Seasonality in fisheries: A bridge between continuous and discrete-time bioeconomic models

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• This research has been supported by a grant from Iceland, Liechtenstein and Norway through the EEA Financial Mechanism. Operated by Universidad Complutense de Madrid. Ref: 021-ABEL-CM-2013.

• Part of the computations of this research was performed in EOLO, the HPC of Climate Change of the International Campus of Excellence of Moncloa (UCM), funded by MECD and MICINN.

INTRODUCTION

• Most of the commercial fisheries are managed on an annual basis where the collection and management of the annual biological and fisheries data are used by management agencies, like the International Council for the Exploration of the Sea (ICES), to provide annual advice regarding the stock status, reference points, and total allowable catches (TACs)

• However, the main stakeholders, fish stocks and fishers, do not show an uniform behavior in real world fisheries in which there are strong seasonal variations in biological parameters (biological seasonality), such as in the population dynamics of migratory fish stocks, and in economic parameters (economic seasonality), such as prices, and costs of harvesting, which imply seasonal variations in harvesting

INTRODUCTION

- In addition, TACs are set annually for most stocks despite the well-known fact that most of them have a distinct seasonal pattern
- Moreover, the TACs allotted to different vessel groups not only are set annually, rather than seasonally (seasonal regulation), but also they are usually based on political decisions, rather than optimal bioeconomic criteria, with the consequent biological and economic implications that may result from suboptimal allocation of TACs
- For the above reasons, the optimal management of seasonal fisheries has become a hot topic in fisheries economics, specially taking into account that both discrete-time (DM) and continuous-time bioeconomic models (CM) are not able to cope with the complex phenomenon of seasonality in fisheries:

-When considering increments in time of one year, DM neglect seasonality. CM also neglect it when considering time-independent optimal feedback policies

INTRODUCTION

- There is a fundamental choice to be made when developing a bioeconomic model: discrete (DM) or continuous-time (CM) modeling:

- CM are based upon the assumption that both biological processes, such as growth, and human activity, such as harvesting, are taking place continuously, while in DM, they are taking place at discrete-time steps (usually annual)

- In DM, the population dynamics of fish stocks is described by the difference equation

$$x_{t+1} = x_t + F_D(x_t) = f_D(x_t), \quad (1)$$

$$F_D(x_t) = r_D x_t^{\alpha}(1 - \frac{x_t}{K_D}). \text{ (LGF)}$$

- While in CM,

$$\dot{x}(t) = dx/dt = F_C(x), \quad (2)$$

$$F_C(x) = r_C x^{\alpha}(1 - \frac{x}{K_C}). \text{ (LGF)}$$

INTRODUCTION

- CM have proved to be useful for analytical purposes (Clark, 2010)

- In addition, CM may serve as a conceptual framework and guideline for DM, despite the difficulty in estimation they present

- However, CM are unable to encompass delay effects, which are commonplace in real world fisheries

- Moreover, both biological processes, such as spawning, and human activity, such as harvesting, could be seasonal rather than continuous over time (Clark, 2010; Bjørndal and Munro, 2012)

- In addition, data are usually available on an annual basis

INTRODUCTION

- Most of the literature on bioconomic modeling of fisheries uses both discrete-time (DM) and continuous-time bioeconomic models (CM) indistinctly without a clear biological and/or economic justification

- Even, in some cases, the choice is a matter of individual taste where DM are generally preferred by biologist while CM are generally preferred by mathematicians

- It is not obvious how DM and CM are related to each other, and consequently this is not a trivial choice, especially in fisheries economics, since methodologies for DM (modeling with difference equations) and CM (modeling with differential equations) are completely different, and consequently the policy advice provided by them can also be different, with significant implications for sustainability of fish stocks

Errors in continuous-time models

• Errors in mathematical modeling of the natural growth function $F_C(x)$ are frequently found in the literature on bioconomic modeling of fisheries in a continuous-time setting

$$\dot{x}(t) = dx/dt = F_C(x), \quad (2)$$

$$F_C(x) = r_C x^{\alpha}(1 - \frac{x}{K_C}). \text{ (LGF)}$$

• Most of the natural growth functions used in CM, which inserted into differential equations, are, however, often estimated in discrete time, which uses difference equations

• Despite the well known fact that the dynamical properties of discrete and continuous-time population dynamics are entirely different

