Multi-scale spacetimes: from theory to phenomenology

Standard Model, gravitational waves and CMB

Gianluca Calcagni

Instituto de Estructura de la Materia (IEM) – CSIC

IEM

CSIC
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

August 30th, 2016

Outline

1 A landscape in QG

Outline

1. A landscape in QG
2. The theory

Outline

1. A landscape in QG
2. The theory
3. SM and gravity

Outline

1. A landscape in QG
2. The theory
3. SM and gravity
4. Phenomenology

Outline

1 A landscape in QG

2 The theory

3 SM and gravity

4 Phenomenology

1/30– Motivations

- Dimensional flow: Changing behaviour of correlation functions, spacetime with scale-dependent "dimension" ($d_{\rm H}$, $d_{\rm S}$, ...). $d < 4$ in the UV. Universal feature in QG ['t Hooft 1993; Carlip 2009; G.C. PRL 2010] (perturbative QG, asymptotic safety, CDT, HL gravity, non-commutative spacetimes, LQG, spin foams, GFT).

1/30– Motivations

- Dimensional flow: Changing behaviour of correlation functions, spacetime with scale-dependent "dimension" ($d_{\rm H}$, $d_{\rm S}$, $\dots$). $d < 4$ in the UV. Universal feature in QG ['t Hooft 1993; Carlip 2009; G.C. PRL 2010] (perturbative QG, asymptotic safety, CDT, HL gravity, non-commutative spacetimes, LQG, spin foams, GFT).
- Dim. flow and UV finiteness? Power-counting renormalizability (gravity)?

1/30– Motivations

- Dimensional flow: Changing behaviour of correlation functions, spacetime with scale-dependent "dimension" ($d_{\rm H}$, $d_{\rm S}$, ...). $d < 4$ in the UV. Universal feature in QG ['t Hooft 1993; Carlip 2009; G.C. PRL 2010] (perturbative QG, asymptotic safety, CDT, HL gravity, non-commutative spacetimes, LQG, spin foams, GFT).
- Dim. flow and UV finiteness? Power-counting renormalizability (gravity)?
- Theory and phenomenology (from particle physics to cosmology) of fractal spacetimes? Very preliminary results in the 1980s [Svozil 1986; Eynk 1989a,b; Müller, Schäfer 1986a,b].

2/30– ABC of multi-scale spacetimes

G.C. EPJ C **76**, 181 (2016) [arXiv:1602.01470]

A. Dimensional flow occurs: [A1] At least two of the dimensions $d_{\rm H}$, $d_{\rm S}$, and $d_{\rm W}$ vary. [A2] Flow is continuous from the IR to a UV cut-off. [A3] Flow occurs locally (prevents false positive).

2/30– ABC of multi-scale spacetimes

G.C. EPJ C **76**, 181 (2016) [arXiv:1602.01470]

A. Dimensional flow occurs: [A1] At least two of the dimensions $d_{\rm H}$, $d_{\rm S}$, and $d_{\rm W}$ vary. [A2] Flow is continuous from the IR to a UV cut-off. [A3] Flow occurs locally (prevents false positive).

B. Integer dimension observed at a finite number of points (e.g., UV and IR extrema).

2/30– ABC of multi-scale spacetimes

G.C. EPJ C **76**, 181 (2016) [arXiv:1602.01470]

A. Dimensional flow occurs: [A1] At least two of the dimensions $d_{\rm H}$, $d_{\rm S}$, and $d_{\rm W}$ vary. [A2] Flow is continuous from the IR to a UV cut-off. [A3] Flow occurs locally (prevents false positive).

B. Integer dimension observed at a finite number of points (e.g., UV and IR extrema).

C. If $d_{\rm W} = 2d_{\rm H}/d_{\rm S}$ and $d_{\rm S} \leq d_{\rm H}$ at all scales, weakly multi-fractal spacetime.

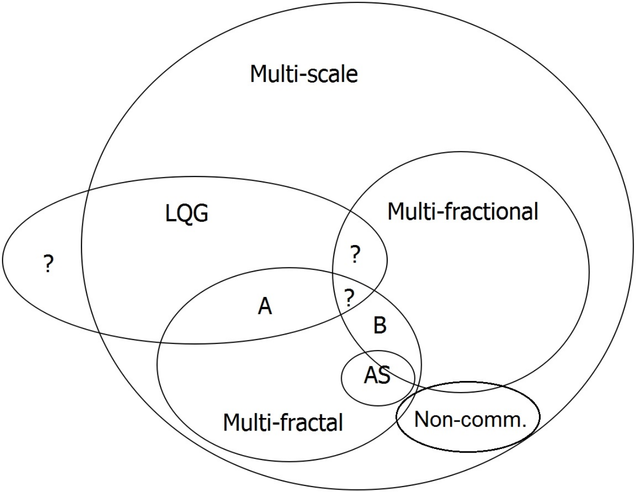
2/30– ABC of multi-scale spacetimes

G.C. EPJ C **76**, 181 (2016) [arXiv:1602.01470]

A. Dimensional flow occurs: [A1] At least two of the dimensions $d_{\rm H}$, $d_{\rm S}$, and $d_{\rm W}$ vary. [A2] Flow is continuous from the IR to a UV cut-off. [A3] Flow occurs locally (prevents false positive).
B. Integer dimension observed at a finite number of points (e.g., UV and IR extrema).
C. If $d_{\rm W}=2d_{\rm H}/d_{\rm S}$ and $d_{\rm S}\leq d_{\rm H}$ at all scales, weakly multi-fractal spacetime.
D. A geometry is a strongly multi-fractal spacetime if, in addition of satisfying A–C, it is nowhere differentiable.

3/30– Landscape of multiscale theories

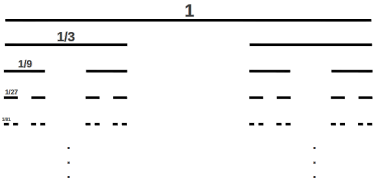
G.C. EPJC 2016 (arXiv:1602.01470)

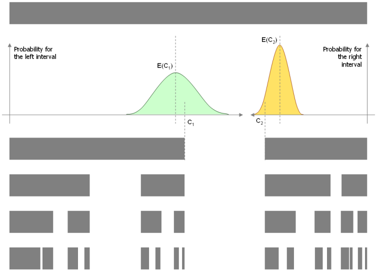


4/30– Example of fractal: Cantor set

Deterministic

Random





Outline

1 A landscape in QG

2 The theory

3 SM and gravity

4 Phenomenology

5/30–
Proposal and results

G.C., Nardelli, Rodríguez, Arzano, Magueijo, Ronco, . . . 2010–2016

5/30– Proposal and results

G.C., Nardelli, Rodríguez, Arzano, Magueijo, Ronco, . . . 2010–2016

- Dimensional flow at structural level via a change of integro-differential structure (“irregular” geometries).

