XXV International Fall Workshop on Geometry and Physics

Madrid, August 29th-September 2nd, 2016

CSIC
Consejo Superior de Investigaciones Científicas

IEM

ICMAT
Instituto de Ciencias Matemáticas

R.S.E.F.

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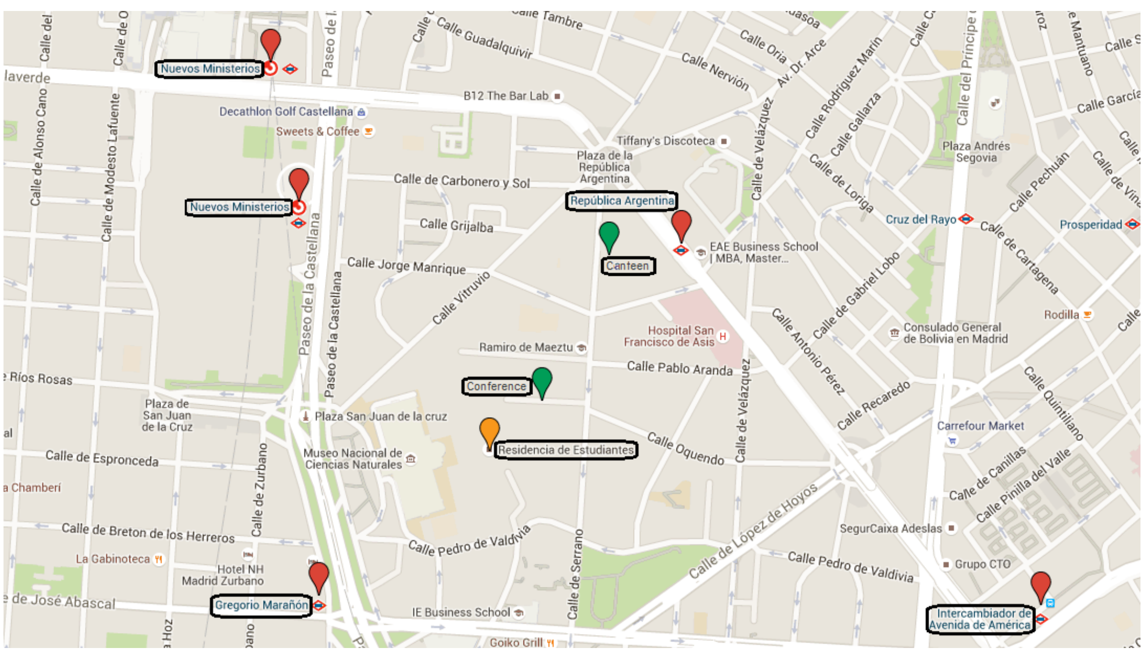
Schedule

	August 29	August 30	August 31	September 1	September 2
8.45-9.00		OPENING			
9.00-10.00		P.W. Michor	P.W. Michor	P.W. Michor	E. Miranda
10.00-11.00		P. Vitale	F. Gay-Balmaz	P.K. Townsend	Awarded poster 1 Awarded poster 1
11.00-11.30		Registration + CB	CB + Poster	CB + poll for 2 posters	Coffee break
11.30-12.30		M. Asorey	M. Asorey	M. Asorey	R. Loja
12.30-13.00		V. Salnikov	Special act	J. Grabowski	M. Domínguez
13.00-14.00		LUNCH	LUNCH	PHOTO + LUNCH	
14.00-14.30	Registration				
14.30-15.00	P.W. Michor Pre-course	M. Ortega		C. Sardón	
15.00-16.00		I. Yudin		M. Crainic	
16.00-16.30	Coffee break	CB + Poster		Coffee break	
16.30-17.30	M. Asorey Pre-course	G. Calcagni		M. Epstein F.M. Ciaglia	
17.30-18.00		J.M. Pérez-Pardo			

Venue

The conference room is located at the *Instituto de Química Física Rocasolano* (Serrano 119, Madrid) on the CSIC main campus while the coffee room and poster sesions will be held in Serrano 121. The *Residencia de Estudiantes* is two blocks away from the main venue.

The closest metro stations are *República Argentina* (line 6) and *Gregorio Marañon* (lines 7 and 10). Both *Intercambiador de transportes de Avenida de América* and *Nuevos Ministerios* commuter train station are nearby, and they are connected to *República Argentina* by metro.



Internet access

If you have an **eduroam** account just turn on the Wi-Fi of your device; it should get online automatically. We also provide a Wi-Fi network for the conference. In order to connect to it please follow these steps:

1. Turn on the Wi-Fi of your device.
2. Connect to the open network called *invitados*.
3. Most of the devices will warn you that you have to log in in order to proceed, if not, just try to access any website from your browser.
4. You should be redirected to the following site

5. Click on *Sign in* (or on *Haga click aquí* in the Spanish version).
6. Introduce the following credentials
   - User: conferenceIFWGP@csic.es
   - Password: It will be provided with the workshop documentation.

Billing information

Those of you who need the receipt for the registration fee but have not requested it yet should send an email to patricia@granadaworkshop.com with the following information:

- Name and surname of the participant.
- Final payer (usually your university or institution).
- Fiscal identification code or number (CIF in Spain) of the final payer.
- Address of the final payer.

Restaurants

From Tuesday 30th to Thursday 1st the lunch is **included** in the inscription fee and will be held at the CSIC canteen (located at Serrano 150, see map). Monday and Friday lunches are not included but you can eat at the same place.

A small list of restaurants within walking distance of the conference venue:

- Residencia de estudiantes (Calle del Pinar, 21-23) www.residencia.csic.es
- Casa de Aragón (Pl. de la República Argentina, 6) www.casadearagonenmadrid.com
- Restaurante Jai Alai (Calle de Balbina Valverde, 2) www.restaurantejaialai.com
- Goiko Grill (Calle de María de Molina, 2) www.goikogrill.com
- Mexican factory (Calle de Serrano, 93) www.mexicanfactory.es
- Garden Navarra (Calle de Zurbano, 95) www.gardennavarra.es
- Ataclub (Calle de Velázquez, 150) www.ataclub.es

Another option is to go to the city center and get into one of the many restaurants that you will find. The best options to get there are

- Take the bus 51 in Serrano 111 and in 30 minutes you will get to the last stop, *Puerta del Sol* (the city center of Madrid).
- Walk 15 minutes to the train station *Nuevos Ministerio* and take the train *Cercanías* C3 (direction *Aranjuez*) or C4 (direction *Parla*) just one stop to *Sol* (5 minutes).
- Walk to the metro station *Gregorio Marañon* and take the line 10 direction *Puerta del Sur* just 2 stops until *Tribunal*. You will be in Fuencarral, one of the most trendy streets in Madrid. Heading to *Gran Vía* you will find lots of restaurants.