• Indeed, it is well known that the discrete-time homologue of the continuous-time LGF is not the discrete-time LGF but the Beverton–Holt growth function which is non-decreasing at high population sizes

AIM

- The aim of this paper is to develop a discretization method of CM (DCM) which allows us to construct a bridge between CM and DM by overcoming the biological and economic weakness and by preserving the strengths of both approaches

- Based on the DCM, the aim of this paper is also to develop a bioeconomic model which allows us to deal with seasonality in fisheries

- The DCM consists of two steps:

First we estimate a proper growth function for the continuous-time model through the Kalman Filter (EnK)

Then we use the Runge-Kutta method to analyze the optimal management of seasonal fisheries in a discrete-time setting

North-East Arctic cod fishery (NEAC)

- NEAC is the largest cod stock in the world and, consequently, one of the most important species in Norwegian fisheries.

- The NEAC fishery is a clear example of seasonal fishery due to its migration pattern. In particular, maturing cod migrate to the Norwegian coast to spawn and back to the Barents Sea after spawning

- 80% of NEAC is harvested during the winter in the area where the stock has gathered and migrated from the Barents Sea to spam

Discretization method of continuous-time bioeconomic models (DCM): NEAC

• NEAC has been analyzed extensively in the literature on fisheries economics by using CM, as defined in (3), which is the starting point of the DCM

$$\max_{h} \int_0^{\infty} e^{-\beta t}\Pi(h,x)dt \quad (3)$$

$$\text{s.t. } \dot{x}(t) = F_C(x) - h; \; x(0) = x_0.$$

• The DCM consists of several stages:

i) The natural growth function (NGF), $\dot{x}(t) = F_C(x)$, is properly estimated in a continuous-time setting by using data assimilation methods. In particular, we use the Kalman filter (EnK)

• In the case of NEAC:

$$\dot{x}(t) = dx/dt = F_C(x), \quad (4)$$

$$F_C(x) = r_C x^2 (1 - \frac{x}{K_C}); \; r_C = 0.00045371; \; K_C = 3,703 \; (1,000 \text{ tons})$$

ii) The NGF estimated in *i)* is discretized by using the fourth order Runge-Kutta method (RKM).

• The RKM is one of the well known robust numerical methods used in temporal discretization for the approximation of solutions of differential equations

• Given a temporal interval $[t, t+\Delta t]$, the RKM allows us to obtain the stock value at period $t+\Delta t$, $x_{t+\Delta t}$, as a function of the stock value at period t

$$x_{t+\Delta t} = f_{RK}(\Delta t) := f_{RK}(x_t), \quad (5)$$

where $f_{RK}(x_t)$ is the proper discrete-time approximation, for incremental time Δt, derived from the continuous-time growth function $F_C(x)$ as estimated in *i)*

• Thus, using the RKM *(ii)*, a proper discrete-time growth function $f_{RK}(x_t)$, for incremental time Δt, is obtained by using an appropriate discretization of the LG $F_C(x)$ estimated in a continuous-time setting *(i)*.

i) EnK

$$\dot{x}(t) = dx/dt = F_C(x), \quad (4)$$

$$F_C(x) = r_C x^2 (1 - \frac{x}{K_C}),$$

$$r_C = 0.00045371; \; K_C = 3{,}703$$

ii) RKM →

$$x_{t+\Delta t} = f_{RK}(x_t) \quad (5)$$

• However, most of the literature on CM estimates the NGF for NEAC in a discrete-time setting (pure discrete-time models, DM)

$$x_{t+1} = x_t + F_D(x_t) = f_D(x_t), \quad (6)$$

$$F_D(x_t) = r_D x_t^2 (1 - \frac{x_t}{K_D}); \; r_D = 0.000665; \; K_D = 2{,}473 \; (1{,}000 \text{ tons})$$

• As expected, the above parameter estimations are very different from those obtained in a continuous-time setting $F_C(x)$, as estimated in *i)*