5/30– Proposal and results

G.C., Nardelli, Rodríguez, Arzano, Magueijo, Ronco, . . . 2010–2016

- Dimensional flow at structural level via a change of integro-differential structure ("irregular" geometries).
- Captures the effective dynamics and the dim. flow of some non-commutative, QG, and VSL models.

5/30–

Proposal and results

G.C., Nardelli, Rodríguez, Arzano, Magueijo, Ronco, . . . 2010–2016

- Dimensional flow at structural level via a change of integro-differential structure ("irregular" geometries).
- Captures the effective dynamics and the dim. flow of some non-commutative, QG, and VSL models.
- Power-counting renormalizability (gravity)?

5/30– Proposal and results

G.C., Nardelli, Rodríguez, Arzano, Magueijo, Ronco, . . . 2010–2016

- Dimensional flow at structural level via a change of integro-differential structure (“irregular” geometries).
- Captures the effective dynamics and the dim. flow of some non-commutative, QG, and VSL models.
- Power-counting renormalizability (gravity)?
- Independent approach with a lot of exotic physics (cosmology, particle physics, discrete geometry,. . .) and easily falsifiable predictions (many experimental constraints), much more easily than QG.

6/30– Multi-fractional theories in a nutshell

G.C. 2012–2016

6/30– Multi-fractional theories in a nutshell

G.C. 2012–2016

$$\int \mathrm{d}^D x\, \mathcal{L}[\partial_x, \phi^i] \to \int \mathrm{d}^D q(x)\, \mathcal{L}[\mathcal{D}_x, \phi^i]$$

6/30– Multi-fractional theories in a nutshell

G.C. 2012–2016

$$\int d^D x\, \mathcal{L}[\partial_x, \phi^i] \to \int d^D q(x)\, \mathcal{L}[\mathcal{D}_x, \phi^i]$$

$$q(x) = \left(x + \frac{\ell_*}{\alpha}\left|\frac{x}{\ell_*}\right|^\alpha\right) F_\omega\left(\ln\left|\frac{x}{\ell_{\rm Pl}}\right|\right), \qquad \ell_*^0 = t_*,\ \ell_*^i = \ell_*$$

6/30– Multi-fractional theories in a nutshell

G.C. 2012–2016

$$\int d^D x\, \mathcal{L}[\partial_x, \phi^i] \to \int d^D q(x)\, \mathcal{L}[\mathcal{D}_x, \phi^i]$$

$$q(x) = \left(x + \frac{\ell_*}{\alpha}\left|\frac{x}{\ell_*}\right|^\alpha\right) F_\omega\left(\ln\left|\frac{x}{\ell_{\rm Pl}}\right|\right)\,, \qquad \ell_*^0 = t_*,\ \ell_*^i = \ell_*$$

Different choices of symmetries:

1. Ordinary derivatives: $\mathcal{D}_x = \partial_x$.
2. Weighted derivatives: $\mathcal{D}_x = (\partial_x q)^{-1/2}\partial_x[(\partial_x q)^{1/2}\,\cdot\,]$.
3. q-derivatives (multifractal): $\mathcal{D}_x = \partial_q = (\partial_x q)^{-1}\partial_x$.
4. Fractional derivatives (multifractal): $\mathcal{D}_x = \partial_x^\alpha$.

7/30– Example 1: Fractional measure

Represents random fractals.

$$\mathrm{d}\varrho_\alpha(x) = \mathrm{d}^D x\, v_\alpha(x) = \mathrm{d}^D x \prod_\mu \frac{|x^\mu|^{\alpha-1}}{\Gamma(\alpha)}$$

7/30– Example 1: Fractional measure

Represents random fractals.

$$\mathrm{d}\varrho_\alpha(x) = \mathrm{d}^D x\, v_\alpha(x) = \mathrm{d}^D x \prod_\mu \frac{|x^\mu|^{\alpha-1}}{\Gamma(\alpha)}$$

"Geometric" coordinates:

$$q^\mu(x^\mu) = \frac{\mathrm{sgn}(x^\mu)|x^\mu|^\alpha}{\Gamma(\alpha+1)} \qquad \Rightarrow \qquad \mathrm{d}\varrho_\alpha = \mathrm{d}^D q$$

8/30– Example 1: Hausdorff dimension

Scaling property:

$$q(\lambda x) = \lambda^{D\alpha} q(x) \qquad \Rightarrow \qquad d_{\rm H} = D\alpha$$

8/30– Example 1: Hausdorff dimension

Scaling property:

$$q(\lambda x) = \lambda^{D\alpha} q(x) \qquad \Rightarrow \qquad d_{\rm H} = D\alpha$$

Same result obtained via self-similarity theorem or via operational definition as the scaling of the volume of a D-ball of radius R: $\mathcal{V}^{(D)}(R) = \int_{D\text{-ball}} \mathrm{d}\varrho_\alpha(x) \propto R^{D\alpha}$.

9/30– Example 2: Multifractional measure

9/30– Example 2: Multifractional measure

$$v_*(x) = \prod_\mu \left[\sum_n g_n v_{\alpha_n}(x^\mu)\right] .$$

9/30– Example 2: Multifractional measure

$$v_*(x) = \prod_\mu \left[\sum_n g_n v_{\alpha_n}(x^\mu)\right] .$$

Scale-dependent Hausdorff dimension.

9/30– Example 2: Multifractional measure

$$v_*(x) = \prod_\mu \left[\sum_n g_n v_{\alpha_n}(x^\mu)\right] .$$

Scale-dependent Hausdorff dimension. Simplest (but not toy) model, two terms (binomial measure):

$$I_D = I_D^{\alpha_1} + \ell_*^{D(\alpha_1-\alpha_2)} I_D^{\alpha_2} , \qquad [I_D] = -D\alpha_1 , \qquad \tfrac{1}{2} \leq \alpha_1 < \alpha_2 \leq 1 .$$

9/30– Example 2: Multifractional measure

$$v_*(x) = \prod_\mu \left[\sum_n g_n v_{\alpha_n}(x^\mu)\right] .$$

Scale-dependent Hausdorff dimension. Simplest (but not toy) model, two terms (binomial measure):

$$I_D = I_D^{\alpha_1} + \ell_*^{D(\alpha_1-\alpha_2)} I_D^{\alpha_2} , \qquad [I_D] = -D\alpha_1 , \qquad \tfrac{1}{2} \leq \alpha_1 < \alpha_2 \leq 1 .$$

$$\mathcal{V}^{(D)}(R) = \ell_*^{D\alpha_1} \left[\Omega_{D,\alpha_1} \left(\frac{R}{\ell_*}\right)^{D\alpha_1} + \Omega_{D,\alpha_2} \left(\frac{R}{\ell_*}\right)^{D\alpha_2}\right] .$$

9/30– Example 2: Multifractional measure

$$v_*(x) = \prod_\mu \left[\sum_n g_n v_{\alpha_n}(x^\mu)\right] .$$

Scale-dependent Hausdorff dimension. Simplest (but not toy) model, two terms (binomial measure):

