**PROGRAM:**

## 60 Years Alberto Ibort Fest Classical and Quantum Physics: Geometry, Dynamics and Control

**5 - 9 March, 2018**

### MANUEL ASOREY

Title: **Bulk-Edge dualities in Topological Matter**

**Abstract:** Topological matter is a new state of matter which includes topological quantum phases, topological insulators and topological semimetals.
The field has undergone an impressive development in the last years, motivated by its potential applications to spintronics and quantum computation. The conducting properties of the topological materials are characterized by topological invariants. These invariants can be formulated in terms of bulk or edge topological properties. The correspondence between both types of topological invariants can be explicitly shown in some cases.

### AIYALAM BALACHANDRAN

**Title:** The Gauss Law: A Tale

**Abstract:** The Gauss law plays a basic role in gauge theories, enforcing gauge invariance and creating edge states and superselection sectors .This talk surveys these aspects of the Gauss law in QED, QCD and nonlinear G/H models.It is argued that nonabelian super selection rules are spontaneously broken. That is the case with SU(3) of color which is spontaneously broken to U(1) x U(1). Nonlinear G/H models are reformulated as gauge theories and the existence of edge states and superselection sectors in these models is also established.

### ALFREDO BAUTISTA

**Title:** A way to build a conformal boundary of a spacetime based on light rays: the three-dimensional case

**Abstract:** Given a spacetime $M$, a *light ray* is the image of a inextensible null geodesic. The set $\mathcal{N}$ of light rays in $M$ is, under some conditions, a differentiable manifold equipped with a canonical contact structure $\mathcal{H} \subset T\mathcal{N}$. In [1], R. Low suggests the construction of a boundary for $M$ as a set of “endpoints” of light rays using objects of $\mathcal{N}$ and $\mathcal{H}$. We study the 3–dimensional case obtaining that, with some general hypotheses, this *L*–boundary permits to extend $M$ as a smooth manifold with boundary $\overline{M} = M \cup \partial M$. Also, we characterize the extensions of $M$ in order to coincide, in some sense, with the *L*–boundary. This study can be used as a first step to face the case of general dimension.

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[3] A. Bautista, A. Ibort, J. Lafuente. On the space of light rays of a space- time and a reconstruction theorem by Low. Class. Quantum Grav. 31 (2014) 075020.

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[5] A. Bautista, A. Ibort, J. Lafuente, R. Low. A conformal boundary for space-times based on light-like geodesics: The 3–dimensional case. J. Math. Phys. 58 (2017) 022503.

**JOSÉ F. CARIÑENA**

**Title:** Killling vector fields and quantisation of natural Hamiltonians

**Abstract:** The usual canonical prescription ordinarily made for the obtention of the quantum Hamiltonian operator for a classical system leads to some ambiguities in situations beyond the simplest ones and these ambiguities arise unavoidably when the configuration space has non-zero curvature, as well as in systems in Euclidean space but with a position-dependent mass. A recently proposed method to circumvent this difficulty for natural Hamiltonians will be described. The idea is not to quantise the coordinates and their (classical) conjugate momenta (which is where the ambiguities could arise), but to work directly with Killing vector fields and associated Noether momenta in order to get in some unambiguous way the corresponding Hamiltonian operator. The examples of one-dimensional position-dependent mass systems and motions on constant curvature surfaces will be used to illustrate the method.

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**DARIUSZ CHRUSCINSKI**

**Title:** Conditions for legitimate memory kernel master equation

**Abstract:** I provide sufficient conditions for the memory kernel governing non-local master equation which guarantee legitimate (completely positive and trace-preserving) dynamical map. It turns out that these conditions provide a natural parameterizations of the dynamical map being a generalization of Markovian semigroup. This parametrization is defined by a pair of maps -- monotonic quantum operation and completely positive map -- and it is shown that such class of maps cover almost all known examples from Markovian semigroup, semi-Markov evolution up to collision models and their generalization.