Conference Dinner

The conference dinner will be held on Thursday 1st at 21.00, at the restaurant *Vargas83* (www.vargas83.com) located at Travesía Trujillos, 2. It is a 5-minutes walk from *Puerta del Sol*, where you can arrive by following the instructions given above.

Tourism

Madrid is one of the biggest cities in Europe with a huge offer for its visitors. We recommend you to visit the official tourism site www.esmadrid.com and check the propaganda brochure included in the folder. Anyhow here we include some of the most iconic places to visit

- Museo del Prado www.museodelprado.es. Open everyday, free the last two hours before closing (from Monday to Saturday it closes at 20.00, on Sundays it closes at 19.00).
- Museo Reina Sofía www.museoreinasofia.es. Closed on Tuesdays, free the last two hours before closing (21.00 everyday).
- Museo Thyssen Bornemisza www.museothyssen.org. Partially closed on Mondays.
- Parque del Retiro.
- Palacio de Cristal (inside the Retiro Park). Free everyday.
- Sol and Plaza Mayor.
- The egiptian temple dedicated to Debod. Free entrance.
- Palacio Real www.patrimonionacional.es

Minicourses

Minicourse I: Introduction to infinite dimensional differentiable manifolds

Peter W. Michor (Universität Wien, Austria)

Overview on Convenient Calculus and Differential Geometry in Infinite dimensions, with Applications to Diffeomorphism Groups and Shape Spaces.

Content:

- A short introduction to convenient calculus in infinite dimensions.
- Manifolds of mappings (with compact source) and diffeomorphism groups as convenient manifolds.
- A zoo of diffeomorphism groups on $\mathbb{R}^n$
- A diagram of actions of diffeomorphism groups.
- Riemannian geometries of spaces of immersions and shape spaces.
- Right invariant Riemannian geometries on Diffeomorphism groups.
- Uniqueness of the Fisher-Rao metric on the space of smooth densities as diffeomorphism invariant Riemannian metric.
- Arnold's formula for geodesics on Lie groups.
- Solving the Hunter-Saxton equation.
- Riemannian geometries on spaces of Riemannian metrics and pulling them back to diffeomorphism groups.
- Approximating Euler's equation of fluid mechanics on the full diffeomorphism group of $\mathbb{R}^n$.
- Robust Infinite Dimensional Riemannian manifolds, Sobolev Metrics on Diffeomorphism Groups, and the Derived Geometry of Shape Spaces.

I can react during the minicourse to wishes and interests of the audience, in the limits of my knowledge and preparation.

Preparatory and background material:

1. Wikipedia: https://en.wikipedia.org/wiki/Convenient_vector_space
2. Peter W. Michor, *Manifolds of mappings and shapes*. In: *The legacy of Bernhard Riemann after one hundred and fifty years*, Advanced Lectures of Mathematics 35, (2016) 459-486. Available at: http://www.mat.univie.ac.at/~michor/convenient-overview.pdf
3. Martin Bauer, Martins Bruveris, Peter W. Michor, *Overview of the Geometries of Shape Spaces and Diffeomorphism Groups*, Journal of Mathematical Imaging and Vision, 50, (2014) 60-97. Available at: http://www.mat.univie.ac.at/~michor/shape-overview.pdf

Minicourse II: Introduction to quantum field theory

Manuel Asorey Carballeira (Universidad de Zaragoza, Spain)

The goal of the series of lectures is to provide an abridged introduction to the foundations of Quantum Field Theory to an audience of theoretical physicists and mathematicians with a background in Geometry and Physics.

The list of topics covered includes:

- Relativistic Classical Field Theories.
- Canonical Quantization.
- The Harmonic Oscillator.
- Fock Space.
- Free Quantum Fields.
- Vacuum Energy.
- Particles versus fields.
- Foundations of Quantum Field Theory I. The Heisenberg Picture.
- Covariant Quantization.
- Causality and Propagators.
- Foundations of Quantum Field Theory II. Interacting Fields.
- Perturbation theory.
- Feynman diagrams.
- Ultraviolet divergences and its renormalization.

Finally, the role of classical fields in the quantum theory will be analyzed having in mind the case of gauge field theories.

Plenary Talks

The Gribov problem in noncommutative quantum field theory

Patrizia Vitale (Università degli Studi di Napoli Federico II, Italy)

The non trivial topology of the space of physical gauge connections for non Abelian gauge theories manifests itself in the so called "Gribov ambiguity", essentially the impossibility of choosing a global section in the principal bundle of gauge connections when the gauge group is non-Abelian. The problem is analyzed in the context of noncommutative gauge theory where, already for the gauge group $U(1)$, Gribov copies appear due to space-time noncommutativity. This is a genuine effect of noncommutative geometry which disappears when the noncommutative parameter vanishes. A similar phenomenon manifests in noncommutative scalar field theory, where local automorphisms of the star product give rise to Gribov-like copies.

Vaismann manifolds and Hard Lefschetz isomorphism

Ivan Yudin (Universidade de Coimbra, Portugal)

It is well known that the global scalar product on the space of k-forms in an oriented compact Riemannian manifold M of dimension m induces an isomorphism between the kth de Rham cohomology group $H^k(M)$ and the dual space of the $(m-k)$th de Rham cohomology group $H^{m-k}(M)$. So, using that the dimension of the de Rham cohomology groups is finite, we deduce the Poincaré-duality: the dimension of $^k(M)$ is equal to the dimension of $H^{m-k}(M)$.

In some special cases, one can define a canonical isomorphism between the vector spaces $H^k(M)$ and $H^{m-k}(M)$. For instance, if M is a compact Kähler manifold of dimension $m = 2n$ then, using the $(n-k)$th exterior power of the symplectic 2-form, one obtains an explicit isomorphism between $H^k(M)$ and $H^{m-k}(M)$.

The class of locally conformally Kähler manifolds is a natural extension of the class of Kähler manifolds and one of its most studied subclasses is that of Vaisman manifolds. In this talk I will explain how one can establish a canonical isomorphism between $H^k(M)$ and $H^{m-k}(M)$ for Vaisman manifolds.

This is a joint work with Beniamino Cappelletti-Montano, Antonio de Nicola, and Juan Carlos Marrero.

Multi-scale spacetimes, from theory to phenomenology: standard model, gravitational waves and CMB

Gianluca Calcagni (IEM-CSIC, Spain)

We review the motivations and main ingredients of multi-scale theories, where the geometry of spacetime changes with the probed scale and shows the typical properties of multi-fractals, as well as a discrete structure at Planckian scales. This paradigm, alternative to popular quantum-gravity scenarios, predicts a characteristic phenomenology that is tested by a number of experiments ranging from particle physics to astrophysics and cosmology. Experiments place tight constraints on the fundamental scales of the geometry.