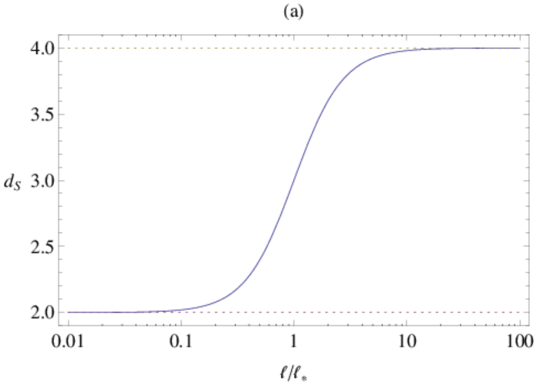
$$I_D = I_D^{\alpha_1} + \ell_*^{D(\alpha_1-\alpha_2)} I_D^{\alpha_2} , \qquad [I_D] = -D\alpha_1 , \qquad \tfrac{1}{2} \leq \alpha_1 < \alpha_2 \leq 1 .$$

$$\mathcal{V}^{(D)}(R) = \ell_*^{D\alpha_1} \left[\Omega_{D,\alpha_1}\left(\frac{R}{\ell_*}\right)^{D\alpha_1} + \Omega_{D,\alpha_2}\left(\frac{R}{\ell_*}\right)^{D\alpha_2}\right] .$$

$R \ll \ell_*$: $\mathcal{V}^{(D)} \sim R^{D\alpha_1}$

$R \gg \ell_*$: $\mathcal{V}^{(D)} \sim \tilde{R}^{D\alpha_2}$, $\tilde{R} = R\ell_*^{-1+\alpha_1/\alpha_2}$

9/30– Example 2: Multifractional measure



10/30– Example 3: Log-oscillating measure

$$q_\alpha(x) \to q_{\alpha,\omega} = c_+|x|^{\alpha+i\omega} + c_-|x|^{\alpha-i\omega}, \qquad \omega = \omega_N := \frac{2\pi\alpha}{\ln N}\,.$$

10/30– Example 3: Log-oscillating measure

$$q_\alpha(x) \to q_{\alpha,\omega} = c_+|x|^{\alpha+\mathrm{i}\omega} + c_-|x|^{\alpha-\mathrm{i}\omega}, \qquad \omega = \omega_N := \frac{2\pi\alpha}{\ln N}.$$

Summing over α, ω and imposing S to be real,

$$S = \int \mathrm{d}\varrho(x)\,\mathcal{L}\,, \qquad \mathrm{d}\varrho(x) = \prod_\mu \left[\sum_n g_n \sum_\omega \mathrm{d}q_{\alpha_n,\omega}(x^\mu)\right]$$

10/30– Example 3: Log-oscillating measure

$$q_\alpha(x) \to q_{\alpha,\omega} = c_+|x|^{\alpha+\mathrm{i}\omega} + c_-|x|^{\alpha-\mathrm{i}\omega}, \qquad \omega = \omega_N := \frac{2\pi\alpha}{\ln N}.$$

Summing over α, ω and imposing S to be real,

$$S = \int d\varrho(x)\,\mathcal{L}\,, \qquad d\varrho(x) = \prod_\mu \left[\sum_n g_n \sum_\omega dq_{\alpha_n,\omega}(x^\mu)\right]$$

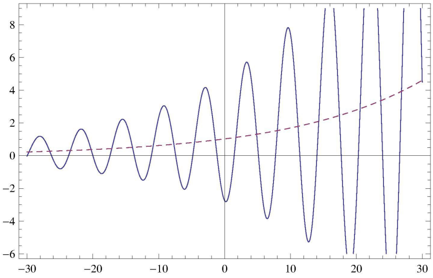
where

$$q_{\alpha,\omega}(x) = \frac{x^\alpha}{\Gamma(\alpha+1)}\left[1 + A_{\alpha,\omega}\cos\left(\omega\ln\frac{|x|}{\ell_\infty}\right) + B_{\alpha,\omega}\sin\left(\omega\ln\frac{|x|}{\ell_\infty}\right)\right]$$

$A_{\alpha,\omega}$ and $B_{\alpha,\omega} \in \mathbb{R}$. Form of measure also dictated by fractal geometry arguments.

10/30– Example 3: Log-oscillating measure

Represents deterministic (multi)fractals (integrals on self-similar fractals can be approximated by fractional integrals with $\alpha \sim d_{\rm H}$ [Ren et al. 1996–2003; Nigmatullin & Le Méhauté 2003]).



11/30– Example 3: Discrete scale invariance

Oscillatory part of ϱ log-periodic under the transformation

$$\ln\frac{|x|}{\ell_\infty}\to\ln\frac{|x|}{\ell_\infty}+\frac{2\pi n}{\omega}\,,\qquad n=0,1,2,\dots$$

11/30– Example 3: Discrete scale invariance

Oscillatory part of ϱ log-periodic under the transformation

$$\ln\frac{|x|}{\ell_\infty}\to\ln\frac{|x|}{\ell_\infty}+\frac{2\pi n}{\omega}\,,\qquad n=0,1,2,\dots$$

implying a DSI:

$$x\to\lambda_\omega^n x\,,\qquad \lambda_\omega=\exp(2\pi/\omega)\,,\qquad n=0,1,2,\dots\,.$$

11/30– Example 3: Discrete scale invariance

Oscillatory part of ϱ log-periodic under the transformation

$$\ln\frac{|x|}{\ell_\infty} \to \ln\frac{|x|}{\ell_\infty} + \frac{2\pi n}{\omega}\,, \qquad n = 0, 1, 2, \dots$$

implying a DSI:

$$x \to \lambda_\omega^n x\,, \qquad \lambda_\omega = \exp(2\pi/\omega)\,, \qquad n = 0, 1, 2, \dots\,.$$

DSIs appear in chaotic systems [Sornette 1998].

12/30– FAQ

- Hausdorff dimension
- Physical measurements
- Frames in FT
- Presentation
- Momentum space
- (Non-)relativistic bodies on fractals

SKIP $\longrightarrow$

12a/30– Hausdorff dimension: generic

$$\int dx \to \int d\varrho(x)$$

12a/30– Hausdorff dimension: generic

$$\int \mathrm{d}x \to \int \mathrm{d}\varrho(x)$$

1. Scaling property:

$$\varrho(\lambda x) = \lambda^{d_\mathrm{H}} \varrho(x)$$

12a/30– Hausdorff dimension: generic

$$\int \mathrm{d}x \to \int \mathrm{d}\varrho(x)$$

1. Scaling property:

$$\varrho(\lambda x) = \lambda^{d_\mathrm{H}} \varrho(x)$$

2. Scaling of the volume of a ball of radius R:

$$\mathcal{V} = \int_{\mathrm{ball}} \mathrm{d}\varrho(x) \propto R^{d_\mathrm{H}}.$$

12b/30– Hausdorff dimension: (multi)scale case

If $\mathrm{d}\varrho(x) = \mathrm{d}q(x) = \mathrm{d}x\, v(x)$, then

$$\mathcal{V} \sim \int_0^R \mathrm{d}x\, v(x) = q(R) \equiv \mathcal{R} = \int_0^{\mathcal{R}} \mathrm{d}q.$$

12b/30– Hausdorff dimension: (multi)scale case

If $\mathrm{d}\varrho(x) = \mathrm{d}q(x) = \mathrm{d}x\, v(x)$, then

$$\mathcal{V} \sim \int_0^R \mathrm{d}x\, v(x) = q(R) \equiv \mathcal{R} = \int_0^{\mathcal{R}} \mathrm{d}q.$$

If $q(x) \sim x^\alpha$, then $\mathcal{V} \sim R^\alpha \equiv \mathcal{R}$.