**FLORIO MARIA CIAGLIA**

**Title:** The space of quantum states, relative entropies and metric tensors.

**Abstract**: The habit does not make the monk....the algebraic dress of quantum mechanics hides a beautiful geometrical lingerie that I will try to uncover during the talk.
In this context, I will briefly outline how we may think of the space of quantum states S as being a non-commutative version of classical probability theory, that is, how to look at quantum states as non-commutative versions of probability distributions.
Then, we will see how the complex general linear group GL(n, C) and the unitary group U(n) act on S partitioning it into the disjoint union of orbits, and we will discover the beautiful and highly rich geometry of the manifolds of isospectral quantum states - the orbits of U(n) - using it as a point of departure in order to look for geometrical structures on the manifold of invertible quantum states - the orbit of GL(n,C) which is the primary object of quantum information theory.
These geometrical structures will be families of quantum metric tensors satisfying the monotonicity property, and we will see how it is possible to extract these families from families of quantum relative entropies satisfying the data processing inequality.
The explicit example of the (huge) two-parameter family of quantum relative entropies known as $\alpha z$-Rényi-relative will be fully worked out.
A covariant, coordinate-free, geometrical formalism will be the background spacetime in which we will move.

**ALBERTO ENCISO**

**Title:** The holographic principle in the AdS/CFT correspondence and Einstein metrics with prescribed conformal infinity

**Abstract:**
In this talk we will consider a couple of mathematical problems about wave propagation in asymptotically AdS spaces. In particular, we will discuss a suitable local wellposedness

theorem for the Einstein equations that ensures the existence of Einstein metrics with prescribed conformal geometry. This is related to the holographic principle in the AdS/CFT correspondence. The talk is based on joint work with Niky Kamran, Arick Shao and Bruno Vergara.

**FERNANDO FALCETO**

**Title:** Toeplitz determinants in Mathematical Physics.

**Abstract:** One can say that Alberto's research originates from the fruitful crossbreeding of Mathematics and Physics. In this talk, on the occasion of his 60th birthday, I will cover a topic that has largely benefited from both, Mathematical and Physical contributions. I will review the role played by Toeplitz matrices and Toeplitz determinants in two particular areas of Physics, the Ising model and the entanglement entropy of fermionic chains.
We will see how, not only the mathematical results have boosted the resolution of these problems, but also the physicist's intuitions have produced new conjectures that have been proven to be true.
We will present new results on block Toeplitz determinants and some conjectures that can be supported by strong numerical evidences.

**JANUSZ GRABOWSKI**

**Title:** $\mathbb{Z}_2^n$**-supermanifolds**

**Abstract:**
The concept of a $\mathbb{Z}_2^n$--supermanifold will be presented with its local model made of formal power series with $\mathbb{Z}_2^n$--gradation and $\mathbb{Z}_2^n$--commutation rules.
The classical Batchelor-Gawedzki theorem says that any smooth supermanifold is (non-canonocally) diffeomorphic to the `superization' $\Pi E$ of a vector bundle $E$
It is also known that this result fails in the complex analytic category.
We show that any smooth $\mathbb{Z}_2^n$--supermanifold is (non-canonocally) diffeomorphic to the `superization' $\Pi E$ of an n-fold vector bundle $E$. The latter can be chosen split.

**JOSÉ M. GRACIA-BONDÍA**

**Title:** The strange simplicity of the last particle's structure

**Abstract:** Wigner's classification of particles in his famed Ann Math 1939 article contains a spectrum of particles (bosons or fermions with unbounded helicity, usually misnomed of "continuous spin", and here called simply WP), which are routinely discarded in the textbooks, in spite of possessing perfectly positive energy.
A reason for that may be Yngvason's theorem (1970): no point-local quantum fields can be associated to them. However, Yngvason himself later (2004) proved that string-local quantum fields can accommodate the WP.
The talk will tackle the basics of WP theory, describing their remarkable kinematical properties, apparently not previously come to the fore.

