Two-phase heat conductors, stationary isothermic surfaces and surfaces with the constant heat flow property

Shigeru Sakaguchi
Research Center for Pure and Applied Mathematics,
Tohoku University

This talk concerns some overdetermined problems which characterize spherical symmetry. We consider a two-phase heat conductor in $\mathbb{R}^N$ with $N \geq 2$ consisting of a core and a shell with different constant conductivities. Suppose that, initially, the conductor has temperature 0 and, at all times, its boundary is kept at temperature 1. It is shown that, if there is a stationary isothermic surface in the shell near the boundary, then the structure of the conductor must be spherical. When the medium outside the two-phase conductor has a possibly different conductivity, we consider the Cauchy problem with the initial condition where the conductor has temperature 0 and the outside medium has temperature 1. Then we show that almost the same proposition holds true. See [Sak1, Sak2]. Moreover, we also mention similar overdetermined problems where a stationary isothermic surface is replaced by a surface with the constant heat flow property. This property has been introduced by G. Alessandrini and N. Garofalo [AG], which is related to J. Serrin's seminal paper [Se], and recently by A. Savo [Sav]. This result contains a recent joint work with R. Magnanini.

References

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