Pure discrete-time bioeconomic models (DM) vs DCM: annual case

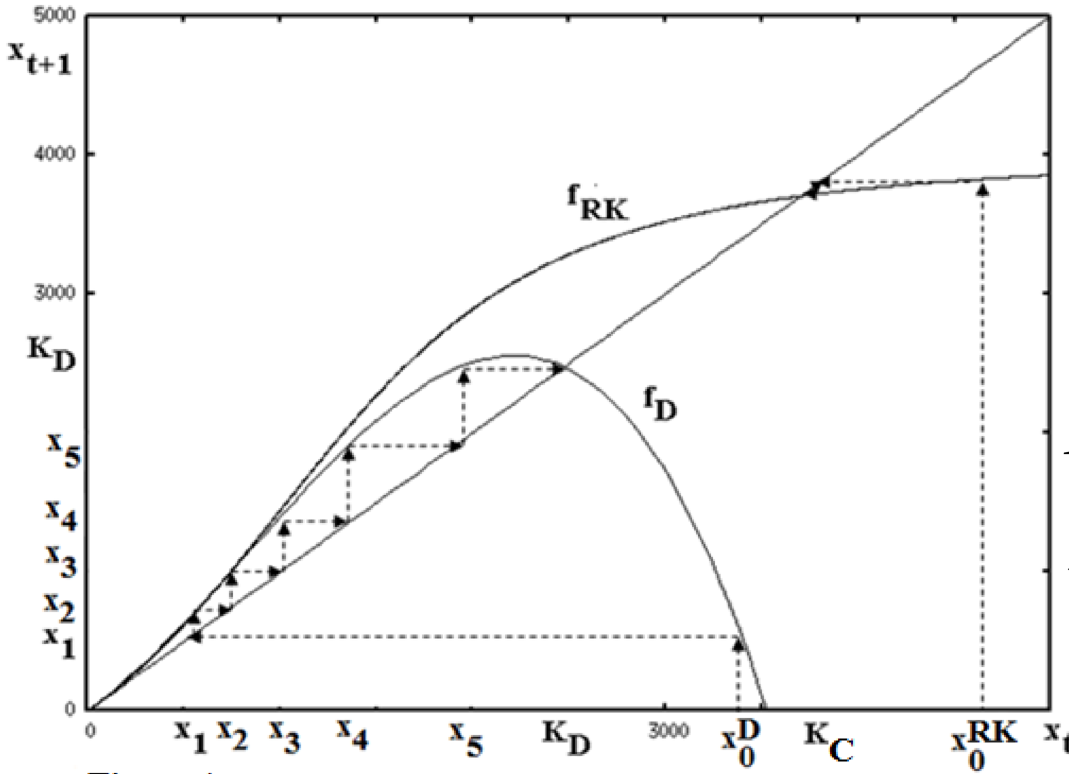

Figure 1.

Biological weakness of pure discrete-time models (DM)

$$DCM: x_{t+\Delta t} = f_{RK}(x_t);\ \Delta t=1\ (5)$$

$$DM: x_{t+1} = x_t + F_D(x_t) = f_D(x_t)\ (6$$

• **DM:** Drastic population decline exhibited by f_D at high population sizes which could only be explained by catastrophic biological events. After only one year, there is a drastic decrease, of about 85%, in population

• **DCM:** f_{RK} is a non-decreasing growth function that reflects a gradual approximation to K_C from high initial stock values. This behavior seems to properly reflect the natural behavior of fish stocks at high population sizes

• **DCM** allows us to overcome biological weakness of **DM**

iii) The net revenue function from the fishery $\Pi(h,x)$ is discretized by considering the temporal interval $[t, t+\Delta t]$, as described in *ii)*

• A generic net revenue function: $$\Pi(h,x) = p(h)h - C(h,x), \quad (7)$$

which is formulated in a continuous-time setting (h represents the harvest rate), and where both the inverse demand function $p(h)$ and the cost function $C(h,x)$ have been estimated on an annual basis, is reformulated to contemplate the temporal interval $[t, t+\Delta t]$ under consideration

• In the case of NEAC:

$$p(h) = (p_1 - p_3 h),\ p_1 = 12.65 \text{ and } p_3 = 0.00839; C(x,h) = c\frac{h^{1.1}}{x}, c = 5{,}848.1.$$

where stock x and harvest h are measured in 1,000 tons, net revenue is measured in million NOK, and prices are measured in NOK/kg