**MAREK KUS**

**Title:** Symplectic geometry of entanglement

**Abstract: I** will give a review of a geometric approach to classification and examination of quantum correlations in composite systems. Since quantum information tasks are usually achieved by manipulating spin and alike systems or, in general, systems with a finite number of energy levels, classification problems are treated in frames of linear algebra. We proposed to shift attention to a geometric description. Using geometric invariant theory and momentum map geometry we propose a way to divide the space of all classes of entanglement of states of composite systems into a finite number of physically meaningful families. We provide an algorithm for classifying entangled states making use of the convexity of an appropriate momentum map and critical sets of its norm square (with a physical interpretation of the total variance of a state). An additional refinement of the classification is provided by the Ness stratification of the null cone by the moment map. Moreover, we note that the Morse index at the critical set of the total variance of state has an interpretation of the number of directions in which entanglement increases. Finally, we introduce a measure of entanglement as a square root of the total variance of state at the critical point and explain its geometric meaning.

**FERNANDO LLEDÓ**

**Title:** Amenability notions around Roe algebras

**Abstracts:** There is a classical mathematical theorem (based on the work by Banach and Tarski) that implies the following shocking statement: An orange can be divided into finitely many pieces, these pieces can be moved and rearranged in such a way to yield two oranges of the same size as the original one. In 1929 J. von Neumann recognized that one of the reasons underlying the Banach-Tarski paradox is the fact that on the unit ball there is an action of a discrete subgroup of isometries that fails to have the property of amenability.
In this talk we will approach the concept of amenability and paradoxcial decompositions from very different perspectives: we will analyze metric, purely algebraic and operator algebraic aspects. In particular, we will define the class of Foelner C*-algebras in terms of a net of unital completely positive maps from the algebra to matrices that are asymptotically multiplicative in a weak sense. The unifying picture for these notions are the so-called uniform Roe-algebras which were introduced by John Roe in the study of index theory on non-compact manifolds.

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2. P. Ara, K. Li, F. Lledó and J. Wu, Amenability and uniform Roe algebras, J. Math. Anal. Appl. 459 (2018) 686-716.

**FRANCO MAGRI**

**Title:** Eighteen years later

**Abstract.** Eighteen years ago I had the opportunity of writing a joint paper with Alberto and Beppe Marmo on “Bihamiltonian structures and Stackel separability” . The paper was a first attempt to unravel the intriguing connection between two different classes of dynamical systems: the bihamiltonian systems on one side, and the separable systems of Classical Mechanics on the other side. The underlying idea was that these two classes of systems coincide in a certain sense. In this talk I wish to make the point on the present understanding of this problem.

**JUAN CARLOS MARRERO**

**Title:** On a new affine formulation of Hamiltonian classical field theories

**GIUSEPPE MARMO**

**Title:** Classical Trajectories to Quantum Commutation Relations: A Scientific Journey

**Abstract:** Starting from classical trajectories one may build a second order vector field.A covariant description in terms of a "potential function"gives rise to the inverse problem in the calculus of variations. The existence of alternative Lagrangian or Hamiltonian descriptions gives rise to alternative quantum commutation relations.Some consequences will be briefly discussed.

**MARGARITA A. MAN’KO and VLADIMIR MAN’KO**

**Title:** Quantum Suprematism with Triadas of Malevich's Squares
identifying spin (qubit) states; new entropic inequalities

**Abstract:** For arbitrary N-level atom states, the density matrix elements are expressed in terms of a set of probability distributions describing the set of ``classical coin'' positions. For one qubit, the states are identified with three probability distributions [1] and illustrated by three squares on the plane called ``Triada of Malevich's Squares'' [2].
Using this approach, called the quantum suprematism picture, new entropic inequalities are obtained for density matrix elements of qudits and N-level atom states. Arbitrary quantum observables are bijectively mapped on the set of classical random variables, and formulas for quantum statistics of the observables are expressed in terms of the classical-like statistics of random variables.

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**EDUARDO MARTÍNEZ**
**Title:** TBA

**MIGUEL A. MARTÍN DELGADO**

**Title:** “Modern Aspects of Quantum Physics and Topology”

**Abstract:** Some modern developments of topology in quantum physics are reviewed, making emphasis on the role of topology in modern approaches to building quantum computers based on quantum error correction and fault-tolerant quantum computation. This provides new perspectives for near-future quantum computers.