12b/30– Hausdorff dimension: (multi)scale case

If $\mathrm{d}\varrho(x) = \mathrm{d}q(x) = \mathrm{d}x\, v(x)$, then

$$\mathcal{V} \sim \int_0^R \mathrm{d}x\, v(x) = q(R) \equiv \mathcal{R} = \int_0^{\mathcal{R}} \mathrm{d}q.$$

If $q(x) \sim x^\alpha$, then $\mathcal{V} \sim R^\alpha \equiv \mathcal{R}$. **What do you measure? R or $\mathcal{R}$?**

$$d_\mathrm{H} = \frac{\mathrm{d}\ln\mathcal{V}}{\mathrm{d}\ln R} \quad \text{or} \quad d_\mathrm{H} = \frac{\mathrm{d}\ln\mathcal{V}}{\mathrm{d}\ln\mathcal{R}}\,?$$

BACK ←

12c/30– Multi-scaling and relational measurements

- Choosing measurement units is a convention. Measuring a size or a velocity by an observer in a multifractal world is not different from measuring them in a normal non-anomalous world.

12c/30– Multi-scaling and relational measurements

- Choosing measurement units is a convention. Measuring a size or a velocity by an observer in a multifractal world is not different from measuring them in a normal non-anomalous world. Example, $v_x = \Delta x/\Delta t$ vs. $v_q = \Delta q(x)/\Delta q(t)$.

12c/30– Multi-scaling and relational measurements

- Choosing measurement units is a convention. Measuring a size or a velocity by an observer in a multifractal world is not different from measuring them in a normal non-anomalous world. Example, $v_x = \Delta x/\Delta t$ vs. $v_q = \Delta q(x)/\Delta q(t)$.
- Measurements of **dimensionless** observables do discriminate! Example, $v_x(O_1)/v_x(O_2)$ vs. $v_q(O_1)/v_q(O_2)$ where $scale(O_1) \ll scale(O_2)$.

BACK ←

12d/30– Field theory: q-derivatives

$$
\begin{aligned}
S_\phi &= \int \mathrm{d}^2 q \left\{ \frac{1}{2}[\partial_{q_0(t)}\phi]^2 - \frac{1}{2}[\partial_{q_1(x)}\phi]^2 - \sum_n \lambda_n \phi^n \right\} \\
&= \int \mathrm{d}^2 x \left\{ \frac{v_1(x)}{2v_0(t)}\dot{\phi}^2 - \frac{v_0(t)}{2v_1(x)}(\partial_x\phi)^2 - \sum_n [v_0(t)v_1(x)\lambda_n]\phi^n \right\}
\end{aligned}
$$

Use integer picture to do calculations in the SM. Any “time” or “spatial” interval or “energy” are measured with q-clocks, q-rods or q-detectors. Observables must be reconverted to the fractional picture to interpret them correctly.

12e/30– Field theory: weighted derivatives

$$\mathcal{D}_x = v^{-1/2}\partial_x[v^{1/2}\cdot\,]$$

$$\begin{aligned} S_\phi &= \int \mathrm{d}^2x\, v\left\{\frac{1}{2}[\mathcal{D}_t\phi]^2 - \frac{1}{2}[\mathcal{D}_x\phi]^2 - \sum_n \lambda_n\phi^n\right\} \\ &= \int \mathrm{d}^2x\left\{\frac{1}{2}[\partial_t\varphi]^2 - \frac{1}{2}[\partial_x\varphi]^2 - \sum_n v^{1-\frac{n}{2}}\lambda_n\varphi^n\right\}, \qquad \varphi = \sqrt{v}\phi \end{aligned}$$

Use integer picture to do calculations in the SM. Time, space and energy are measured with physical clocks, rods and detectors. Observables must be reconverted to the fractional picture to interpret them correctly. BACK ←

12f/30– Presentation problem: flat space

Weight $\bar{v}(x) = v(x-\bar{x})$ has integrable sing. at $x=\bar{x}$. “Where” are we?

- Different presentations correspond to different theories (in the same class) predicting different observables. Example:

$$\Delta\bar{q}(t) = \Delta t|1+\mathcal{T}|\,,\qquad \mathcal{T} = \frac{1}{\alpha}\frac{t_*}{\Delta t}\left(\left|\frac{t_{\rm B}-\bar{t}}{t_*}\right|^{\alpha} - \left|\frac{t_{\rm A}-\bar{t}}{t_*}\right|^{\alpha}\right).$$

- Observations can discriminate among different presentations (similar to Ito vs. Stratonovich).
- Minkowski has a global “volcano” detectable by repeating the same experiment.

12g/30– Presentation problem: with gravity

- A global notion of *irregularity* exists for Minkowski embedding only in the theory with fractional derivatives:

$$\int_{x_i}^{x} dx'\, \partial_{x'} q(x'-\bar{x}) f(x') = \lim_{n\to\infty} \sum_{j=0}^{n-1} f(\tilde{x}_j+\bar{x})\,[q(x_{j+1})-q(x_j)]$$
$$\neq \lim_{n\to\infty} \sum_{j=0}^{n-1} f(\tilde{x}_j+\bar{x}_j)\,[q(x_{j+1})-q(x_j)]\,.$$

- Cosmology (gravity on a solution): $\bar{t}$ is no more special than the big bang at $t=0$.
- Gravity: a LIF centered on the observer is locally isomorphic to multi-scale Minkowski spacetime and each and every LIF has its own “volcano.”