• The DCM, ***i)-iii)***, allows us to obtain a discrete-time approximation of the CM, as defined in (3). In particular, the DM obtained by the DCM is given by (9):

$$\max_{h} \int_0^\infty e^{-\beta t} \Pi(h,x)dt \quad (3)$$
$$\text{s.t. } \dot{x}(t) = F_C(x) - h$$
$$x(0) = x_0,$$

DCM →

← t⟶0

$$\max_{\{x_{t+\Delta t}\}_{t=0}^\infty} \sum_{t=0}^\infty \delta^{\Delta t} \Pi(x_t, x_{t+\Delta t}, \Delta t) \quad (9)$$
$$\text{s.t. } 0 \le x_{t+\Delta t} \le f_{RK}(x_t),\ t = 0,1,...,$$
$$x_0 > 0 \text{ given}, \Pi(x_t, x_{t+\Delta t}, \Delta t) \ge 0,$$
$$t = 0,1,...,$$

• Using the dynamic programming approach, we can define the following Bellman equation associated with (9):

$$V(x) = \max_{0 \le y \le f_{RK}(x)} [\Pi(x, y, \Delta t) + \delta^{\Delta t} V(y)], \quad (10)$$

• The DCM allows us to construct a bridge between CM and DM due to the fact that, a CM, as defined in (3), may be thought of as the limiting case of the DM, as defined in (9), in which the interval between times Δt in the discrete-time frame $t+\Delta t$ becomes vanishingly small. In other words, the DM converges to the CM when t⟶0

• This allows for increments in time less than one year $\Delta t \le 1$ and consequently it allows us to analyze seasonal fisheries

Pure discrete-time bioeconomic models (DM) vs DCM: NEAC

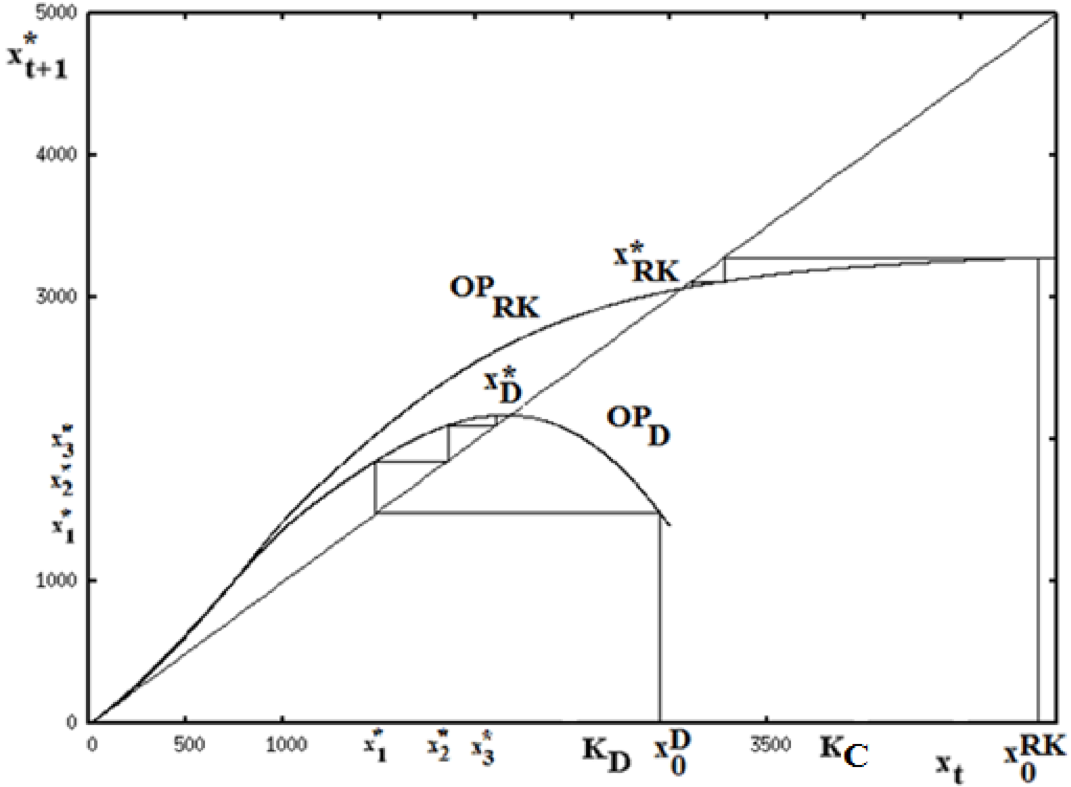

Figure 2.