**EVA MIRANDA**

**Title:** Alberto Ibort, $\omega^99$ and Jacobi manifolds

**Abstract:** In June 99, a small group of symplectic geometers "made in Spain" met in Lisbon in a conference called $\omega^99$. During that conference Alberto Ibort fascinated us all again with his unifying ideas in Geometry and Physics combining old and new techniques. The group GESTA (Geometria Simpléctica con Técnicas Algebraicas) was born in this spirit that summer under his enthusiastic lead. Some of the dreams we had in 99 have become true and many interesting collaborations have blossomed in the meanwhile.

In this talk, I will present joint work with Cédric Oms strongly influenced by several conversations with Alberto Ibort and which tries to reconcile the "old and new".
The study of singular symplectic manifolds was probably initiated by Arnold in 87. Later on, Radko classified stable Poisson structures in dimension 2 and in later joint work with Guillemin and Pires, we observed the higher dimensional case can be treated as a generalization of symplectic geometry by extending the de Rham complex including log-type singularities. In this talk we will explore the odd-dimensional analogue of these geometries by extending contact manifolds to the singular setting and considering them as Jacobi manifolds.

**FEDELE LIZZI**

**Title:** Dimensional Deception

**Abstract:** We study the dimensional aspect of the geometry of quantum spaces. Introducinga physically motivated notion of the scaling dimension, we study in detail the model based on a fuzzy torus. We show that for a natural choice of a deformed Laplace operator, this model demonstrates quite non-trivial behaviour: the scaling dimension flows from 2 in IR to 1 in UV. Unlike another model with the similar property, the so-called Horava-Lifshitz model, our construction does not have any preferred direction.
The dimension flow is rather achieved by a rearrangement of the degrees of freedom. In this respect the number of dimensions is deceptive. Some physical consequences are discussed.

**JOSÉ G. LLAVONA**

**Title:** Representation of polynomials on Banach lattices and their application to the multilinear Hausdorff problem of moments

**Abstract:** The main result of this talk is a representation theorem for homogeneous orthogonally additive polynomials on Banach lattices. The representation theorem is used to study, the linear span of the set of zeros of homogeneous real-valued orthogonally additive polynomials, and the multilinear Hausdorff problem of moments.

**JUAN MANUEL PÉREZ-PARDO**

**Title:** Quantum Control on the Boundary

**Abstract:** Schroedinger equation is a linear evolution equation. The problem of controlling a finite dimensional quantum system is therefore a well understood problem where one can apply the classical theory of control. However, applying such ideas to the infinite dimensional setting is far from being straightforward. For instance, one immediately encounters difficulties with the definition of the dynamical Lie algebra due to the appearance of unbounded operators.

In spite of the technical difficulties, the latter bring also new and interesting possibilities to the theory of quantum control. We will introduce notions of controllability suited for the infinite-dimensional situation and discuss how one can control the state of a system by means of changing its boundary conditions.

**EDITH PADRÓN**

**Title:** Encoding symplectic fiberwise actions on complete G-Lagrangian fibrations

**Abstract:** It is well-known that an action of a Lie group $G$ on a manifold $M$ can be lifted to a fiberwise $G$-action on the cotangent bundle $T^*Q$ of $Q$ such that this new action is symplectic.But, it is the only possibility? The answer is not. We will show that there are other possibilities and they are encoded by una certain cohomology group. We will extend this result for fiberwise actions on complete $G$-Lagrangian fibrations.

