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1st New Developments in Momentum Polytope Theory Symposium and School

Program & abstracts

	Monday 26	Tuesday 27	Wednesday 28	Thursday 29	Thursday 30
8:30	Opening				
9:00	Pelayo	Dullin	Cupit-Foutou	(notalk)	Bürgisser
9:30					
10:00	Koshevoy	Castillo	Christandl	(no talk)	Mir
10:30	Break	Break	Break	Break	Break
11:30	Bolsinov	Cardona	Miranda	(notalk)	Zoom Party
12:00		Larotonda			
12:30	Break	End	End	(no talk)	End
14:00	Woodward			Meinrenken	
14:30					
15:00				Palmer	
15:30				Break	
16:00				Weitsman	
16:30					

	Monday 2	Tuesday 3	Wednesday 4	Thursday 5
8:30	Opening			
9:00	Pelayo	Prato	Prato	Posters
9:30				
10:00				
10:30	Break	Break	Break	Break
11:00	Cupit-Foutou	Pelayo	Cupit-Foutou	Posters
11:30				
12:00				
12:30	End	End	End	End

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Presentation

The convexity theorem for the momentum polytope was widely generalised by Fields medal and Abel prize winner Michael Atiyah, Leroy P. Steele prize winner Victor Guillemin, and Savillion Professor of Geometry at Oxford, Frances Kirwan and celebrated mathematician Shlomo Sternberg.

One of the reasons to which we can attribute this concentration of talent is that describing the image of a momentum map defined on a symplectic manifold has generated a large amount of research and remains, to this day, one of the most active areas in symplectic geometry.

The Instituto de Ciencias Matemáticas (ICMAT) is pleased to host the first Symposium on New Developments in Momentum Polytope Theory from July 26 to July 30, 2021 - followed by the first Summer School on New Developments in Momentum Polytope Theory from August 2 to August 5, 2021.

There have not been any summer schools solely focussed on momentum polytope theory, but there are researchers across the world who are or have been working on it. Therefore, this symposium and school is to bring everyone up to speed with current state-of-the-art developments, methods and applications.

We have speakers from all variety of backgrounds with one focal point: momentum polytope theory.

We hope you enjoy the symposium and the school

Scientific committee

Manuel de Leon
ICMAT-CSIC, Spain

Amna Shaddad
ICMAT-Marie Curie, Spain

Organising committee

Amna Shaddad
ICMAT-Marie Curie, Spain

Manuel de Leon
ICMAT-CSIC, Spain

Abstracts of courses

Stéphanie Cupit-Foutou

Ruhr-Universität Bochum, Germany

Hamiltonian multiplicity-free manifolds

The first lecture deals with some basic definitions and results on Hamiltonian multiplicity-free manifolds acted on by a connected compact Lie group. In the second lecture, we shall focus on the momentum polytopes of the compact connected Hamiltonian multiplicity-free manifolds and some of their generalizations.

Alvaro Pelayo

Universidad Complutense de Madrid, Spain

Introduction to symplectic geometry of integrable Hamiltonian Systems

this minicourse will give an overview of some of the basic concepts of symplectic geometry and the theory of integrable Hamiltonian systems, with an emphasis on the construction of symplectic invariants.

Elisa Prato

Università degli Studi di Firenze, Italy

Nonrational convex polytopes from a symplectic viewpoint

The Delzant construction assigns a symplectic toric manifold to each smooth convex polytope. If the polytope is simple and rational, but not smooth, the Delzant construction yields symplectic toric orbifolds. Symplectic toric quasifolds, on the other hand, were introduced in 1999 in order to extend this construction to any simple convex polytope. If the polytopes are not rational, these spaces are highly singular, but still retain a lot of the properties of their smooth/orbifold counterparts. In particular, they possess a beautiful atlas, whose charts are given by quotients of open subsets of $\mathbb{R}^{2n}$ modulo the action of countable subgroups of the standard torus T^n.

In these lectures, we begin by recalling the Delzant construction and we apply it to a number of examples. We pass on to describing the extension of this construction to the nonrational case. We discuss several examples: the quasisphere, a one parameter family of spaces that generalise Hirzebruch surfaces, the quasifolds associated to the tiles of certain nonperiodic tilings, and to the regular dodecahedron. We conclude by describing the symplectic reduction and the symplectic cutting operations in this setting.