$$V(x) = \max_{0 \le y \le f_{RK}(x)} [\Pi(x, y, \Delta t) + \delta^{\Delta t} V(y)]$$

- $DCM : x_{t+\Delta t} = f_{RK}(x_t);\ \Delta t=1$
- $DM : x_{t+1} = x_t + F_D(x_t) = f_D(x_t)$

- The solution obtained in the case of DM is very different from that obtained by the DCM $x^*_{RK} > x^*_D$

- An example of two well-established discrete-time bioconomic models, which however give rise to quite different optimal policies for the same species, with the consequent uncertainty on what is the appropriate model for management advice

- According to the biological weakness of DM, the DCM is the most appropriate one

DCM allows us to construct a bridge between CM and DM by overcoming the biological and economic weakness and by preserving the strengths of both approaches

Seasonal fisheries

- DCM allows for increments in time less than one year $\Delta t \leq 1$ and consequently it allows us to analyze seasonal fisheries

- Specifically, if the one-year interval is divided in N periods, then this can be done by defining $\Delta t = 1/N \leq 1$

- In this way, the DCM allows us to analyze the phenomenon of seasonality in fisheries for different values of $\Delta t \leq 1$

Seasonal vs. annual harvesting

- We analyze both the case of quarterly harvest, $\Delta t=0.25$ (N=4), and the case of monthly harvest, $\Delta t=0.083$ (N=12), and we compare these to the case of annually harvest, $\Delta t=1$ (N=1)

$$V(x) = \max_{0 \le y \le f_{RK}(x)} [\Pi(x, y, \Delta t) + \delta^{\Delta t} V(y)]$$

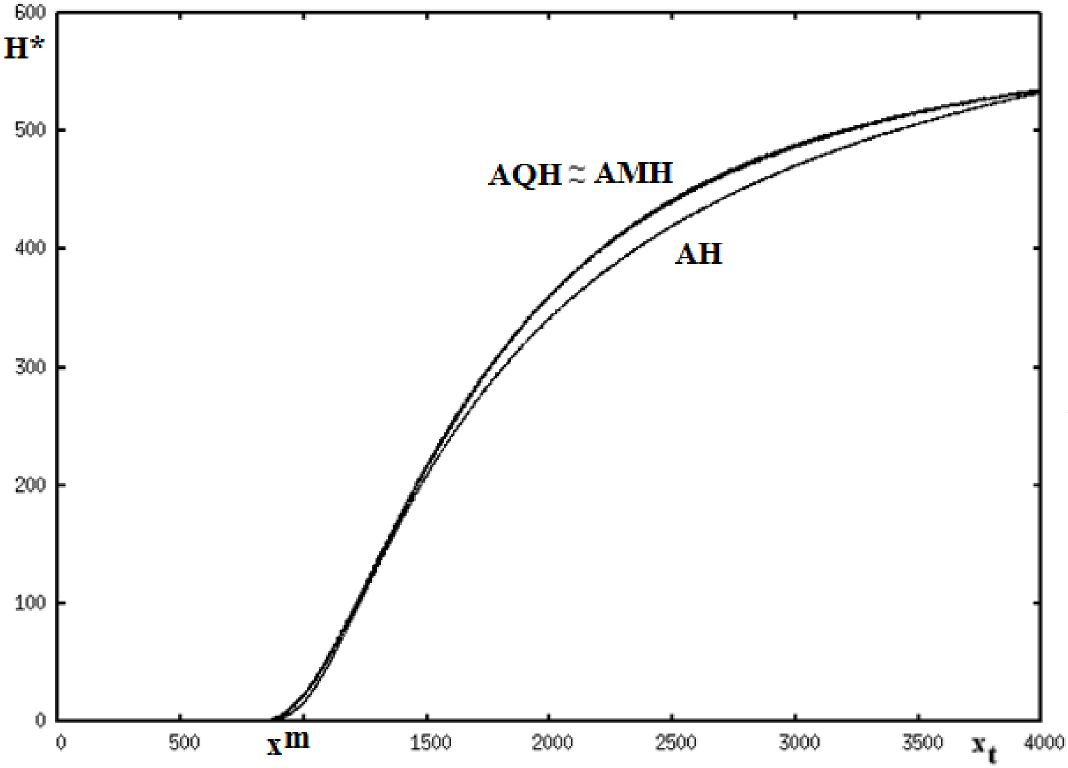

Figure 3.