A variational Lagrangian formulation for nonequilibrium thermodynamics

François Gay-Balmaz (CNRS-École Normale Supérieure, France)

We present a Lagrangian variational formulation for nonequilibrium thermodynamics. This formulation extends the Hamilton principle of classical mechanics to include irreversible processes in both discrete and continuum systems. The irreversibility is encoded into a nonlinear nonholonomic constraint given by the expression of entropy production associated to the irreversible processes involved. The introduction of the concept of thermodynamic displacement allows the definition of a corresponding variational constraint. We also present the geometric structures underlying nonequilibrium thermodynamics. We illustrate our theory with both finite and infinite dimensional examples, including mechanical systems with friction, chemical reactions, electric circuits, and reacting fluids.

Worldline time reversal and massless supermultiplets

Paul K. Townsend (DAMTP, University of Cambridge, UK)

The $4D$ Minkowski space massless superparticle action, with N-extended supersymmetry, has a worldline time-reversing symmetry corresponding to CPT invariance of the quantum theory. However, this symmetry is anomalous for odd N. The anomaly is a close analog of the parity anomaly of $3D$ electrodynamics coupled to an odd number of Majorana spinor fields. For even N it is possible to preserve CPT but there is a Kramers degeneracy when $N/2$ is odd. These results explain some well-known features of massless $4D$ supermultiplets:

1. No irreducible $N = 1$ supermultiplet is CPT self-conjugate.
2. The $N = 2$ hypermultiplet has $2^{N+1} = 8$ states whereas the $N = 4$ Maxwell supermultiplet has only $2^N = 16$ states.

Poisson manifolds of compact type

Marius Crainic (University of Utrecht, The Netherlands)

b^m-Symplectic manifolds: going to infinity and coming back

Eva Miranda Galcerán (Universitat Politècnica de Catalunya, Spain)

Several problems from celestial mechanics (like the elliptic restricted 3-body problem) and their singularities ("collisions") can be described using symplectic forms away from a critical set (known in the literature of celestial mechanics as the line at infinity or the collision manifold). In these examples the symplectic form either vanishes or goes to infinity along the critical set. It is possible to give a global description of these objects using b^m-symplectic forms (see [2]) and folded symplectic forms.

We will present some of these examples [1, 4] and we will quickly review some results concerning dynamics on these manifolds like action-angle coordinates for integrable systems and KAM results [2, 4, 5].

We will end up this talk explaining a desingularization procedure called deblogging (joint work with Victor Guillemin and Jonathan Weitsman [3]) which associates a family of symplectic forms or folded symplectic forms to a given b^m-symplectic form depending on the parity of m. Time permitting, several applications of this procedure will be discussed.

References

[1] A. Delshams, A. Kiesenhofer and E. Miranda, *Examples of integrable and non-integrable systems on singular symplectic manifolds* [arXiv:1512.08293].

[2] V. Guillemin, E. Miranda and A.R. Pires, *Symplectic and Poisson geometry on b-manifolds*, Adv. Math. 264, pp. 864-896 (2014).

[3] V. Guillemin, E. Miranda, J. Weitsman, *Desingularizing b^m-symplectic structures* [arXiv:1512.05303].

[4] A. Kiesenhofer, E. Miranda and G. Scott, *Action-angle variables and a KAM theorem for b-Poisson manifolds*, J. Math. Pures Appl. (9) 105 (2016), no. 1, 66-85.

[5] A. Kiesenhofer and E. Miranda, *Cotangent models for integrable systems*, to appear at Communications in Mathematical Physics [arXiv:1601.05041].

Isoparametric submanifolds of complex space forms

Miguel Domínguez Vázquez (ICMAT-CSIC, Spain)

A hypersurface in a Riemannian manifold is called isoparametric if it and its nearby equidistant hypersurfaces have constant mean curvature. These objects have been investigated in the 30s by Levi-Civita, Segre and Cartan, and since the 70s by many mathematicians. Their study has revealed interactions with different areas of mathematics, such as the theory of isometric actions, symmetric spaces, and mathematical physics.

The first part of the talk will review some of the basic notions and results in this area. We will then go on to discuss some recent advances towards a classification of isoparametric submanifolds in the complex projective and hyperbolic spaces.

This talk is based on joint works with José Carlos Díaz Ramos, Víctor Sanmartín López and Cristina Vidal Castiñeira.

Talks

Graded geometry in physics - above and beyond

Vladimir Salnikov (University of Luxembourg)

In this talk I will describe various instances of graded and generalized geometry, appearing naturally in theoretical physics (sigma models, gauging, symmetries of functionals) and classical mechanics.

For the first part, the key idea is that one can reformulate the property of gauge invariance in the language of equivariant Q-cohomology [1] This permits to exhibit obstructions to gauging using a nice geometric picture, and describe the symmetries of some sigma models, including the Dirac sigma model [2], which is universal in the space-time dimension 2 [3]. The formalism can be also applied to supersymmetric theories.

The second part is a work in progress related to a natural generalization of Hamiltonian systems to so-called port-Hamiltonian, that include dissipative systems and interaction. Generalized geometry, and in particular Dirac structures, turn out to be useful in the context – I will briefly sketch the direction.

References

[1] V.Salnikov, *Graded geometry in gauge theories and beyond*, Journal of Geometry and Physics, Volume 87 (2015).

[2] V.Salnikov and T.Strobl, *Dirac Sigma Models from Gauging*, Journal of High Energy Physics, (2013).

[3] A.Kotov, V.Salnikov and T.Strobl, *2d Gauge Theories and Generalized Geometry*, Journal of High Energy Physics, (2014).

Translating solitons, semi-Riemannian manifolds and Lie groups

Miguel Ortega (Universidad de Granada)

Famous solutions to the Mean Curvature Flow in Euclidean and Minkowski spaces are the translating solitons, which are submanifolds such that their mean curvature vector $\vec{H}$ satisfy $\vec{H} = v^{\perp}$, where $v \in \mathbb{R}^n$ is a fixed constant unit vector. For simpleness, it is very common to choose $v = (1, 0, \ldots, 0)$. These objects have been extensively studied.

Now, let (M, g) be a semi-Riemannian manifold, and $\varepsilon \in \{1, -1\}$ a constant. Given a map $u : M \to \mathbb{R}$, we say that its graph $F : M \to (M \times \mathbb{R}, g + \varepsilon dt^2)$ is a (vertical) translating soliton if the mean curvature vector $\vec{H}$ of F satisfies $\vec{H} = \partial_t^{\perp}$. As a first result, when the graph is semi-Riemannian, we obtain the PDE that function u must satisfy.

