indicate several properties which the free complex Banach lattice $FBL_{\mathbb{C}}[E]$ shares with its real counterpart. Moreover, we show that examples of non-isomorphic complex Banach spaces E and F can be given so that $FBL_{\mathbb{C}}[E]$ and $FBL_{\mathbb{C}}[F]$ are lattice isometric: It is worth noting that in the real case it remains unknown whether two Banach spaces E and F are isomorphic whenever their corresponding free Banach lattices are lattice isometric.

Invariant subspaces for positive operators on Banach lattices with unconditional basis

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Invariant subspace problem is one of the most studied, open problems on Operator Theory. In spite of the existence of counterexamples on spaces such as ℓ_1 ([5]), the problem is still open for positive operators, even on ℓ_p spaces for $1 \leq p < \infty$, ([1] and [2]). In this talk, we consider the invariant subspace problem for positive operators on Banach lattices whose order is induced by an unconditional basis. We will present results regarding the existence of non-trivial invariant subspaces (and non-trivial invariant ideals) for certain classes of positive operators, such as lattice homomorphisms and band-diagonal operators, and we will discuss the limits of our methods. These results are part of joint works with Eva A. Gallardo-Gutiérrez and Pedro Tradacete ([3] and [4]).

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Variable Lebesgue spaces versus L_p spaces when it comes to Fixed Point Theory

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Variable Lebesgue spaces $L^{p(\cdot)}(\Omega)$ can be considered as a generalization of classical Lebesgue spaces $L^p(\Omega)$ when the exponent p is allowed to be a variable measurable function $p(\cdot)$. Although variable Lebesgue spaces can be included within the larger family of Musielaz-Orlicz spaces, during the past three decades many researches have felt attracted to this variable extension of Lebesgue spaces, in particular, since the discovery of many applications of $L^{p(\cdot)}(\Omega)$ to PDEs connected with different problems of physical nature (see references [1, 3, 6]). In fact, properties of variable Lebesgue spaces related to some areas such as harmonic analysis or partial differential equations have been widely studied in the references included above, where it is shown that variable Lebesgue spaces share many standard properties from their classical counterparts but, at the same time, differ from them in several subtle and interesting ways.

There is a vast literature studying the fixed point property for nonexpansive operators in classical L_p spaces. We would like to emphasize that all $L_p(\Omega)$ with $1 < p < +\infty$ have the so-called *fixed point property*, while both $L_1(\Omega)$ and $L_\infty(\Omega)$ fail this condition. The reflexivity plays an essential role in the previous statements to the extent that a closed subspace X of $L_1(\Omega)$ has the *fixed point property* if and only if it is reflexive (see [2, 4]).

In this talk we will study the *fixed point property* for the family of variable Lebesgue spaces $L^{p(\cdot)}(\Omega)$. We will check that reflexivity also plays its particular role but, unlike the classical case, it is not crucial for determining the *fixed point property.* Although some of our results may resemble those known for the standard L^p spaces, we discover some unexpected dissimilarities that allow us to raise new open problems within the scope of metric fixed point theory. Most of the content of this talk can be found in [5].

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Essentially disjoint operator ranges

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In this talk, we will review some results on operator ranges and present several extensions of a theorem of Shevchik which asserts that if R is a proper dense operator range in a separable Banach space E, then there exists a compact, one-to-one and dense range operator $T : E \to E$ such that $T(E) \cap R = \{0\}$, and several results of Chalendar and Partington concerning the existence of compact endomorphisms on a separable Banach space E which leaves invariant a given closed subspace $Y \subset E$, or more generally, a countable chain of closed subspaces of E. Joint work with S. Lajara.

Generating bounded operators

Pedro José Miana Sanz (pjmiana@unizar.es)
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In this talk we study the solution of the quadratic equation $TY^2 - Y + I = 0$ where T is a linear and bounded operator on a Banach space X. We describe the spectrum set and the resolvent operator of Y in terms of operator T. In the case that $4T$ is a power-bounded operator, we show that a solution (named Catalan generating function) is given by the Taylor series

$$C(T) := \sum_{n=0}^{\infty} C_n T^n,$$

where the sequence $(C_n)_{n\geq 0}$ is the well-known Catalan numbers. We express $C(T)$ by means of an integral representation which involves the resolvent operator $(\lambda - T)^{-1}$. Some particular examples to illustrate our results are given, in particular an iterative method defined for square matrices T which involves Catalan numbers. This is a joint paper with Natalia Romero from the Universidad de La Rioja.

Banach spaces consisting of strongly norm-attaining Lipschitz functions

Óscar Roldán Blay (oscar.roldan@uv.es)
Universitat de València

We study the existence of non-trivial Banach subspaces Y of $\mathrm{Lip}_0(M)$ consisting of strongly norm-attaining Lipschitz functionals, where M is a pointed metric space (that is, with a distinguished point 0), and $\mathrm{Lip}_0(M)$ is the space of Lipschitz functionals f on M satisfying that $f(0) = 0$. We characterize when such subspaces can have dimension n, for $n \in \mathbb{N}$, and show that infinite-dimensional such subspaces can also exist in many cases. We also study the possible sizes of the metric space M given that such a subspace Y exists. Similar questions are studied for some particular classes of metric spaces. This talk is based on a recent joint work with Vladimir Kadets.

Isomorphisms of Banach spaces of continuous functions

Jakub Rondoš (jakub.rondos@gmail.com)
Charles University Prague

The talk will be about several recent results and open problems in the isomorphic theory of Banach spaces of continuous functions on compact Hausdorff spaces.

C(K)-spaces in the twisted realm

Alberto Salguero Alarcón (salgueroalarcon@unex.es)
Universidad de Extremadura

A twisted sum of two Banach spaces Y and X is another space Z containing Y as a closed subspace so that $Z/Y = X$. In this talk, we intend to present spaces of continuous functions within the context of twisted sums of Banach spaces. We shall focus on spaces of continuous functions on Alexandroff-Urysohn compacta, which arise from almost disjoint families of subsets of $\mathbb{N}$. Several classical results will be treated, as well as their role in the recent counterexample for the complemented subspace problem. This talk is based on joint works with F. Cabello, J. M. F. Castillo, W. Marciszewski and G. Plebanek.

Weakly compact sets in variable exponent spaces

Mauro Sanchiz (msanchiz@ucm.es)
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In this talk we will see recent characterizations of the weakly compact sets of variable exponent Lebesgue spaces $L^{p(\cdot)}(\Omega)$. To do so, we recall different characterizations of weakly compact sets in Banach lattices, such as Dunford-Pettis Theorem on L_1 or Ando's criteria on Orlicz spaces, which serve as reference. One of the characterizations allows us to prove that the $L^{p(\cdot)}(\Omega)$ spaces are weakly Banach-Saks.

Boundedness of the Hilbert transform in Marcinkiewicz spaces and applications

Kanat Tulenov (tulenov@math.kz)

Institute of Mathematics and Mathematical Modeling & Al-Farabi Kazakh National University

We deal with characterizing optimal range for the Calderón operator and the Hilbert transform in Marcinkiewicz function spaces. These results are further used as a sufficient condition to obtain Lipschitz estimates for commuting tuples in fully symmetric (quasi-)Banach operator spaces.

Noetherian solvability and explicit solution of a singular integral equation with weighted Carleman shift in Besov space

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We prove Noetherian solvability and obtain the index formula for a singular integral equation with a Carleman shift in Besov spaces. We also obtain the existence of solution of the singular integral equation as well as their explicit representations in Besov spaces.