- Annual optimal harvest if the stock is: annually harvested $\Delta t=1$, AH(x_t), quarterly harvested $\Delta t=0.25$, AQH(x_t), and monthly harvested $\Delta t=0.083$, AMH(x_t)
- x_t (1,000 tons) represents the stock value at the beginning of the year
- Harvest moratorium at low stock levels, with a gradual increase in harvest at high enough stock values

- AQH≈AMH which implies that, at least for this fishery, there is a fast convergence

Seasonal vs. annual harvesting

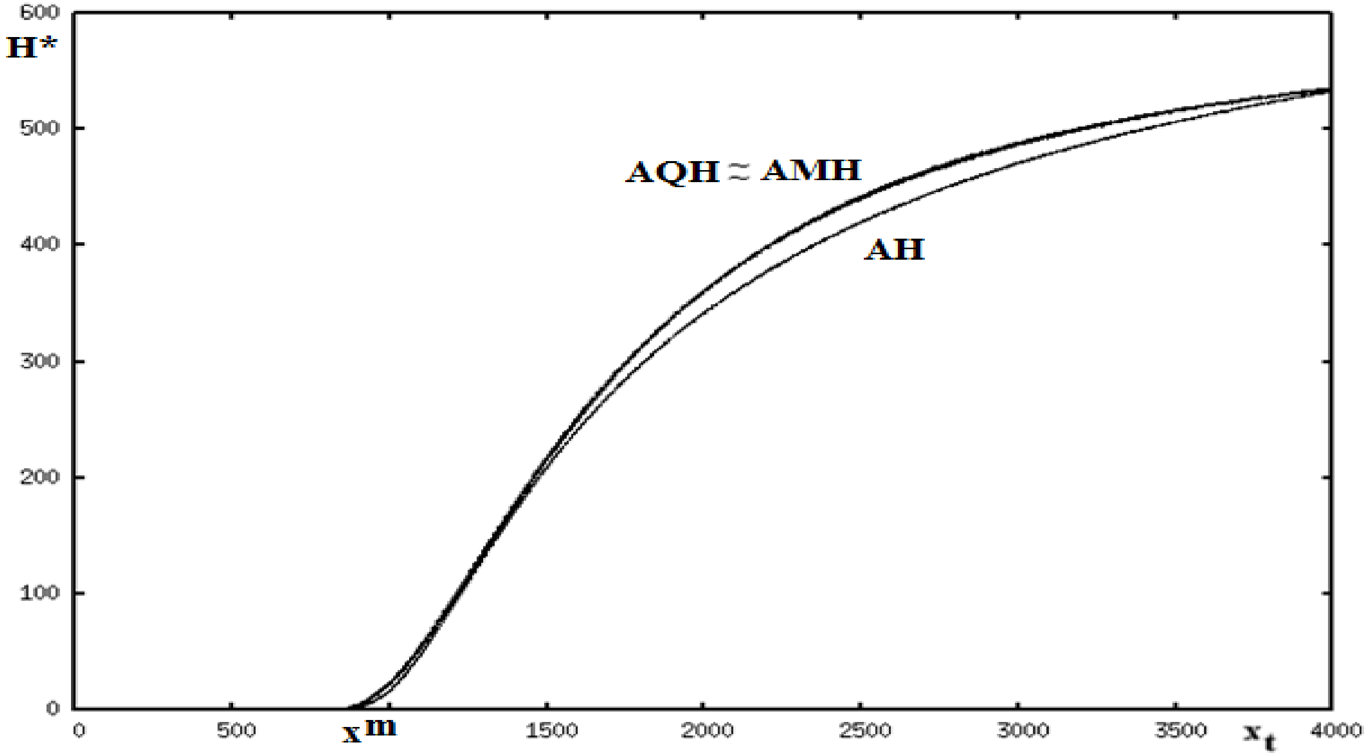

Figure 3.