12h/30– Momentum space: theory with q-derivatives

Invertible Fourier transform:

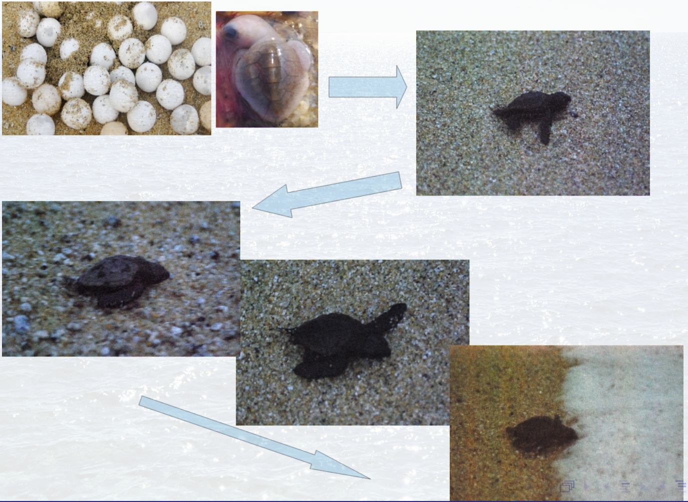
$$p(k) = \frac{1}{q(1/k)} = \frac{k}{1 + \frac{1}{\alpha}\left|\frac{E_*}{k}\right|^{\alpha-1} F_\omega(-\ln|k|)}$$

Fundamental energy scale(s), no reference to special points in space!

▸ BACK ←

12i/30– How do bodies move in fractal spacetimes?

G.C. EPJC 2016 (arXiv:1602.01470) BACK ←



13/30– Status

G.C. et al. 2012–2016 (PRL, ATMP, JHEP, JCAP, PRD, ...)

	$\Box$, $\Box^\dagger$	$\mathcal{D}^2$	$\Box_q$	$\partial^{2\alpha}$
Momentum transform	✗?	✓	✓	?
Relativistic mechanics	✓	✓	✓	?
Perturbative field theory	✓	✓	✓	✓?
QFT and SM	?	✓	✓	?
Perturbative renormalizability	?	✗	✗	✓?
Phenomenology (particles)	?	✓	✓	?
Gravity and cosmology	✓	✓	✓	?
Phenomenology (astro & cosmo)	?	✓	✓	?

Outline

1. A landscape in QG
2. The theory
3. SM and gravity
4. Phenomenology

14/30– Standard Model: weighted derivatives

G.C., Nardelli, Rodríguez-Fernández, PRD **94**, 045018 (2016) [arXiv:1512.06858]

SM usual action with the replacements

$$\partial_\mu \to \mathcal{D}_\mu\,, \qquad \text{coupling} \to \sqrt{v}\,\text{coupling}$$

14/30– Standard Model: weighted derivatives

G.C., Nardelli, Rodríguez-Fernández, PRD **94**, 045018 (2016) [arXiv:1512.06858]

SM usual action with the replacements

$$\partial_\mu \to \mathcal{D}_\mu\,, \qquad \text{coupling} \to \sqrt{v}\,\text{coupling}$$

Time and lengths in the integer picture are the usual ones: no physical changes in any sector except in QED (e only coupling mesured directly).

14/30– Standard Model: weighted derivatives

G.C., Nardelli, Rodríguez-Fernández, PRD **94**, 045018 (2016) [arXiv:1512.06858]

SM usual action with the replacements

$$\partial_\mu \to \mathcal{D}_\mu \,, \qquad \text{coupling} \to \sqrt{v}\,\text{coupling}$$

Time and lengths in the integer picture are the usual ones: no physical changes in any sector except in QED (e only coupling mesured directly).
Examples: fine-structure constant and Lamb shift

$$\alpha_{\text{QED}}(t) = \frac{\tilde{\alpha}_{\text{QED}}}{v(t)}$$

$$\Delta E \simeq \Delta E^{(0)} + \left[5\ln\frac{k_0(2,1)}{\alpha_{\text{QED}}^2 k_0(2,0)} + \frac{43}{24}\right]\frac{\alpha_{\text{QED}}^5 m}{6\pi}\left|\frac{t_*}{t}\right|^{1-\alpha_0}$$

15/30– Standard Model: q-derivatives

G.C., Nardelli, Rodríguez-Fernández, PRD **93**, 025005 (2016) [arXiv:1512.02621]

SM usual action with the replacement

$$\partial_\mu \to \partial_{q^\mu}$$

everywhere. Theory invariant under q-Poincaré transformations

$$q'^\mu(x^\mu) = \Lambda_\nu^{\ \mu} q^\nu(x^\nu) + a^\mu$$

and also under CPT.

15/30– Standard Model: q-derivatives

G.C., Nardelli, Rodríguez-Fernández, PRD **93**, 025005 (2016) [arXiv:1512.02621]

SM usual action with the replacement

$$\partial_\mu \to \partial_{q^\mu}$$

everywhere. Theory invariant under q-Poincaré transformations

$$q'^\mu(x^\mu) = \Lambda_\nu^{\ \mu} q^\nu(x^\nu) + a^\mu$$

and also under CPT.
Times and lengths in the integer picture are composite.
Example: muon lifetime

$$q^0(\tau_{\rm mu}) = \frac{1}{\Gamma} = \tau_0, \qquad \Gamma = \frac{G_{\rm F}^2 m_{\rm mu}^5}{192\pi^3} + \cdots$$

16/30– Gravity: weighted derivatives

G.C., JCAP 1312 (2013) 041 [arXiv:1307.6382]

$$ {}^1\Gamma^\rho_{\mu\nu}[g] := \tfrac{1}{2} g^{\rho\sigma}\left({}_1\mathcal{D}_\mu g_{\nu\sigma} + {}_1\mathcal{D}_\nu g_{\mu\sigma} - {}_1\mathcal{D}_\sigma g_{\mu\nu}\right), \qquad {}_1\mathcal{D} = v^{-1}\partial[v\,\cdot\,] $$

$$ \mathcal{R}^\rho{}_{\mu\sigma\nu} := \partial_\sigma {}^1\Gamma^\rho_{\mu\nu} - \partial_\nu {}^1\Gamma^\rho_{\mu\sigma} + {}^1\Gamma^\tau_{\mu\nu}\,{}^1\Gamma^\rho_{\sigma\tau} - {}^1\Gamma^\tau_{\mu\sigma}\,{}^1\Gamma^\rho_{\nu\tau} $$

16/30– Gravity: weighted derivatives

G.C., JCAP 1312 (2013) 041 [arXiv:1307.6382]

$$
{}^1\Gamma^\rho_{\mu\nu}[g] := \tfrac{1}{2} g^{\rho\sigma}\left({}_1\mathcal{D}_\mu g_{\nu\sigma} + {}_1\mathcal{D}_\nu g_{\mu\sigma} - {}_1\mathcal{D}_\sigma g_{\mu\nu}\right), \qquad {}_1\mathcal{D} = v^{-1}\partial[v\,\cdot]
$$

$$
\mathcal{R}^\rho{}_{\mu\sigma\nu} := \partial_\sigma {}^1\Gamma^\rho_{\mu\nu} - \partial_\nu {}^1\Gamma^\rho_{\mu\sigma} + {}^1\Gamma^\tau_{\mu\nu}{}^1\Gamma^\rho_{\sigma\tau} - {}^1\Gamma^\tau_{\mu\sigma}{}^1\Gamma^\rho_{\nu\tau}
$$

