The free Banach lattice generated by a lattice

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A Banach lattice is a vector lattice L that is also a Banach space and for all $x, y \in L$, $|x| \leq |y| \ \Rightarrow\ \|x\| \leq \|y\|$

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- $L^p(\mu)$ with $f \leq g$ iff $f(x) \leq g(x)$ for almost x.

Sublattices, ideals and quotients

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- Y is a Banach sublattice if it is closed linear subspace that is moreover closed under operations $\vee$, $\wedge$.

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- Y is an ideal if moreover, if $f \in Y$ and $|g| \leq |f|$ then $g \in Y$. This makes X/Y a Banach lattice.

The free Banach lattice generated by a set A

Definition (de Pagter, Wickstead 2015)

We say that $F = FBL(A)$ if there is an inclusion map $A \longrightarrow F$ such that every bounded map $A \longrightarrow X$ extends to a unique Banach lattice homomorphism $FBL(A) \longrightarrow X$ of the same norm.

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It exists and is unique up to isometries.
For $a \in A$, take $\delta_a : [-1,1]^A \longrightarrow \mathbb{R}$ the evaluation function.

Theorem (de Pagter, Wickstead; Avilés, Rodríguez, Tradacete)

The free Banach lattice generated by a set A is the closure of the vector lattice generated by $\{\delta_a : a \in A\}$ in $\mathbb{R}^{[-1,1]^A}$ under the norm

$$\|f\| = \sup \left\{ \sum_{i=1}^{m} |f(x_i^*)| \ : \ \sup_{a\in A} \sum_{i=1}^{m} |x_i^*(a)| \leq 1 \right\}$$

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Definition (Avilés, Rodríguez, Tradacete 2018)

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For $x \in E$, take $\delta_x : E^* \longrightarrow \mathbb{R}$ the evaluation function.

Theorem (Avilés, Rodríguez, Tradacete)

The free Banach lattice generated by E is the closure of the vector lattice generated by $\{\delta_x : x \in E\}$ in $\mathbb{R}^{E^*}$ under the norm

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The free Banach lattice generated by a lattice $\mathbb{L}$

Definition

A lattice $\mathbb{L}$ is distributive if $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$ and $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ for every $x, y, z \in \mathbb{L}$.

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Definition (Avilés, R. A. 2018)

Given a lattice $\mathbb{L}$, the free Banach lattice generated by $\mathbb{L}$ is a Banach lattice F together with a lattice homomorphism $\phi : \mathbb{L} \longrightarrow F$ such that for every Banach lattice X and every bounded lattice homomorphism $T : \mathbb{L} \longrightarrow X$, there exists a unique Banach lattice homomorphism $\hat{T} : F \longrightarrow X$ such that $T = \hat{T} \circ \phi$ and $||\hat{T}|| = ||T||$.

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- For the existence one can take the quotient of $FBL(\mathbb{L})$ by the closed ideal $\mathscr{I}$ generated by the set

$$\{\delta_{x\vee y} - \delta_x \vee \delta_y,\ \delta_{x\wedge y} - \delta_x \wedge \delta_y : x, y \in \mathbb{L}\},$$

where, for $x \in \mathbb{L}$,

$$\delta_x : [-1,1]^{\mathbb{L}} \longrightarrow [-1,1]$$

is the map given by $\delta_x(x^*) = x^*(x)$ for every $x^* \in [-1,1]^{\mathbb{L}}$, together with the lattice homomorphism

$$\phi : \mathbb{L} \longrightarrow FBL(\mathbb{L})/\mathscr{I}$$

given by $\phi(x) = \delta_x + \mathscr{I}$ for every $x \in \mathbb{L}$.

A description of the free Banach lattice generated by a lattice

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Given $f \in \mathbb{R}^{\mathbb{L}^*}$, define

$$\|f\|_* = \sup\{\sum_{i=1}^{n} |f(x_i^*)| : n \in \mathbb{N},\ x_1^*, \dots, x_n^* \in \mathbb{L}^*,\ \sup_{x \in \mathbb{L}} \sum_{i=1}^{n} |x_i^*(x)| \leq 1\}.$$

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Theorem (Avilés, R. A.)

Consider $FBL_*\langle\mathbb{L}\rangle$ to be the Banach lattice generated by $\{\dot{\delta}_x : x \in \mathbb{L}\}$ inside the Banach lattice of all functions $f \in \mathbb{R}^{\mathbb{L}^*}$ with $\|f\|_* < \infty$, endowed with the norm $\|\cdot\|_*$ and the pointwise operations. Then $FBL_*\langle\mathbb{L}\rangle$, together with the assignment $\phi(x) = \dot{\delta}_x$ is the free Banach lattice generated by $\mathbb{L}$.