- Both AQH and AMH are greater than AH, specially at high enough stock values

Seasonal vs. annual harvesting

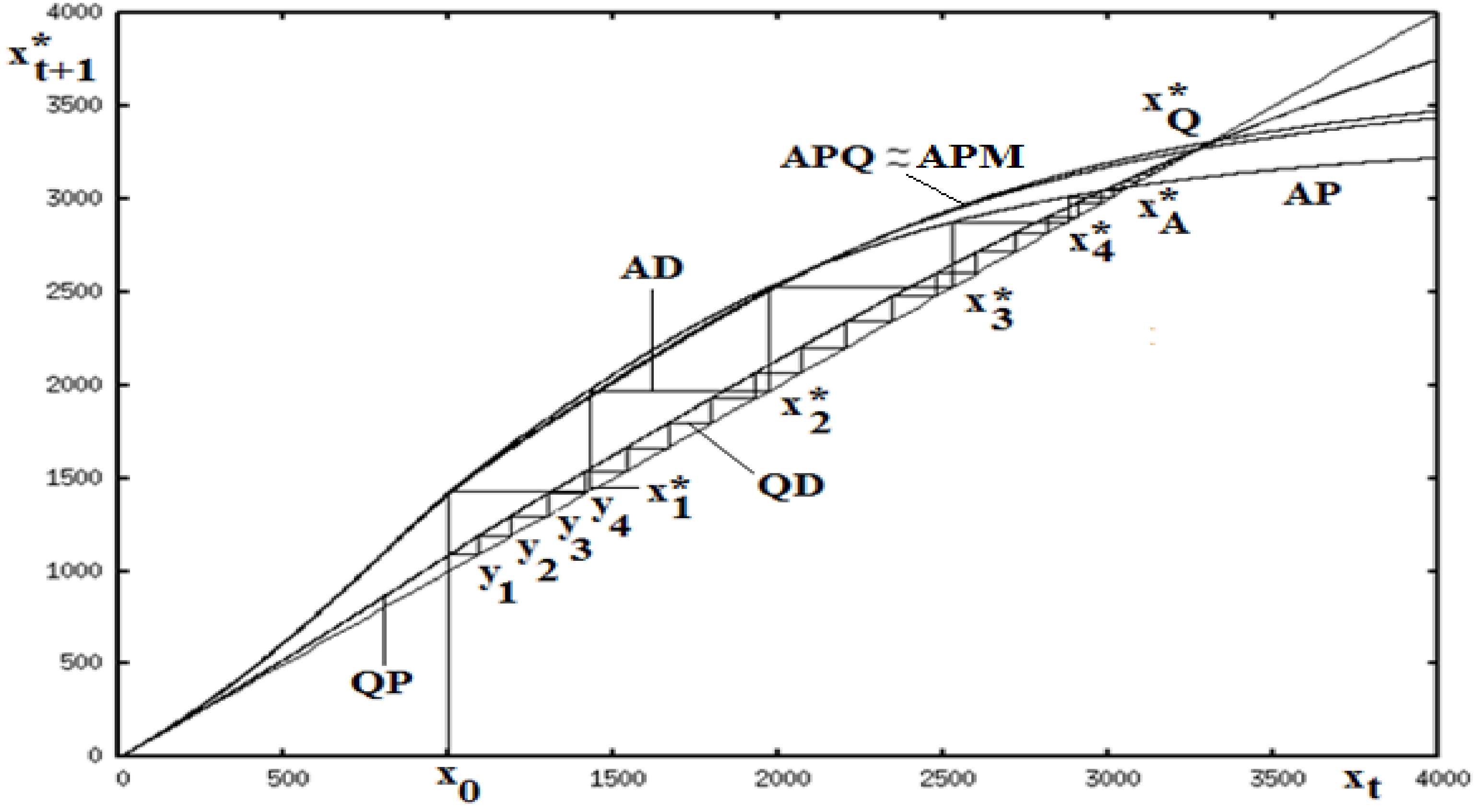

Figure 4.