Abstracts of talks

Alexey Bolsinov

Loughborough University, UK

Integrable systems with spherical singularities

Integrable systems with spherical singularities possess two remarkable properties: 1) they admit a torus action generated by continuous functions which are smooth everywhere except for singular fibers, 2) singular fibes are smooth Lagrangian submanifolds (e.g. Lagrangian spheres). The image of the momentum map for such a system is a polytop whose (combinatorial and affine) properties are essentially different from those of Delzant polytops. The main issue to discuss is quite tradiional in this area: can one reconstruct the system from this polytop?

Peter Bürgisser

Technische Universität Berlin, Germany

Optimization, Complexity and Invariant Theory

Invariant and representation theory studies symmetries by means of group actions and is a well established source of unifying principles in mathematics and physics. Recent research suggests its relevance for complexity and optimization through quantitative and algorithmic questions. The goal of the talk is to give an introduction to new algorithmic and analysis techniques that extend convex optimization from the classical Euclidean setting to a general geodesic setting. We also point out surprising connections to a diverse set of problems in different areas of mathematics, statistics, computer science, and physics.
The talk is mainly based on this joint article with Cole Franks, Ankit Garg, Rafael Oliveira, Michael Walter and Avi Wigderson: http://arxiv.org/abs/1910.12375

Robert Cardona Aguilar

Universitat Politècnica de Catalunya, Spain

Integrable systems on singular symplectic manifolds

In this talk, we address the problem of local and global existence of action-angle coordinates for integrable systems in singular symplectic manifolds. Two types of singular geometric structures, whose singular locus is a hypersurface, are considered: b-symplectic manifolds [4] and folded symplectic manifolds [1]. In the b-symplectic case, integrable systems were already introduced and studied in [7]. For folded singu-larities, we introduce the corresponding Hamiltonian dynamics and the notion of integrability, generalizing folded toric actions [2, 6]. We will present some examples and prove an action-angle coordinate theorem around a regular fiber that lies in the singular hypersurface. Time permitting, we will construct examples in certain b-symplectic manifolds and exhibit topological obstructions to the existence of semi- local action-angle coordinates. The obstructions arise from the topolog- ical monodromy of the singular locus of a b-symplectic manifold. By applying a desingularization technique [5], similar statements about folded integrable systems can be deduced. This is joint work with Eva Miranda.

Federico Castillo

Universidad Católica de Chile, Chile

Effective linear restrictions on the spectrum of 1-reduced constrained operators

In the 1960's several authors characterized the set of 1-reduced density operators via a spectral characterization: the spectra must lie in a hypersimplex. We study a constrained version: we focus of 1-reduced density operators arising from density operators with a prescribed spectrum. The spectra of these operators form a moment polytope. We consider a convex relaxation where we symmetrize the polyotpe and obtain inequalities for this bigger set. Since these inequalities are essentially independent of the number of particles involved, they may be interpreted as generalized Pauli constraints. This is joint work with JP. Labbe, J. Liebert, A. Padrol, E. Philippe, and C. Schilling.

Matthias Christandl

University of Copenhagen, Denmark

Tensors: rank, entropy and entanglement

We wish to understand when a tensor s can be transformed into a tensor t by application of linear maps to its tensor legs (we then say s restricts to t). In the language of restrictions, the rank of a tensor t is given by the minimal size of a diagonal tensor restricting to t. The study of rank and restrictions are motivated by algebraic complexity theory, where the rank corresponds to the computational complexity of a bilinear map (e.g. matrix multiplication) which then is viewed as a tensor with three legs.
Interestingly, some important open problems can be formulated in terms of asymptotic properties of restriction, among them the exponent of matrix multiplication. Following the seminal work of Volker Strassen, we will therefore study whether for large n the (n+o(n))'th tensor power of s can be restricted to the n'th tensor power of t. The information-theoretic flavor of the problem is apparent and was heavily used by Strassen in conjunction with the discovery of algebraic structures (his spectral theorem).
Identifying k-leg-tensors with states of quantum systems of k particles allows us to bring tools and ideas from quantum information theory to the table, among them entanglement polytopes and quantum entropy. I will use these to construct a family of functionals - the quantum functionals - that can serve as obstructions to asymptotic restrictions. The functionals are the first of their kind applicable to all tensors, thereby solving a problem by Strassen from 1986.