A description of the free Banach lattice generated by a lattice

Idea of the proof

The Banach lattice homomorphism

$$R_{\mathscr{I}} : FBL(\mathbb{L})/\mathscr{I} \longrightarrow FBL_*\langle\mathbb{L}\rangle$$

given by $R_{\mathscr{I}}(f + \mathscr{I}) = f|_{\mathbb{L}^*}$ for every $f + \mathscr{I} \in FBL(\mathbb{L})/\mathscr{I}$ is an isometry such that $R(\delta_x + \mathscr{I}) = \dot{\delta}_x$.

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$$\|f|_{\mathbb{L}^*}\|_* \leq \|f\|_{\mathscr{I}} := \inf\{\|g\| : f - g \in \mathscr{I}\}.$$

- How to prove that $\|f\|_{\mathscr{I}} \leq \|f|_{\mathbb{L}^*}\|_*$?.

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Idea of the proof

- $\mathbb{L}$ **finite:** $FBL(\mathbb{L})$ consists exactly of all the positively homogeneous continuous functions on $[-1,1]^{\mathbb{L}}$ (De Pagter and Wickstead).

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- $\mathbb{L}$ **finite:** $FBL(\mathbb{L})$ consists exactly of all the positively homogeneous continuous functions on $[-1,1]^{\mathbb{L}}$ (De Pagter and Wickstead).
- $\mathbb{L}$ **infinite:**

- Reduction to the finite case supposing that f can be written as $f = P(\delta_{x_1}, \ldots, \delta_{x_n})$ for some $x_1, \ldots, x_n \in \mathbb{L}$, where P is a formula that involves linear combinations and the lattice operations $\vee$ and $\wedge$.

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- If $\mathbb{F}_0 \subset \mathbb{L}$, with $\mathbb{L}$ distributive and $\mathbb{F}_0$ finite, then there exists a finite sublattice $\mathbb{F}_1 \subset \mathbb{L}$ such that for every lattice $\mathbb{M}$ and every lattice homomorphism $y^* : \mathbb{F}_1 \longrightarrow \mathbb{M}$ there exists a lattice homomorphism $z^* : \mathbb{L} \longrightarrow \mathbb{M}$ such that $z^*|_{\mathbb{F}_0} = y^*|_{\mathbb{F}_0}$.

Chain conditions on the free Banach lattice of a linear order

Definition

A Banach lattice X satisfies the countable chain condition (ccc), if whenever $\{f_i : i \in I\}$ are positive elements and $f_i \wedge f_j = 0$ for all $i \neq j$, then we must have that $|I|$ is countable.

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Let $\mathbb{L}$ be a **linearly ordered set** and $FBL\langle\mathbb{L}\rangle = FBL_*\langle\mathbb{L}\rangle$ the free Banach lattice generated by $\mathbb{L}$. Then,

Theorem (Avilés, R. A.)

$FBL\langle\mathbb{L}\rangle$ has the countable chain condition if and only if $\mathbb{L}$ is order-isomorphic to a subset of the real line.

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The key of the proof is the following lemma:

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1. $\mathbb{L}$ is order-isomorphic to a subset of the real line.
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3. For every uncountable family of triples
$$\mathscr{F} = \{\{x_1^i, x_2^i, x_3^i\} : x_1^i, x_2^i, x_3^i \in \mathbb{L},\ x_1^i < x_2^i < x_3^i,\ i \in J\}$$
there exist $i \neq j$ such that $x_1^i \leq x_2^j \leq x_3^i$ and $x_1^j \leq x_2^i \leq x_3^j$.

Projective Banach lattices

Definition

A Banach lattice P is projective if whenever X is a Banach lattice, J a closed ideal in X and $Q : X \longrightarrow X/J$ the quotient map, then for every Banach lattice homomorphism $T : P \longrightarrow X/J$ and $\varepsilon > 0$, there is a Banach lattice homomorphism $\hat{T} : P \longrightarrow X$ such that $T = Q \circ \hat{T}$ and $\|\hat{T}\| \leq (1+\varepsilon)\|T\|$.

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- $FBL\langle\mathbb{L}\rangle$ for a finite lattice $\mathbb{L}$ (Avilés, R. A.).

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- $FBL\langle\mathbb{L}\rangle$ for an infinite linear order $\mathbb{L}$ (Avilés, R. A.).
- c_0 and $FBL[c_0]$ (Avilés, Martínez-Cervantes, R. A.).

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