Length of vectors changes under parallel transport:

$$
\nabla_\sigma g_{\mu\nu} = W_\sigma g_{\mu\nu}\,, \qquad W_\mu = \partial_\mu \Phi\,, \qquad \Phi := \ln v\,.
$$

16/30– Gravity: weighted derivatives

G.C., JCAP 1312 (2013) 041 [arXiv:1307.6382]

$$ {}^1\Gamma^\rho_{\mu\nu}[g] := \tfrac{1}{2} g^{\rho\sigma} \left({}_1\mathcal{D}_\mu g_{\nu\sigma} + {}_1\mathcal{D}_\nu g_{\mu\sigma} - {}_1\mathcal{D}_\sigma g_{\mu\nu} \right) , \qquad {}_1\mathcal{D} = v^{-1}\partial[v\,\cdot\,] $$

$$ \mathcal{R}^\rho{}_{\mu\sigma\nu} := \partial_\sigma {}^1\Gamma^\rho_{\mu\nu} - \partial_\nu {}^1\Gamma^\rho_{\mu\sigma} + {}^1\Gamma^\tau_{\mu\nu} {}^1\Gamma^\rho_{\sigma\tau} - {}^1\Gamma^\tau_{\mu\sigma} {}^1\Gamma^\rho_{\nu\tau} $$

Length of vectors changes under parallel transport:

$$ \nabla_\sigma g_{\mu\nu} = W_\sigma g_{\mu\nu} , \qquad W_\mu = \partial_\mu \Phi , \qquad \Phi := \ln v . $$

$$ \begin{aligned} S &:= \frac{1}{2\kappa^2} \int \mathrm{d}^D x\, v \sqrt{-g} \left[\mathcal{R} - \omega \mathcal{D}_\mu v \mathcal{D}_\nu v - U(v) \right] + S_\mathrm{m} \\ &= \frac{1}{2\kappa^2} \int \mathrm{d}^D x\, \mathrm{e}^\Phi \sqrt{-g} \left(\mathcal{R} - \frac{9\omega}{4} \mathrm{e}^{2\Phi} \partial_\mu \Phi \partial^\mu \Phi - U \right) + S_\mathrm{m} . \end{aligned} $$

17/30– $D = 4$ Einstein and Friedmann equations

Einstein frame: $\bar{g}_{\mu\nu} = \mathrm{e}^{\Phi} g_{\mu\nu}$. $\Omega = (9\omega/4)\mathrm{e}^{2\Phi} - 3/2$.

$$\kappa^2 \bar{T}_{\mu\nu} = \bar{R}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}(\bar{R} - \mathrm{e}^{-\Phi}U) - \Omega\left(\partial_\mu\Phi\partial_\nu\Phi + \frac{1}{2}\bar{g}_{\mu\nu}\partial_\sigma\Phi\bar{\partial}^\sigma\Phi\right)$$

17/30– $D = 4$ Einstein and Friedmann equations

Einstein frame: $\bar{g}_{\mu\nu} = e^{\Phi} g_{\mu\nu}$. $\Omega = (9\omega/4)e^{2\Phi} - 3/2$.

$$\kappa^2 \bar{T}_{\mu\nu} = \bar{R}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}(\bar{R} - e^{-\Phi}U) - \Omega\left(\partial_\mu\Phi\partial_\nu\Phi + \frac{1}{2}\bar{g}_{\mu\nu}\partial_\sigma\Phi\bar{\partial}^\sigma\Phi\right)$$

Cosmology:

$$H^2 = \frac{\kappa^2}{3}\,\bar{\rho} + \frac{\Omega}{2}\frac{\dot{v}^2}{v^2} + \frac{U(v)}{6v} - \frac{\mathsf{K}}{a^2}\,,$$

$$2\dot{H} - \frac{2\mathsf{K}}{a^2} + \kappa^2(\bar{\rho} + \bar{P}) = -\Omega\frac{\dot{v}^2}{v^2}\,.$$

18/30– Gravity: q-derivatives

G.C., JCAP 1312 (2013) 041 [arXiv:1307.6382]

$$
{}^q\Gamma^\rho_{\mu\nu} := \tfrac{1}{2} g^{\rho\sigma}\left(\frac{1}{v_\mu}\partial_\mu g_{\nu\sigma} + \frac{1}{v_\nu}\partial_\nu g_{\mu\sigma} - \frac{1}{v_\sigma}\partial_\sigma g_{\mu\nu}\right),
$$

$$
{}^q R^\rho{}_{\mu\sigma\nu} := \frac{1}{v_\sigma}\partial_\sigma {}^q\Gamma^\rho_{\mu\nu} - \frac{1}{v_\nu}\partial_\nu {}^q\Gamma^\rho_{\mu\sigma} + {}^q\Gamma^\tau_{\mu\nu}\, {}^q\Gamma^\rho_{\sigma\tau} - {}^q\Gamma^\tau_{\mu\sigma}\, {}^q\Gamma^\rho_{\nu\tau}\,.
$$

Action:

$$
S = \frac{1}{2\kappa^2}\int \mathrm{d}^D x\, v\, \sqrt{-g}\,({}^q R - 2\Lambda) + S_{\rm m}\,.
$$

18/30– Gravity: q-derivatives

G.C., JCAP 1312 (2013) 041 [arXiv:1307.6382]

$$ {}^q\Gamma^\rho_{\mu\nu} := \tfrac{1}{2} g^{\rho\sigma}\left(\frac{1}{v_\mu}\partial_\mu g_{\nu\sigma} + \frac{1}{v_\nu}\partial_\nu g_{\mu\sigma} - \frac{1}{v_\sigma}\partial_\sigma g_{\mu\nu}\right) , $$

$$ {}^q R^\rho{}_{\mu\sigma\nu} := \frac{1}{v_\sigma}\partial_\sigma {}^q\Gamma^\rho_{\mu\nu} - \frac{1}{v_\nu}\partial_\nu {}^q\Gamma^\rho_{\mu\sigma} + {}^q\Gamma^\tau_{\mu\nu}\,{}^q\Gamma^\rho_{\sigma\tau} - {}^q\Gamma^\tau_{\mu\sigma}\,{}^q\Gamma^\rho_{\nu\tau} . $$

Action:

$$ S = \frac{1}{2\kappa^2}\int \mathrm{d}^D x\, v\,\sqrt{-g}\,({}^q R - 2\Lambda) + S_\mathrm{m} . $$

Einstein equations:

$$ {}^q R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}({}^q R - 2\Lambda) = \kappa^2\, {}^q T_{\mu\nu} . $$

19/30– Cosmology

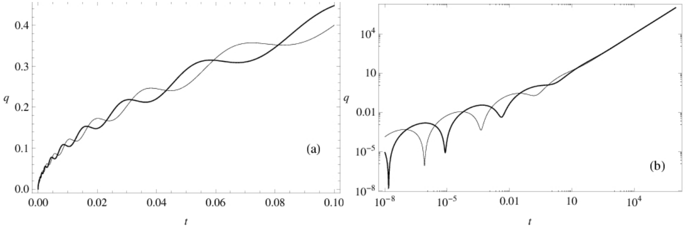
$$\frac{H^2}{v^2} = \frac{\kappa^2}{3}\rho + \frac{\Lambda}{3} - \frac{\mathsf{K}}{a^2}\,,$$
$$\dot{\rho} + 3H(\rho + P) = 0\,.$$

Ordinary slow-roll approximation unnecessary.