Stéphanie Cupit-Foutou

Ruhr-Universität Bochum, Germany

Momentum polytopes of Kähler compact Hamiltonian multiplicity-free manifolds

As is well-known, the momentum polytope of a compact connected Hamiltonian manifold acted on by a connected compact Lie group is a rational convex polytope. My talk deals with these manifolds which are Kähler and multiplicity-free; I shall outline how they can be classified by combinatorial representation theoretical invariants involving their momentum polytopes and in particular how the latter can be characterized.

Holger R. Dullin

University of Sydney, Australia

Momentum Polytopes related to geodesic flows on spheres

Separation of variables for the geodesic flows on spheres leads to a large family of integrable systems whose integrals are defined through the separation constants. Reduction by the periodic flow of the Hamiltonian leads to integrable systems on Grassmanians. Specifically for the geodesic flown on the S^3 the reduced system defines a family of integrable systems on $S^2 \times S^2$. We show that the image of these systems under a continuous momentum map defined through the action variables has a triangle as its image. The image is rigid and does not change when the integrable system is changed within the family. Each member of the family can be identified with a point inside a Stasheff polytope. Corners of the polytope correspond to toric systems (possibly with degenerations), edges correspond semi-toric systems (in various meanings of the word), and the face cor- responds to "generic" integrable systems. A fundamental difference of this momentum map to that of a toric or semi-toric system is that the number of tori in the preimage of a non critical point may be 1, 2, or 4. The momentum map is continuous but not smooth along the images of hyperbolic singularities. The corresponding quantum problem and generalisations to higher dimensional spheres will be discussed.
This is joint work with Diana Nguyen, Sean Dawson

Gleb Koshevoy

Russian Academy of Sciences, Russia

Redundancy in string cone inequalities and potential functions on Cluster Varieties

We give a conjectural equivalence between redundancy of string cone inequalities and multiplicities in potential functions. We prove the if part of the conjecture in general and provide several examples and applications. Join work with Bea Schumann

Gabriel Larotonda

Universidad de Buenos Aires, Argentina

Finsler geometry of a compact Lie group by means of a momentum polytope.

In this talk we will discuss the geometry of a compact Lie group K which acts by Hamiltonian diffeomorphisms on a symplectic manifold M. Generalized Hofer norms on the Lie algebra of K are a nice way of giving the group K a Finsler geometry by means of the momentum polytope $\mu(M) \subset Lie(K)$. We will discuss the correspondence between geodesics in the group K and geodesics in the (infinite dimensional) Lie group of Hamiltonian diffeomorphisms of M, with its Hofer metric. We will also discuss the correspondence between the geometry of the polytope and certain qualities of the geodesics in K. This is joint work with Martin Miglioli.

Eckhard Meinrenken

University of Toronto, Canada

Convexity for group-valued moment maps

I will review the convexity theorem for Lie group valued moment maps, proved around 1998, and describe some of the more recent developments and open questions.

Pau Mir

Universitat Politècnica de Catalunya, Spain

Rigidity of Contangent lifts and Integrable Systems

The theorem by Palais on the rigidity of compact group actions can be extended to cotangent lifts. I will use this result to prove rigidity for integrable systems and their momentum polytopes on symplectic manifolds, including systems with degenerate singularities which are invariant under a torus action. This is joint work with Eva Miranda.

Eva Miranda

Universitat Politècnica de Catalunya, Spain

From a b-Delzant theorem to geometric quantization and back

In this talk I will present a Delzant theorem for b-Hamiltonian toric actions on compact symplectic manifolds.
The only 2-dimensional toric manifolds are the b-sphere and the b-torus and the only invariants of their classification are the connected components of the critical set, the regularized Liouville volume and the modular weights associated to each connected component. All these invariants can be read off the image of the b-moment map. For higher dimensions, the classification of b-toric manifolds is also given in terms of b-polytopes.
One of the applications of this classification scheme is that of quantization. In this set-up, we will generalize a theorem by Guillemin-Sternberg which identifies Bohr-Sommerfeld leaves with fibers of the integral points in the image of the moment map and connect it to a sheaf-theoretical approach to Geometric Quantization suggested by Kostant.
Last but not least, we will retake the formal geometric quantization scheme for b-manifolds (see Jonathan Weitsman's talk).