Next, we let a Lie group Σ act on M. On one hand, we will show the relation with principal fiber bundles with structure group Σ and totally geodesic fibers. On the other hand, if the Lie group acts on M in such a way that the space of orbits M/Σ is diffeomorphic to an open interval $I \subset \mathbb{R}$, the PDE can be tranformed in a ODE, and we develop a technique to obtain new examples. Some of them were already known, like the rotationally symmetric ones in the Euclidean and Minkowski spaces, but others are new.

Quantum mechanics and information geometry

Juan Manuel Pérez-Pardo (INFN-Sezione Napoli)

It was realised by S. Amari and cowokers, cf. [1] and references therein, that differential geometry is a useful means to study statistical models. In particular, they possess the structure of a Riemannian manifold, with metric the Fisher-Rao metric, and a pair of so-called dual connections. The geometric formulation of Quantum Mechanics allows to use those ideas and, therefore, to study its statistical structure from a geometric viewpoint. It turns out that the Fisher-Rao metric is "built-in" in a natural way within Quantum Mechanics. We will exploit these geometric relationship and study the role that well-developed notions such as complete integrability and totally geodesic embeddings play in this context.

References

[1] S. Amari and H. Nagaoka, *Methods of Information Geometry*, Oxford University Press (2000).

Hamilton-Jacobi theory on Nambu-Poisson manifolds

Cristina Sardón (ICMAT-CSIC)

The Hamilton-Jacobi theory is a formulation of Classical Mechanics equivalent to other formulations as Newton's equations, Lagrangian or Hamiltonian Mechanics. It is particulary useful for the identification of conserved quantities of a mechanical system. The primordial observation of a geometric Hamilton-Jacobi equation is that if a Hamiltonian vector field X_H can be projected into the configuration manifold by means of a 1-form dW, then the integral curves of the projected vector field X_H be transformed into integral curves of X_H provided that W is a solution of the Hamilton-Jacobi equation. This interpretation has been applied to multiple settings: in nonholomic, singular Lagrangian Mechanics and classical field theories.

Our aim is to apply the geometric Hamilton-Jacobi theory to systems endowed with a Nambu-Poisson structure. The Nambu-Poisson structure has shown its interest in the study physical systems described in terms of several, compatible Hamiltonian descriptions. In this way, we will apply our theory to two interesting examples in the Physics literature: the third-order Kummer-Schwarz equations and a system of n copies of a first-order differential Riccati equation. From these examples, we retrieve the original Nambu bracket in three dimensions and a generalization of the Nambu bracket to n dimensions, respectively.

References

[1] J.F. Cariñena, J. Grabowski, J. de Lucas, C. Sardón, LieDirac systems and Schwarzian equations. J. Differential Equations 257, 23032340 (2014).

[2] T. Curtright, C. Zachos, Classical and Quantum Nambu Mechanics, Phys. Rev. D 68, 085001 (2003).

[3] R. Ibañez, M. de León, J.C. Marrero, D. Martín de Diego, Dynamics of generalized Poisson and Nambu-Poisson brackets, J. Math. Phys. 38, 2332 (1997).

[4] Y. Nambu, Generalized Hamiltonian dynamics, Phys. Rev. D 7, 2405-2412 (1973).

[5] L. Takhtajan, On foundation of the generalized Nambu Mechanics, Comm. Math. Phys. 160, 295-315 (1994).

Mechanics on graded bundles

Janusz Grabowski (Institute of Mathematics Polish Academy of Sciences)

We introduce the concept of a *graded bundle*, generalizing that of a vector bundle and being a purely even version of an N-manifold of Ševera, its *linearization* and the concept of *holonomic vectors* therein, and a double structure of this kind. The latter is based on the fundamental observation that graded bundles have a convenient equivalent description as *homogeneity structures*, i.e. manifolds with a smooth action of the monoid $(\mathbb{R}, \cdot)$ of multiplicative reals. Then, we study *weighted groupoids* and *weighted algebroids*, being Lie groupoids and algebroids, respectively, equipped with compatible graded bundle structures. Finally, using the fact that standard examples of graded bundles are higher tangent bundles $T^n M$, we present applications in mechanics with higher order Lagrangians.

References

[1] A.J. Bruce, K. Grabowska and J. Grabowski, *Higher order mechanics on graded bundles*, J. Phys. A 48 (2015), 205203 (32pp).

[2] A.J. Bruce, K. Grabowska and J. Grabowski, *Graded bundles in the category of Lie groupoids*, SIGMA 11 (2015), 090.

[3] A.J. Bruce, K. Grabowska and J. Grabowski, *Linear duals of graded bundles and higher analogues of (Lie) algebroid*, J. Geom. Phys. 101 (2016), 71-99.

[4] J. Grabowski and M. Rotkiewicz, *Higher vector bundles and multi-graded symplectic manifolds*, J. Geom. Phys. 59 (2009), no. 9, 1285-1305.

[5] J. Grabowski and M. Rotkiewicz, *Graded bundles and homogeneity structures*, J. Geom. Phys. 62 (2012), no. 1, 21-36.

Material groupoids and algebroids

Marcelo Epstein (University of Calgary)

Although not widely used, the notion of *material groupoid* is one of the most intuitive ideas in Continuum Mechanics. Given a material body $\mathcal{B}$ and considering some material property, we ask the following question: Do two points $X, Y \in \mathcal{B}$ share this particular material property? If they do, we draw an arrow from X to Y . If they don't, no arrow is drawn. Since "having the same property" is surely an equivalence relation, every point $X \in \mathcal{B}$ should be assigned a loop-shaped arrow. Moreover, for every arrow drawn from X to Y there should also be an arrow drawn from Y to X. Finally, if there is an arrow from X to Y and another arrow from Y to Z, there should also be an arrow from X to Z. The set $\mathcal{Z}$ of all these arrows

constitutes the *material groupoid associated with the chosen material property.* The body B is the base of the groupoid. As a collection of arrows, the groupoid is un- doubtedly a concrete geometric realization of the constitutive nature of a material body. For example, if every pair of points is joined by an arrow, the body is *uniform* with respect to the chosen property. In the mathematical terminology, we speak of a *transitive* groupoid. At the other extreme, if no two points are connected by an arrow (so that the only surviving arrows are the loops), we have a *totally intransitive groupoid.*

Clearly, these rough intuitive notions can be properly formalized; but, for now, let us assume that in some sense the arrows depend smoothly on the pairs of points. By this we mean that (thinking of a transitive groupoid, for definiteness) if X is close to X' and Y is close to Y' then the arrow from X to Y is also close to the arrow from X' to Y' and that this dependence of the arrow on the source-target pair is also differentiable, in some sense to be made precise later. We speak then of a *Lie groupoid.* We can look at the vicinity of a loop and calculate thereat the derivative of the arrow, that is, the linear part of the change in the arrow. In a manner completely and justifiably reminiscent of the passage from a Lie group to its Lie algebra (of "infinitesimal generators"') we obtain a new geometric structure called the *Lie algebroid* associated with the Lie groupoid. The elucidation of the physical meaning of this construct is one of the objectives of this work. In particular, we investigate the relation between the material algebroid and the notion of *material connection* used in the theory of continuous distributions of dislocations in crystalline solids.