19/30– Cosmology

$$\frac{H^2}{v^2} = \frac{\kappa^2}{3}\rho + \frac{\Lambda}{3} - \frac{\mathsf{K}}{a^2}\,,$$
$$\dot{\rho} + 3H(\rho + P) = 0\,.$$

Ordinary slow-roll approximation unnecessary.



20/30– Inflationary spectra

General behaviour:

$$P_{\rm s,t} = \mathcal{A}_{\rm s,t}\tilde{k}^{n-1} \sim \mathcal{A}_{\rm s,t}\left(\frac{k}{k_*}\right)^{\alpha(n-1)}[F_\omega(\ln k)]^{1-n}\,.$$

Scale invariance without slow-roll approximation and a log-oscillating pattern.

$\to$ Spacetime discrete at scales $\sim \ell_\infty$ (totally disconnected?).

Visible effect of this geometry: not "holes" in the fabric of spacetime but a logarithmic modulation of the power spectrum of primordial fluctuations!

Outline

1. A landscape in QG
2. The theory
3. SM and gravity
4. Phenomenology

21/30– Observational constraints (absolute, $\alpha_0, \alpha \ll \frac{1}{2}$)

WEIGHTED DER.	t_* (s)	ℓ_* (m)	E_* (eV)	source
$\alpha_{\rm QED}$ quasars	—	—	—	G.C., Magueijo, Rodríguez, PRD 2014
CMB black-body	—	—	—	G.C., Kuroyanagi, Tsujikawa, JCAP 2016
Lamb shift	$< \mathbf{10^{-23}}$	$< 10^{-14}$	$> 10^{7}$	G.C., Nardelli, Rodríguez, PRD 2016(b)
measured $\alpha_{\rm QED}$	$< \mathbf{10^{-26}}$	$< 10^{-17}$	$> 10^{10}$	G.C., Nardelli, Rodríguez, PRD 2016(b)
GW and GRB	—	—	—	G.C., arXiv:1603.03046

q-DER.	t_* (s)	ℓ_* (m)	E_* (eV)	source
CMB primordial (!)	weak	weak	weak	G.C., Kuroyanagi, Tsujikawa, JCAP 2016
CMB black-body	—	—	—	G.C., Kuroyanagi, Tsujikawa, JCAP 2016
muon lifetime	$< \mathbf{10^{-13}}$	$< 10^{-5}$	$> 10^{-3}$	G.C., Nardelli, Rodríguez, PRD 2016(a)
Lamb shift	$< 10^{-23}$	$< 10^{-15}$	$> \mathbf{10^{7}}$	G.C., Nardelli, Rodríguez, PRD 2016(a)
measured $\alpha_{\rm QED}$	—	—	—	G.C., Nardelli, Rodríguez, PRD 2016(b)
GW	$< 10^{-22}$	$< 10^{-14}$	$> \mathbf{10^{7}}$	G.C., arXiv:1603.03046
GRB $\sim$	$< 10^{-32}$	$< 10^{-24}$	$> \mathbf{10^{26}}$	G.C., arXiv:1603.03046

22/30– Observational constraints ($\alpha = 1/2 = \alpha_0$)

WEIGHTED DER.	t_* (s)	ℓ_* (m)	E_* (eV)	source
$\alpha_{\rm QED}$ quasars	$< \mathbf{10^{6}}$	$< 10^{15}$	$> 10^{-23}$	G.C., Magueijo, Rodríguez, PRD 2014
CMB black-body	$< 10^{-21}$	$< 10^{-12}$	$> \mathbf{10^{3}}$	G.C., Kuroyanagi, Tsujikawa, JCAP 2016
Lamb shift	$< \mathbf{10^{-29}}$	$< 10^{-20}$	$> 10^{13}$	G.C., Nardelli, Rodríguez, PRD 2016(b)
measured $\alpha_{\rm QED}$	$< \mathbf{10^{-36}}$	$< 10^{-27}$	$> 10^{20}$	G.C., Nardelli, Rodríguez, PRD 2016(b)
GW and GRB	—	—	—	G.C., arXiv:1603.03046

q-DER.	t_* (s)	ℓ_* (m)	E_* (eV)	source
CMB primordial (!)	weak	weak	weak	G.C., Kuroyanagi, Tsujikawa, JCAP 2016
CMB black-body	$< 10^{-26}$	$< 10^{-18}$	$> \mathbf{10^{10}}$	G.C., Kuroyanagi, Tsujikawa, JCAP 2016
muon lifetime	$< \mathbf{10^{-18}}$	$< 10^{-9}$	$> 10^{2}$	G.C., Nardelli, Rodríguez, PRD 2016(a)
Lamb shift	$< 10^{-27}$	$< 10^{-19}$	$> \mathbf{10^{11}}$	G.C., Nardelli, Rodríguez, PRD 2016(a)
measured $\alpha_{\rm QED}$	—	—	—	G.C., Nardelli, Rodríguez, PRD 2016(b)
GW	$< 10^{-39}$	$< 10^{-30}$	$> \mathbf{10^{23}}$	G.C., arXiv:1603.03046
GRB $\sim$	$< 10^{-50}$	$< 10^{-42}$	$> \mathbf{10^{44}}$	G.C., arXiv:1603.03046

23/30– Example: fine-structure constant

Measured with accuracy $\frac{\delta\alpha_{\rm QED}}{\alpha_{\rm QED}} \sim 10^{-10}$.

Weighted derivatives: $\alpha_{\rm QED}(t) = \frac{\tilde{\alpha}_{\rm QED}}{v_*(t)} \Rightarrow$
$\Delta\alpha_{\rm QED} = \alpha_{\rm QED}(t)|t_*/t|^{1-\alpha_0}$

$\Delta\alpha_{\rm QED} < \delta\alpha_{\rm QED}$, $t = t_{\rm QED} = 10^{-16}$ s,

$$t_* < 10^{-16-10/(1-\alpha_0)}\ {\rm s}$$

24/30– Multi-scale spacetimes vs. QG: dispersion relations and GWs

QG and string theory (IR limit $k \ll M$):

$$E^2 \simeq k^2 \left[1 + b\left(\frac{k}{M}\right)^n\right], \qquad \Delta v = \frac{\mathrm{d}E}{\mathrm{d}k} - 1 \sim \left(\frac{E}{M}\right)^n, \qquad n = 1, 2$$

24/30– Multi-scale spacetimes vs. QG: dispersion relations and GWs

QG and string theory (IR limit $k \ll M$):

$$E^2 \simeq k^2 \left[1 + b\left(\frac{k}{M}\right)^n\right], \qquad \Delta v = \frac{\mathrm{d}E}{\mathrm{d}k} - 1 \sim \left(\frac{E}{M}\right)^n, \qquad n = 1, 2$$

GW150914 event: $|\Delta v| < 1.7 \times 10^{-18}$, $M(n=1) > 4 \times 10^4\,\mathrm{eV}$, $M(n=2) > 10^{-4}\,\mathrm{eV}$ [Arzano, G.C., PRD 2016].

