On the level curves and the critical points of the solutions to the $H_R = H_L$ surface equation

Magdalena Caballero

Spacelike surfaces in the Lorentz-Minkowski 3-dimensional space $\mathbb{L}^3$ can be endowed with another Riemannian metric, the one induced by the Euclidean space $\mathbb{R}^3$. Those surfaces are locally the graph of a smooth function $u(x, y)$ satisfying $|Du| < 1$. If in addition they have the same mean curvature with respect to both metrics, they are the solutions to a certain partial differential equation, the $H_R = H_L$ surface equation.

It is well known that the only surfaces that are simultaneously minimal in $\mathbb{R}^3$ and maximal in $\mathbb{L}^3$ are open pieces of helicoids and of spacelike planes, [3]. The key of the proof of this result consists in showing that the level curves of such surfaces are lines.

In this talk we consider the general case of spacelike graphs with the same mean curvature with respect to both metrics, obtaining several interesting results by focusing on the behavior of its level curves and its critical points.

References

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