Non-uniqueness of closed non-smooth hypersurfaces with constant anisotropic mean curvature and self-shrinkers of anisotropic mean curvature flow.

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We study variational problems for surfaces in the euclidean space with an anisotropic surface energy. An anisotropic surface energy is the integral of an energy density which depends on the surface normal over the considered surface. It was first introduced by Gibbs to model the equilibrium shape of a small crystal. If the energy density is constant one, the anisotropic surface energy is the usual area of the surface. The minimizer of an anisotropic surface energy among all closed surfaces enclosing the same volume is unique (up to translations) and it is called the Wulff shape. Equilibrium surfaces of a given anisotropic surface energy functional for volume-preserving variations are called surfaces with constant anisotropic mean curvature (CAMC surfaces). In general, the Wulff shape and CAMC surfaces are not smooth. If the energy density satisfies the so-called convexity condition, the Wulff shape is a smooth convex surface and closed embedded CAMC surfaces are only homotheties of the Wulff shape. In this talk, we show that if the convexity condition is not satisfied, such a uniqueness result is not always true, and also the uniqueness for self-shrinkers with genus zero for anisotropic mean curvature flow does not hold in general. These concepts and results are naturally generalized to higher dimensions.