24/30– Multi-scale spacetimes vs. QG: dispersion relations and GWs

QG and string theory (IR limit $k \ll M$):

$$E^2 \simeq k^2 \left[1 + b\left(\frac{k}{M}\right)^n\right], \qquad \Delta v = \frac{\mathrm{d}E}{\mathrm{d}k} - 1 \sim \left(\frac{E}{M}\right)^n, \qquad n = 1, 2$$

GW150914 event: $|\Delta v| < 1.7 \times 10^{-18}$, $M(n=1) > 4 \times 10^4\,\mathrm{eV}$, $M(n=2) > 10^{-4}\,\mathrm{eV}$ [Arzano, G.C., PRD 2016].

Viable fundamental mass $M > 10\,\mathrm{TeV}$ only if $0 < n < 0.68$ [Arzano, G.C., PRD 2016].

24/30– Multi-scale spacetimes vs. QG: dispersion relations and GWs

QG and string theory (IR limit $k \ll M$):

$$E^2 \simeq k^2 \left[1 + b\left(\frac{k}{M}\right)^n\right], \qquad \Delta v = \frac{\mathrm{d}E}{\mathrm{d}k} - 1 \sim \left(\frac{E}{M}\right)^n, \qquad n = 1, 2$$

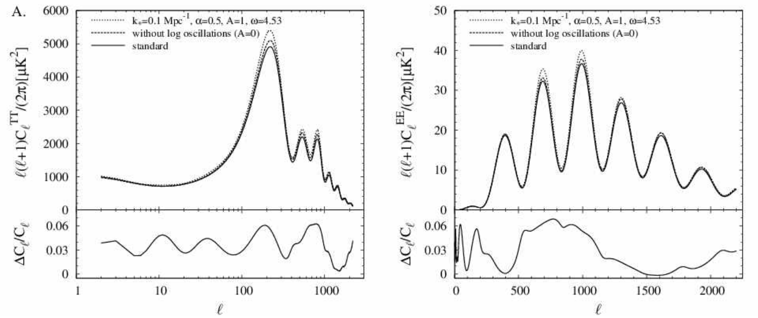
GW150914 event: $|\Delta v| < 1.7 \times 10^{-18}$, $M(n=1) > 4 \times 10^4\,\mathrm{eV}$, $M(n=2) > 10^{-4}\,\mathrm{eV}$ [Arzano, G.C., PRD 2016].

Viable fundamental mass $M > 10$ TeV only if $0 < n < 0.68$ [Arzano, G.C., PRD 2016].

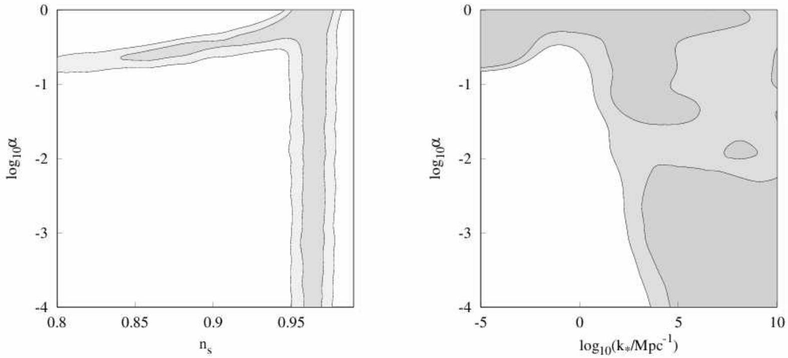
Multi-scale theory with q-derivatives can be constrained by GW alone! [G.C., arXiv:1603.03046]

$$E^2 \simeq k^2 \left[1 \pm O(1)\left(\frac{k}{E_*}\right)^{1-\alpha}\right], \qquad 0 < 1 - \alpha < 1.$$

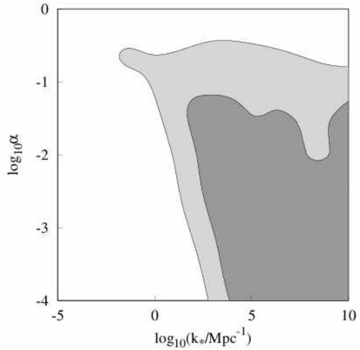
25/30– q-derivatives: CMB spectra



26/30– 2D contours without log-oscillations



27/30– 2D contours with log-oscillations



E.g. $N = 4$: **Upper bound $\alpha < 0.1$**. In general, $\alpha \lesssim 0.1 - 0.6$.

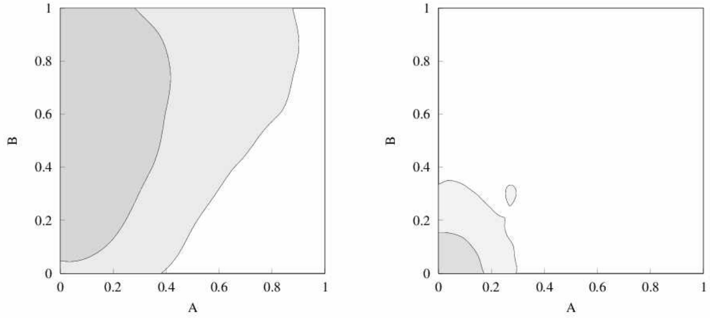
28/30– Consequences for spacetime dimension

The Hausdorff dim. $d_{\rm H}^{\rm space}=3\alpha$ of space in the UV cannot exceed

$$
\begin{aligned}
N=2:&\qquad d_{\rm H}^{\rm space}\lesssim 0.3 \qquad \text{(UV)}\\
N=3:&\qquad d_{\rm H}^{\rm space}\lesssim 1.9 \qquad \text{(UV)}\\
N=4:&\qquad d_{\rm H}^{\rm space}\lesssim 1.7 \qquad \text{(UV)}
\end{aligned}
$$

Counter-intuitive!

29/30– 2D contours with log-oscillations



Example: $N = 2$, $\alpha = 0.1, 0.5$

30/30– Upper bounds on A, B

Amplitudes of log oscillations of geometry cannot exceed

$$
\begin{aligned}
N = 2: &\qquad A < 0.3,\ B < 0.4\\
N = 3: &\qquad A < 0.3,\ B < 0.2\\
N = 4: &\qquad A < 0.4,\ B < 1.0
\end{aligned}
$$

First constraints of this kind.

どうもありがとうございました！

Thank you!

¡Muchas gracias!

Muito obrigado!

Grazie!

Danke schön!