Joseph Palmer

University of Illinois at Urbana-Champaign, US

Extending circle actions to integrable systems

A Hamiltonian circle action on a symplectic manifold is generated by a single real valued function. Given a compact four dimensional symplectic manifold with a Hamiltonian circle action generated by a function J, when can we find another function H such that (J,H) forms an integrable system? What can we say about the properties of the resulting integrable system (J,H)?
We show that such an H always exists, and moreover that H can be chosen so that all singularities of (J,H) are non-degenerate except for some finite number of parabolic orbits. The case that (J,H) is a toric integrable system was studied by Karshon, and the case that (J,H) is a semitoric integrable system was studied by Hohloch-Sabatini-Sepe-Symington. Due to the topic of this symposium we will emphasize the role that moment polytopes play in this story. This work is joint with S. Hohloch.

Alvaro Pelayo

Universidad Complutense de Madrid, Spain

Symplectic and Spectral Geometry of Integrable Systems

Integrable systems form a special class of dynamical systems with many conserved quantities and they play a prominent role in both mathematics and physics. In this talk first I will discuss some classical and recent results on symplectic geometry of finite dimensional classical integrable Hamiltonian systems, and then I will make some connections with the spectral geometry of their quantum counterparts.

Jonathan Weitsman

Northeastern University, US

Towards a polarization-free quantization

We show how a variant of geometric quantization which is free of a choice of polarization may be defined, and also possible problems with this method.

Chris Woodward

Rutgers University, US

Counting disks in a cubic surface via moment polytopes

The count of twenty-seven spheres in a cubic surface by Cayley-Salmon was a prominent result of nineteenth century mathematics. I will explain this computation, and a more modern computation of the count of twenty-one disks in the cubic surface bounding a Lagrangian torus from the point of view of moment polytopes and tropical geometry. The count plays a role in the construction of the mirror to the cubic surface. This is joint with Sushmita Venugopalan.

Abstracts of posters

Sean Dawson

University of Sydney, Australia

Monodromy in Prolate Spheroidal Harmonics

We show that separating the free particle in R3in prolate spheroidal coordinates gives an integrable system that exhibits quantum monodromy. Using an analogue of the Laplace-Runge-Lenz vector we show that this classical Liouville integrable system is symplectically equivalent to the C. Neumann system.
We explore the various quantum and classical properties of this system. By finding the momentum map, we show that this is a generalised semi-toric system with a non-degenerate focus-focus point and so there is monodromy in both the classical and the quantum systems. Additionally, we investigate the spheroidal wave functions (the eigenfunctions of the corresponding two commuting operators on L2(S2)) as well as the resulting joint spectrum and action map. To our knowledge, this is the first explicit demonstration that quantum monodromy exists in a class of classically known special functions. A similar phenomena also appears when separating the geodesic flow in prolate coordinates on S3. To appear in Stud. Appl. Math. (2021), arXiv:2001.11270
This is joint work with Holger Dullin and Diana Nguyen.

Ronan Kerr

Loughborough University, UK

Geodesic Flow on Spheres and Bending Flows on Polygon Spaces

https://www.icmat.es/congresos/2021/mompolytopes_school/Poster/Ronan_Kerr_Poster.pdf

Anastasia Matveeva

UPC, Spain

Singular symplectic geometry: moment maps and reduction

In this poster we will start some introduction to singular symplectic geometry. In particular, we study b- and bm-symplectic manifolds (that can be seen as symplectic manifolds with boundary) together with the associated group actions. Focusing on moment map theory we are going to obtain an analogue of Hamiltonian reduction for bm-symplectic manifolds. This is joint work with prof. Eva Miranda.