Time, classical and quantum

Florio Maria Ciaglia (Università di Napoli Federico II)

We propose a new point of view regarding the problem of time in quantum mechanics, based on the idea of replacing the usual time operator T with a suitable real-valued function T on the space of physical states. The proper characterization of the function T relies on a particular relation with the dynamical evolution of the system rather than with the infinitesimal generator of the dynamics (Hamiltonian). We first consider the case of classical hamiltonian mechanics, where observables are functions on phase space and the tools of differential geometry can be applied. The idea is then extended to the case of the unitary evolution of pure states of finite-level quantum systems by means of the geometric formulation of quantum mechanics. It is found that T is a function on the space of pure states which is not associated to any self-adjoint operator. The link between T and the dynamical evolution is interpreted as defining a simultaneity relation for the states of the system with respect to the dynamical evolution itself. It turns out that different dynamical evolutions lead to different notions of simultaneity, i.e., the notion of simultaneity is a dynamical notion.

Non-commutative integrable systems and isotropic realizations

Rui Loja Fernandes (University of Illinois)

A non-commutative integrable system (NCIS) on a symplectic manifold (X^{2n}, Ω) is given by a collection of functions $\{f_1, \dots, f_k\}$ where $(k \geq n)$, satisfying the following two assumptions:

1. Involutivity: the first $r = 2n - k$ functions Poisson commute with all k functions
$$\{f_i, f_j\} = 0, \quad (i = 1, \dots, r;\ j = 1, \dots, k).$$
2. Independence: the functions are independent almost everywhere
$$df_1 \wedge \cdots \wedge df_k \neq 0 \text{ on a dense open set.}$$

When $k = n$ one recovers the classical notion of a commutative integrable system (CIS). The same way a CIS is related with a Lagrangian fibrations, NCIS are related with isotropic fibrations. In this lecture I will recall this reationship and I will explain some beautiful connections with Poisson geometry, integral affine geometry and symplectic groupoids.

Posters

All poster sessions will be held at Serrano 121 during some of the coffee breaks, as you can check on the schedule. During the first coffee break on Thursday, a poll will take place among all the participants to choose the two posters whose authors will be kindly invited to give a talk on Friday.

Quantum splines as a quantum optimal control problem

L. Abrunheiro, M. Camarinha, J. Clemente-Gallardo, J.C. Cuchí and P. Santos (University of Aveiro, University of Coimbra, Universidad de Zaragoza, Universitat de Lleida and Instituto Politécnico de Coimbra)

Quantum splines, introduced in 2012 by Brody, Holm and Meier [Phys. Rev. Lett. 109, 100501], are curves on a Hilbert space, which generalize the notion of Riemannian splines to the quantum domain. In this work, we present an optimal control approach to these objects, using a geometrical formulation of quantum mechanics. As physical magnitudes are represented by self-adjoint operators on a Hilbert space, we know that magnitudes associated with a finite dimensional Hilbert space can be identified with the unitary Lie algebra $\mathfrak{u}(n)$ and, via its canonical scalar product, with the elements of the dual space $\mathfrak{u}^*(n)$. Our first goal is to formulate an optimal control problem for a nonlinear system on $\mathfrak{u}^*(n) \times \mathfrak{u}^*(n)$ which encodes the variational problem of quantum splines. Then, the corresponding Hamiltonian equations and the interpolation conditions are derived. Our results are illustrated with some examples and the corresponding quantum splines are computed with the implementation of a suitable numerical method. Our formalism includes as particular cases the other formulations of the problem and offers the potential to extend it to more general dynamical situations, such as the case of open systems.

On L_∞-algebras and Courant algebroids

Paulo Antunes and Joana Nunes da Costa (Universidade de Coimbra)

We establish new constructions of L_∞-algebras in relation with Courant algebroids.

Poisson homogeneous spaces: the Anti-de Sitter case

Ángel Ballesteros, Catherine Meusburger and Pedro Naranjo (Universidad de Burgos and FAU Erlangen-Nürnberg)

The correspondence between Poisson homogeneous spaces of a given Poisson-Lie group G and Lagrangian subalgebras of the Drinfel'd double of G is revisited. Connections with the construction of the corresponding quantum homogeneous spaces are studied, and the particular case in which G is a Drinfel'd double by itself is explored in detail. In particular, Poisson homogeneous spaces for the AdS group in $(1+1)$ dimensions are systematically constructed. In the $(2+1)$ case, the two Poisson homogeneous spaces connected with the AdS Poisson-Lie structures coming from two Drinfel'd double structures of the AdS Lie algebra are presented, and the role of the cosmological constant is emphasized.

Projective geometry and the KdV equations

Roisin Braddell (University of Bordeaux)

Given a smooth function f, the Schwarzian derivative is the rather fearful looking operator

$$S(f) = \frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2$$

which is invariant under all Mobius transformations. It is unsurprising, then, that the Schwarzian plays a central role in the theory of the projective line. It also pops up in some less expected places, such as the KdV equations - one of the most famous equations of hydrodynamics, and the first infinite dimensional system which was proven to be completely integrable. The talk will sketch the connections between hydrodynamical systems and objects more typically found in theory on projective spaces, beginning with the Schwarzian.

Bi-Jacobi fields and the biharmonic flow

Margarida Camarinha (Universidade de Coimbra)

Biharmonic curves on a Riemannian manifold M are the critical points of the bienergy functional

$$J(\gamma) = \frac{1}{2}\int_0^T \left\langle \frac{D^2\gamma}{dt^2}, \frac{D^2\gamma}{dt^2} \right\rangle dt$$

and are seen as a natural generalization of geodesics [J. Eells and J.H. Sampson, Am. J. Math. 86 (1964), H. Urakawa, Symmetry, 7 (2015)]. The theory of Jacobi fields and conjugate points along geodesics can be extended to biharmonic curves. Two characterizations of Jacobi fields along geodesics can be used: the null vectors of the second variation of the energy functional and the variational vector fields associated with a one-parameter family of geodesics. The generalized Jacobi fields along biharmonic curves are called bi-Jacobi fields (see [M. Camarinha et al. Differential Geom. Appl., 15 (2001)] for the theory of bi-Jacobi fields in general and [L. Noakes and T. Ratiu, Commun. Math. Sci., 14 (2016)] for bi-Jacobi fields along geodesics).

