The main theme of the talk is the dynamics of Hamiltonian pseudo-rotations, i.e., Hamiltonian diffeomorphisms of a closed symplectic manifold M having the minimum number of periodic points. This is an interesting and important class of maps and, as has been established recently, there is a strong relation between the symplectic topological properties of M and the dynamics of pseudo-rotations of M, going far beyond periodic orbits. In this talk we discuss new connections between pseudo-rotations and holomorphic curves, recently found in a joint work with Erman Cineli and Viktor Ginzburg.