**DANIEL PERALTA**

**Title:** A problem of Berry and knotted zeros in the eigenfunctions of the harmonic oscillator.

**Abstract:** In 2001, motivated by finding eigenfunctions of the hydrogen atom whose nodal sets form torus knots, Sir Michael Berry conjectured the same was true for the harmonic oscillator and asked whether any finite link could be
realized as the nodal set of an eigenfunction to some quantum system. In this talk we give a proof that answers both these questions in the affirmative. Specifically, we show that any

finite link in Euclidean space can be realized (up to a global diffeomorphism) as the union of connected components of the nodal set of a harmonic oscillator eigenfunction. We will see that the high energy asymptotics of the harmonic oscillator's eigenfunctions and the wealth of different solutions to the Helmholtz equation play key roles in this proof. Time permitting, I will also show that an analogous result holds for eigenfunctions of the hydrogen atom, which matches Berry's original setting. This is joint work with Alberto Enciso and David Hartley.

**FRANCISCO PRESAS**

**Title:** Open books in contact geometry.

**Abstract:** We will explain the famous Giroux' correspondence between open book decompositions modulus stabilizations and contact structures modulus diffeomorphisms. It will be apparent that it was really close to Ibort-Martínez-Presas previous results. Some special position results and applications of them will be introduced.

**MIGUEL ÁNGEL RODRÍGUEZ**

**Title:** Spin chains: thermodynamics and criticality

**Abstract:** The thermodynamics and critical behavior of su(m,n) supersymmetric spin chains of Haldane–Shastry type with a chemical potential term are analyzed in detail for some specific examples. The partition function, which is expressed in a closed form, allows us to write an analytic expression for the free energy per site in the thermodynamic limit and its low-temperature behavior determines the critical regions and central charges. It is also observed that the bosonic or fermionic densities can undergo first-order phase transitions at T = 0. Joint work with F. Finkel, A. González-López and I. León.

**NARCISO ROMÁN-ROY**

**Title:** Properties of multisymplectic manifolds.

**Abstract:** This lecture is devoted to review some of the main properties of multisymplectic geometry.
After reminding the standard definition of "multisymplectic manifold" and its relevance, we introduce its characteristic submanifolds, the canonical models, and other relevant kinds of multisymplectic manifolds, such as those where the existence of Darboux-type coordinates is assured.
The Hamiltonian structures that can be defined in these manifolds are also studied, as well as other important properties; in particular, their invariant forms and the characterization by automorphisms.

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**LUIS VELÁZQUEZ**

**Title**: Recurrence and topological phases in quantum walks

**Abstract:** Quantum walks appeared in the early nineties as a quantum version of random walks. Their increasing interest was initially tied to their role in the design of quantum computation algorithms. Nevertheless, the simplicity and tuneability of quantum walks has also made them versatile platforms for the study of complex quantum systems, from which they capture essential aspects such as dynamics in random environments, transport properties in many body systems, topological phases of matter, or the nanoscale behaviour of biological systems.

This talk will highlight this second aspect of quantum walks, illustrated with two different but surprisingly related problems: the analysis of quantum recurrence (return properties) and the study of symmetry protected topological phases. The communicating vessel for this connection is a very well known mathematical object: Schur functions. The ability of Schur functions in codifying recurrence and topological phases not only serves as bridge between these two areas, but also uncovers the geometrical and topological meaning of certain recurrence notions with unexpected consequences, and allows us to completely classify the topological phases of 1D quantum walks of wide interest, even in the absence of translational invariance.

Furthermore, the quantum interpretation of Schur functions has returns to mathematics itself, since it opens up novel approaches to such functions, leading to new results in harmonic analysis which, for instance, have been key to close open problems in the theory of orthogonal polynomials.

The talk will summarize some of these results, fruit of joint collaborations with R.F. Werner, C. Cedzich, C. Stahl, T. Geib (U Hannover), A.H. Werner (U Copenhagen), F.A. Grünbaum, J. Wilkening (UC Berkeley) and J. Bourgain (IAS Princeton).

**PATRIZIA VITALE**

**Title:** A toy model for Double Field Theory and its generalization to Principal Chiral Models

**Abstract:** I will discuss a double geometry formulation of the isotropic rigid rotator, as a dynamical system whose configuration space is usually chosen to be the rotation group.
I will generalize the carrier space of the dynamics in terms of the double group of SU(2) and discuss the emerging geometry.

The construction is thus extended to principal chiral models as an instance of 1+1 field theory with non-Abelian duality.