In this work we extend some properties of Jacobi fields along geodesics to bi-Jacobi fields. We relate bi-Jacobi fields to the biharmonic flow on the third order tangent bundle T^3M. We consider a connection map on T^3M and the corresponding nonlinear connection defined by its kernel [I. Bucataru, J. Geom. Mech., 5 (2013)]. We describe the differential of the biharmonic flow in terms of bi-Jacobi fields and the decomposition of TT^3M into vertical and horizontal subbundles.

Variational/Hamiltonian integrators for the sampling of molecules

Cédric M. Campos (Yachay Tech)

Simulation of big molecules or large sets of molecules is a challenging problem. I shall present how variational/Hamiltonian integrators can be applied to such problems, that is the Hybrid/Hamiltonian Monte Carlo method (HMC), and how the numerical integrators or the sampling method can be tailored to improve results. As results we shall see numerical comparisons between several HMC numerical integrators recently suggested in the literature. The problems used for benchmarking include a wide range of degrees of freedom.

Jacobi last multiplier and quasi-velocities

José F. Cariñena a and Patrícia Santos (Universidad de Zaragoza and Universidade de Coimbra)

In this work we will establish the relation between the Jacobi last multiplier [C. Jacobi et al. Texts and Readings in Mathematics (2009)] and [M.C. Nucci and P.G.L. Leach, J. Math. Phy. 49 (2008)], that is a geometrical tool in the solution of problems in mechanics and that provides Lagrangians descriptions and constants of motion for systems of ordinary differential equations, and non-holonomic Lagrangian mechanics where the dynamics can be determined by the Hamel's equations [J.G. Papastavridis, Analytical Mechanics (2002)] and [J.F. Cariñena, et al. J. Phys. A: Math. Theor. 40 (2007)].

The Modular Class of a Lie n-algebroid

Raquel Caseiro (Universidade de Coimbra)

We introduce and study the notion of modular class of a Lie n-algebroid.

On a problem of Singer about plane curves: curvature and distance from a point

Ildefonso Castro, Ildefonso Castro-Infantes, Jesús Castro-Infantes (Universidad de Granada and Universidad de Jaén)

The fundamental existence and uniqueness theorem in the theory of plane curves states that a curve is uniquely determined up to rigid motion by its curvature κ given as a function of its arc-length. However, in most cases such curves are impossible to find explicitly in practice, due to the difficulty in solving the three quadratures appearing in the integration process. In [D.A. Singer, Amer. Math. Monthly 106 (1999)], David A. Singer posed a different problem:

Can a plane curve be determined if its curvature is given in terms of its position?

Singer started to deal with the posed problem by studying the condition $\kappa(r) = r := \sqrt{x^2+y^2}$, but only the very pleasant special case of the classical Bernoulli lemniscate, $r^2 = 3\cos 2\theta$ in polar coordinates, was solved explicitly by him. Probably, the most interesting solved problem in this setting corresponds to the Euler elastic curves, whose curvature is proportional to one of the coordinate functions, say $\kappa(x, y) = cy$.

Inspired by the above question and by the classical elasticae, the first two named authors recently studied the plane curves whose curvature depends on distance to a line. Motivated again by the problem posed by David A. Singer and by the classical Bernoulli lemniscate and the Norwich spiral, we study the plane curves whose curvature is expressed in terms of the distance from a point, say the origin and so $\kappa = \kappa(r)$. By introducing the notion of *radial primitive curvature*, we provide new characterizations of some well known curves, like the mentioned Bernoulli lemniscate, the inverse Norwich spiral, the anti-clothoid, the cardioid, the sinusoidal spirals and the Cassini ovals. We also find out several new families of spiral curves whose intrinsic equations are expressed in terms of elementary functions and we are able to get arc length parametrizations of them and they are depicted graphically.

Some remarks on Dirac-Kahler operator: Lie groups and homogeneous spaces

Fabio Di Cosmo (University of Naples Federico II)

Dirac equation was introduced in 1928. In its paper Dirac wanted to write an equation which squared to the Klein-Gordon equation. In order to achieve this result he introduced a Hamiltonian operator depending on a set of matrices which satisfy certain algebraic rules. Nowadays we interprete these matrices as a representation of a Clifford algebra and the elements on which it acts are understood as spinors. Therefore a possible generalization of Dirac operator on general Riemannian manifold is related to the theory of Spin manifolds and Ssinor bundles.

Another approach to generalize Dirac operator was proposed by Kahler in 1962. Starting from the isomorphism between exterior algebras and Clifford algebras, he was able to find a representation of the Clifford product on the space of differential forms; let us call this product $\vee$-product. Therefore if one considers the exterior algebra $\Lambda(M)$ over a Riemannian manifold (M, g), this is an algebra with respect to both $\vee$ and $\wedge$ products. Eventually it is also possible to write a Dirac-type operator acting on $\Lambda(M)$ and interestingly it assumes the following form:

$$D_K = d + \delta$$

where d is the exterior derivative and δ is the codifferential. In the same spirit of Dirac equation, the square of this operator, the so called Dirac- Kahler operator, is the Laplace-Beltrami operator

$$\Delta = d\delta + \delta d$$

and in the case of $\mathbb{R}^4$ equipped with the Minkowsky metric it can be de composed as the direct sum of four copies of the well known Dirac operator $D = \gamma^\mu \partial_\mu$.

In this talk I would like to show some examples where Dirac-Kahler operator can be easily written, which are Lie groups and homogeneous spaces. In the first case, indeed, one can use left (or right) invariant defferential forms in order to write equations globally, even if in terms of a non-holonomic basis. As for the second one, I will exploit the fact that the exterior algebra on a homogeneous space can be considered as a subalgebra of the exterior algebra of the group which acts on it. Therefore one can still continue to use some of the results of the previous case. In this setting Dirac-Kahler operator is very useful because it is written in terms of the exterior derivative and the codifferential, and one knows their behaviour with respect to the pull-back of differential forms.

In particular I will focus on two examples: the abelian case, where it is possible to write both Dirac equation and its classical limits by enlarging the manifold R^4 to a $U(1)$-bundle. As for the non abelian case I will consider the group $SU(2)$ and the sphere S^2 as homogeneous space. Eventually I will also expose a possible way to write in this framework a Dirac-Kahler equation in presence of a magnetic field generated by a monopole. Concluding remarks will be devoted to possible extensions and outlooks.

