Sasakian manifolds are odd-dimensional counterparts of Kahler manifolds in even dimensions, with K-contact manifolds corresponding to symplectic manifolds. It is an interesting problem to find obstructions for a compact manifold to admit such types of structures and in particular, to construct K-contact manifolds which do not admit Sasakian structures.

In dimension 5, we translate the question about K-contact 5-manifolds to constructing symplectic 4-manifolds containing disjoint symplectic surfaces of positive genus spanning the homology. We give a new construction that produces a simply connected K-contact 5-manifold.

The question on Sasakian 5-manifolds translates to the existence of algebraic surfaces containing disjoint complex curves of positive genus spanning the homology. We conjecture that this is not possible for $b_1=0$, $b_2>1$. We prove this conjecture for some cases where the curves have small genus.

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