Martin Miglioli

University of Buenos Aires, Argentina

TBC

TBC

Diana Nguyen

University of Sydney, Australia

Integrable Systems arising from Separation on S^3

It is known from Schöbel and Veselov [CMP 337, p1255] that the algebraic variety of separable coordinates on S^3 can be represented as a Stascheff polytope. The most general of these are the ellipsoidal coordinates and the 5 degenerate systems are prolate, oblate, Lame-subgroup reduction, spherical and cylindrical. In this presentation we investigate the various integrable systems arising from separating the geodesic flow on the round sphere S^3 in these coordinates. By symplectic reduction of the geodesic Hamiltonian we obtain corresponding integrable systems on the reduced space $S^2 \times S^2$. We show that the image of the corresponding action map for each system has a Delzant triangle as the boundary even though most of the systems is the family are not toric.
Resounding similarities are found between the momentum maps of our systems and those from the geodesic flow on an ellipsoid and the Neumann problem. We also show preliminary results from our study of the corresponding quantum systems.
This is joint work with Sean Dawson and Holger Dullin.

Ood Shabtai

Tel Aviv University, Israel

Pairs of spectral projections of spin operators

We study the semiclassical behavior of an arbitrary bivariate polynomial, evaluated on certain spectral projections of spin operators, and contrast it with the behavior of the polynomial when evaluated on random pairs of projections.

Xiudi Tang

University of Toronto, Canada

Symplectic excision

We study which subsets can be excised from a symplectic manifold symplectically. Joint work with Yael Karshon.

PRESENTED BY THE ICMAT DIFFERENTIAL - GEOMETRY AND GEOMETRIC MECHANICS GROUP

The Instituto de Ciencias Matemáticas is a mixed research center consisting of the Consejo Superior de Investigaciones Científicas – CSIC and three Madrid universities: the Universidad Autónoma de Madrid (UAM), the Universidad Carlos III de Madrid (UC3M), and the Universidad Complutense de Madrid (UCM).

Participant list

Casey Blacker
Euler International Mathematical Institute and Saint Petersburg State University

Alexey Bolsinov
Loughborough University

Narasimha Chary Bonala
Ruhr University of Bochum

Peter Buergisser
TU Berlin

Robert Cardona
Universitat Politècnica de Catalunya

Federico Castillo
Pontificia Universidad Catolica

Stéphanie Cupit-Foutou
Ruhr-Universität Bochum

Martin de Diego David
ICMAT, CSIC Spain

Sean Dawson
The University of Sydney

Manuel de León
ICMAT-CSIC

Holger Dullin
University of Sydney

Marta Farre Puiggali
Antwerp University

Patrick Iglesias-Zemmour
The Hebrew University of Jerusalem

Ronan Kerr
Loughborough University

Manuel Lainz
ICMAT

Gabriel Larotonda
Universidad de Buenos Aires and CONICET

Peter Littelmann
University of Cologne

José Manuel López Alonso
University Complutense of Madrid

Asier López Gordón
Instituto de Ciencias Matemáticas (ICMAT)

José Manuel López Alonso
University Complutense of Madrid

David Martínez Campos
Universidad Autónoma de Madrid

Anastasia Matveeva
UPC

Eckhard Meinrenken
University of Toronto

Pau Mir
Universitat Politècnica de Catalunya

Eva Miranda
UPC-CRM-Observatoire de Paris

Henrik Müller
University of Cologne

Diana Nguyen
The University of Sydney

Joseph Palmer
University of Illinois at Urbana-Champaign and University of Antwerp

Elisa Prato
Università degli Studi di Firenze

Francisco Santos Leal
Universidad de Cantabria

Ood Shabtai
Tel Aviv University

Amna Shaddad
ICMAT - Marie Curie

Kanak Sharma
Jawaharlal Nehru University

Kevin Shu
Georgia Insitute of Technology

Valdemar Tsanov
Ruhr-University Bochum

Tobias Waedt
Ruhr-University Bochum

Jonathan Weitsman
Northeastern University

Chris Woodward
Rutgers University

Registration: https://www.icmat.es/congresos/2021/mompolytopes_symposium/registration.php
https://www.icmat.es/congresos/2021/mompolytopes_school/registration.php

Zoom Details: ICMAT Uhlenbeck is inviting you to a scheduled Zoom meeting.
Topic: Zoom meeting invitation - New Developments in Momentum Polytope Theory Time: 26 Jul. 2021 09:00 a.m. Madrid
Join the Zoom meeting
https://us06web.zoom.us/j/86011784538?pwd=c1VHd3lCLzhtREt3SFgyd2Y3ME10QT09
Meeting ID: 860 1178 4538 Access code: 839473






MARIE CURIE
ACTIONS