Some applications of the inverse problem of the calculus of variations

Marta Farré Puiggalí (ICMAT-CSIC)

The inverse problem of the calculus of variations consists in determining whether a given system of second order differential equations is equivalent to some regular Lagrangian system.

I will explain some results and applications of the inverse problem to hamiltonization of nonholonomic systems and stabilization of controlled Lagrangian systems.

Completeness of certain inextensible accelerated observers in General Relativity

Daniel de la Fuente (Universidad de Granada)

We present the notions of uniformly accelerated, unchanged direction and circular motion in General Relativity in the realm of the Lorentzian Geometry. We analyse the completeness of the inextensible trajectories of observers which obey one of these motions, when the ambient spacetime has certain symmetries. The completeness of inextensible electromagnetic trajectories is also studied in some static spacetimes.

Symmetries and conservation laws in multisymplectic higher-order field theories: the gravitational field

Jordi Gaset and Narciso Román-Roy (Universitat Politècnica de Catalunya)

First integrals (and conservation laws), symmetries, Cartan (Noether) symmetries and gauge symmetries are stated in the ambient of fiber bundles which are endowed with a (pre)multisymplectic structure. As a particular case, we study all these kinds of symmetries and conservation laws for classical second order Lagrangian field theories. Finally we apply these results to analyze symmetries and conservation laws for the Hilbert-Einstein formulation of the gravitational field.

Dimensional analysis of relativistic field equations: applications to multi-scalar-tensor and bi-metric field theories

Adriń Gordillo and José Navarro (Universidad de Extremadura)

Recently, there is renewed interest in elucidating all possible field equations of modified theories of gravity.

In this talk, I will explain how to apply the ideas of [J. Navarro and J.B. Sancho, J. Geom. Phys. 58 (2008)] and [J. Navarro and J.B. Sancho, J. Math. Phys. 53 (2012)], that esentially consist on taking care of the dimensional analysis of the fields under consideration, in order to refine the current knowledge of these field equations. I will pay special attention to multi-scalar-tensor and bi-metric theories of gravity.

On moduli spaces for finite-order jets of linear connections

Adrián Gordillo and José Navarro (Universidad de Extremadura)

In this talk, I will describe the ringed-space structure of moduli spaces of jets of linear connections (at a point) as orbit spaces of certain linear representations of the general linear group.

Then, I will use this fact to prove that the only (scalar) differential invariants associated to linear connections are constant functions, as well as to recover various expressions appearing in the literature regarding the dimensions of generic strata of these moduli spaces.

Hamiltonian structures of Lienard equation revisited: From non standard to contact Hamiltonian

Partha Guha (S. N. Bose National Centre for Basic Sciences)

Drinfel'd doubles for $(2+1)$ Poincaré gravity

Iván Gutiérrez-Sagredo, Ángel Ballesteros, Francisco José Herranz (Universidad de Burgos)

By starting from the complete classification of three-dimensional Lie bialgebra structures given in [X. Gomez, J. Math. Phys. 41 (2000)], all possible Drinfel'd double structures for the Poincaré algebra $\mathfrak{iso}(2,1)$ are constructed in a physically adapted basis. The corresponding classical r-matrices are given, together with the resulting pairing between basis generators. Our results are compared with the corresponding ones in [A. Ballesteros et al., Class. Quantum Grav. 30 (2013)], where some cases are obtained by a contraction process. We explore the relations between the associated quantum Poincaré groups and $(2+1)$ gravity with vanishing cosmological constant, where the former appear naturally as symmetries of the corresponding non-commutative spacetimes.

The anisotropic oscillator on curved spaces: a new exactly solvable model

Francisco J. Herranz (Universidad de Burgos)

We present a new exactly solvable (classical and quantum) model that can be interpreted as the generalization to the two-dimensional sphere and to the hyperbolic space of the two-dimensional anisotropic oscillator with any pair of frequencies ω_x and ω_y. The new curved Hamiltonian H_κ depends on the curvature κ of the underlying space as a deformation/contraction parameter, and the Liouville integrability of H_κ relies on its separability in terms of geodesic parallel coordinates, which generalize the Cartesian coordinates of the plane. Moreover, the system is shown to be superintegrable for commensurate frequencies $\omega_x : \omega_y$, thus mimicking the behaviour of the flat Euclidean case, which is always recovered in the $\kappa \to 0$ limit. The additional constant of motion in the commensurate case is, as expected, of higher-order in the momenta and can be explicitly deduced by performing the classical factorization of the Hamiltonian. The known $1:1$ and $2:1$ anisotropic curved oscillators are recovered as particular cases of H_κ , meanwhile all the remaining $\omega_x : \omega_y$ curved oscillators define new superintegrable systems. Furthermore, the quantum Hamiltonian $\hat{H}_\kappa$ is fully constructed and studied by following a quantum factorization approach. In the case of commensurate frequencies, the Hamiltonian $\hat{H}_\kappa$ turns out to be quantum superintegrable and leads to a new exactly solvable quantum model. Its corresponding spectrum, that exhibits a maximal degeneracy, is explicitly given as an analytical deformation of the Euclidean eigenvalues in terms of both the curvature κ and the Planck constant $\hbar$. In fact, such spectrum is obtained as a composition of two one-dimensional (either trigonometric or hyperbolic) Pösch-Teller set of eigenvalues.

Lie groupoids and algebroids applied to the study of uniformity and homogeneity of material bodies

Víctor Manuel Jiménez (ICMAT-CSIC)

A Lie groupoid, called *material Lie groupoid*, is associated in a natural way to any elastic material. The corresponding Lie algebroid, called *material Lie algebroid*, is used to characterize the homogeneity property of the material. We also relate these results with the previously ones in terms of G-structures.

Homothetic solitons for the inverse mean curvature flow

Ana M. Lerma (Universidad de Jaén)

Let $\phi : M^n \to \mathbb{R}^m$ be an isometric immersion of an n-dimensional manifold in Euclidean m-space with nonvanishing mean curvature vector H. The submanifold M in $\mathbb{R}^m$ is called a homothetic soliton for the inverse mean curvature flow (in short IMCF) if there exists a constant $a \neq 0$ satisfying

$$-\frac{H}{|H|^2} = a\phi^{\perp} \tag{$*$}$$

where $\perp$ stands for the projection onto the normal bundle of M.

In this poster we study solutions to (*) with arbitrary dimension and codimension. We first show that the closed ones are necessarily spherical minimal immersions and so we reveal the strong rigidity of the Clifford torus in this setting. Mainly we focus on the Lagrangian case, obtaining numerous examples and uniqueness results for some products of circles and spheres that generalize the Clifford torus to arbitrary dimension. We also characterize the pseudoumbilical ones in terms of soliton curves to IMCF, as well as minimal Legendrian immersions in odd-dimensional spheres. As a consequence, we classify the rotationally invariant Lagrangian homothetic solitons to IMCF.