- Annual optimal policy functions, AP ($\Delta t=1$), and APQ ($\Delta t=0.25$), if the stock is annually and quarterly harvested
- Quarterly optimal policy (QP): optimal stock level in the next quarter (after harvesting) as a function of the stock level in the current quarter

- AD from the initial stock level x_0: $x_0 \rightarrow x_1^* \rightarrow x_2^* \rightarrow x_3^* \rightarrow x_4^* \rightarrow ... \rightarrow x_A^*$
- QD from the initial stock level x_0: $x_0 \rightarrow y_1 \rightarrow y_2 \rightarrow y_3 \rightarrow y_4 \rightarrow ... \rightarrow x_Q^*$

$$x_Q^* > x_A^*$$

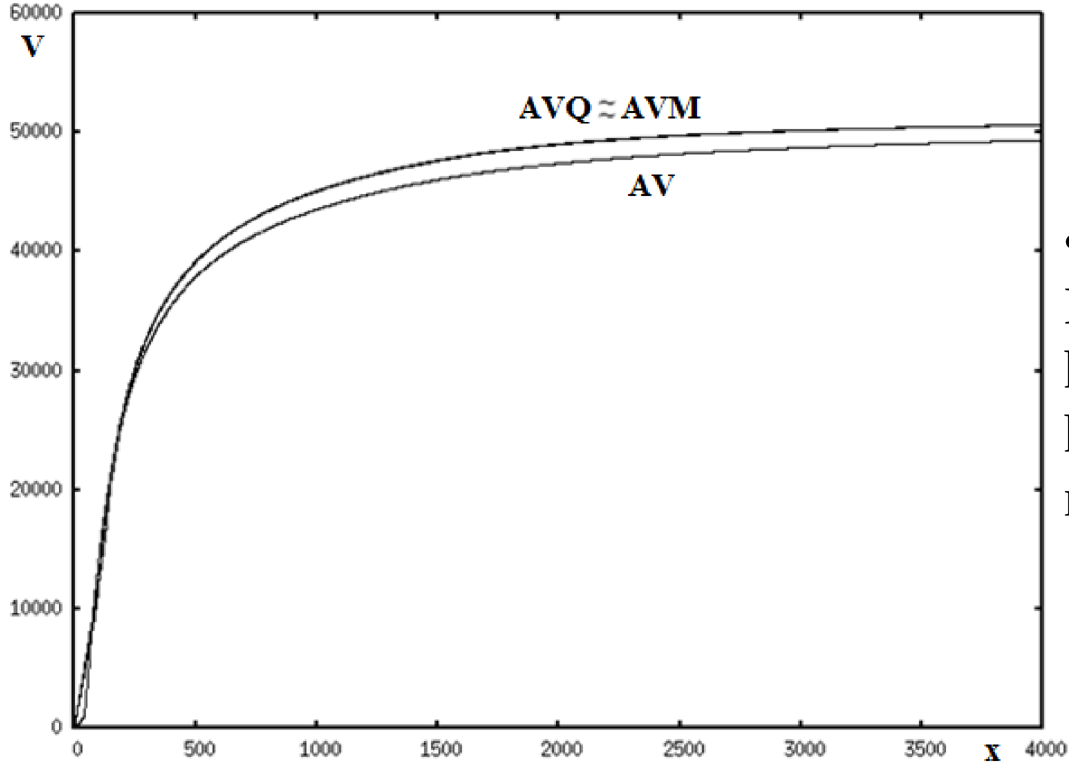

Figure 6.

$$V(x) = \max_{0 \le y \le f_{RK}(x)} [\Pi(x, y, \Delta t) + \delta^{\Delta t} V(y)]$$

- Annual value function (million NOK) if the stock is: annually harvested AV ($\Delta t=1$), quarterly harvested AVQ ($\Delta t=0.25$), and monthly harvested AVM ($\Delta t=0.083$)

- AVQ≈AVM>AV

- DCM is able to deal with seasonality in fisheries

- In the case of the NEAC fishery, we have shown that seasonal harvesting is a win-win optimal solution with higher harvest, higher optimal steady state equilibrium, and higher economic value than those obtained in the case of annual harvesting

Discussion and conclusions

- Knowledge of the relation between CM and DM is crucial in order to avoid biologically and economically meaningless models that can lead to erroneous (suboptimal) policy advice, with the consequent uncertainty regarding the appropriate bioconomic model which should be used to ensure long term sustainability
- DCM allows us both to overcome the biological and economic weakness of DM, and to properly estimate the population dynamics of fish stocks in a continuous-time setting

- DCM can provide a useful tool for testing the safety of biological reference points by analyzing the risk of collapse of seasonal fisheries