Finally, we also want to emphasize two very important applications of IMCF: on the one hand, Huisken and Ilmanen ([G. Huisken and T. Ilmanenm Int. Math. Res. Not. 20 (1997)] and [G. Huisken and T. Ilmanen, J. Differential Geom. 59 (2001)]) used a level set approach to the IMCF and were able to prove the Riemannian Penrose inequality; on the other hand, using IMCF, Bray and Neves [H. Bray and A. Neves, Ann. of Math. 159 (2004)] proved the Poincaré conjecture for 3-manifolds with Yamabe invariant greater than real projective 3-space.

Surfaces in Riemannian and Lorentzian 3-manifolds admitting a Killing vector field

José M. Manzano (Universidad de Jaén)

A Killing submersion is a Riemannian submersion from an orientable connected 3-manifold onto a Riemannian surface, such that the fibres of the submersion are the integral curves of a Killing vector field without zeroes. A Killing submersion can be Riemannian or Lorentzian, depending on whether the Killing vector field is assumed spacelike or timelike. The geometry of the total space is characterized by two geometric functions in the base surface, namely the *bundle curvature* τ and the Killing length μ. These two functions also yield restrictions to the topology of the 3-manifold. If the base is simply connected, τ and μ can be prescribed arbitrarily giving rise to a unique Killing submersion structure, but uniqueness fails if simple connectedness is dropped.

We will also show the existence of a constant $C(M,\mu)$ such that the total space of any Lorentzian Killing submersion over M with Killing length μ and bundle curvature τ satisfying $\inf_M |\tau| > C(M,\mu)$ does not admit complete spacelike surfaces. In other words, if the space is twisted enough, then it is not distinguishing from the point of view of causality. We will see that the constant $C(M,\mu)$ can be understood as the Cheeger constant of M with density μ, and it is sharp in some specific examples.

Black holes, quantum gravity and mock moonshine

Kishore Marathe (City University of New York)

We will discuss some recent developments in the relation of classical and quantum theory of black holes and Ramanjujan's mock theta functions.

Nearly Sasakian and nearly cosymplectic manifolds

Antonio De Nicola, Giulia Dileo and Ivan Yudin (University of Coimbra and Università degli Studi di Bari Aldo Moro)

We prove that every nearly Sasakian manifold of dimension greater than five is Sasakian. This provides a new criterion for an almost contact metric manifold to be Sasakian. Moreover, we classify nearly cosymplectic manifolds of dimension greater than five.

Nijenhuis forms on Lie-infinity algebras

Joana Nunes da Costa (Universidade de Coimbra)

We investigate Nijenhuis deformations of Lie-infinity algebras, a notion that unifies several Nijenhuis deformations, namely those of Lie algebras, Lie algebroids, Poisson structures and Courant structures. Some examples are included.

Invariant solutions to the heterotic equations of motion on compact quotients of Lie groups

Antonio Otal, Luis Ugarte and Raquel Villacampa (Centro Universitario de la Defensa de Zaragoza and Universidad de Zaragoza)

A model for string theory was proposed in [P. Candelas et al., Nucl. Phys. B 258 (1985)] involving a ten dimensional space $\mathbb{R}^{1,3} \times M^6$, where $\mathbb{R}^{1,3}$ is a Lorentzian spacetime and M^6 is a compact Calabi-Yau manifold. Strominger [Nucl. Phys. B 274 (1986)] generalized the previous construction allowing a background M^6 with a non-zero torsion. This led to a complicated system of PDEs known as the *Strominger system* which is written in terms of the fermionic and bosonic fields relevant in the physical theory. But the system can be reformulated in a geometrical way involving linear connections defined on several bundles over the background M^6.

Several works have been devoted since then to find solutions to this sysmem. In this talk we present compact manifolds providing many solutions to the Strominger system with respect to a 2-parameter family of metric connections $\nabla^{\varepsilon,\rho}$. All our solutions are compact and invariant in the sense that are constructed as quotients of Lie groups. The family $\nabla^{\varepsilon,\rho}$ is a natural extension of the canonical 1-parameter family of Hermitian connections [P. Gauduchon, Boll. Un. Mat. Ital. B (7) 11 (1997)] that includes other metric connections that are of interest in the anomaly cancellation condition (for instance the Levi-Civita connection or the so denoted ∇^- connection). Some of the examples solve in addition the most restrictive system of heterotic equations of motion with respect to the Bismut connection [A. Otal et al. arxiv: 1604.02851].

Concretely, we construct invariant solutions to the Strominger system with respect to the Chern connection ∇^c , with non-flat instanton and positive α' on compact complex solvmanifolds with holomorphically trivial canonical bundle found in [A. Fino et al., Int. Math. Res. Not. IMRN 2015]. Furthermore, invariant solutions to the heterotic equations of motion with respect to the Bismut connection ∇^+, non-flat instanton and $\alpha' > 0$ are also obtained.

In the semisimple case we find the first known solutions to the heterotic equations of motion on a compact quotient of $SL(2,\mathbb{C})$. We also provide many invariant solutions to the Strominger system on this manifold recovering the ones obtained in [T. Fei, S.-T. Yau, Commun. Math. Phys. 338 (2015)].

Minimal hypersurfaces in manifolds with density

Juan J. Salamanca (Universidad de Granada)

Given a manifold with density, the critical points of the weighted area functional are the ϕ-minimal hypersurfaces. Note that this notion generalizes properly the classical minimal hypersurfaces. We focus on the case the manifold is a warped product with density over a parabolic manifold. In this setting, the class of ϕ-minimal graphs is associated to a wide family of PDEs. Using geometrical techniques, we provide several uniqueness results for ϕ-minimal graphs. As application, uniqueness for new Moser-Bernstein type problems are shown. Classical minimal graphs are also considered as a special case.

Calabi-Bernstein results for complete maximal hypersurfaces in Robertson-Walker spacetimes with flat fiber

José Antonio Sánchez Pelegrín (Universidad de Granada)

In this work, we study maximal hypersurfaces in spatially open Robertson-Walker spacetimes of arbitrary dimension whose fiber is flat and obey the Null Convergence Condition.

We mainly focus here on global properties of maximal hypersurfaces. Therefore, some uniqueness and nonexistence results for complete maximal hypersurfaces in these spacetimes by means of a Liouville-type theorem are given.

Furthermore, we also apply these results to some relevant spacetimes, such as the steady state spacetime, Einstein-de Sitter spacetime and certain radiation models.