Effects of harvesting and strength of competition on the spatial scales of population fluctuations of two competing species

Javier Jarillo Díaz

Universidad Complutense de Madrid (Spain)

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Spatial population synchrony scale

- One species models. Effects of:
  - Environmental fluctuations (Moran effect).
  - Dispersal.
- Two species models. Effects of:
  - Harvesting.
  - Competition.
- Conclusions.

Spatial population synchrony

- Spatial population synchrony: correlation of temporal fluctuations in population size between neighbor localities.
- Causes:
  - Environmental factors (e.g., temperature and humidity).
  - Dispersal.
  - Inter-specific interactions (e.g., competition and predation).
- Implications:
  - Estimation of global extinction risk.
  - Species conservation.
  - Sustainable harvesting strategies.

Moran effect

- Moran (1953) analyzed a linear model of two closed populations, subjected to environmental stochasticity, with no dispersal.
- He found correlations between populations equals environmental correlation,

$$\rho(y) = \rho_e(y)$$

- Thus, population synchrony scale equals the environmental correlation length,

$$l = l_e$$

Dispersal effect

- Observations: cohabitant related species with different dispersal abilities show different synchrony scales.

  Local dynamics

  $$\frac{dN}{dt} = r\,N\,\frac{K - N}{K}$$

  Linear evolution of population fluctuations around equilibrium,

  $\epsilon = \left(N - N^{(eq)}\right)/N^{(eq)}$, with $N^{(eq)} = K$

  $$d\epsilon(z,t) = -(r + m)\,\epsilon(z,t)\,dt + m\,dt\int \epsilon(z - x, t)\,f(x)\,dx + \sigma_e\,dB(z,t)$$

  Spatial population synchrony scale (Lande, Engen, and Sæther, 1999)

  $$l^2 = l_e^2 + \frac{m\,l_m^2}{r}$$

Two competing harvested species: Local dynamics

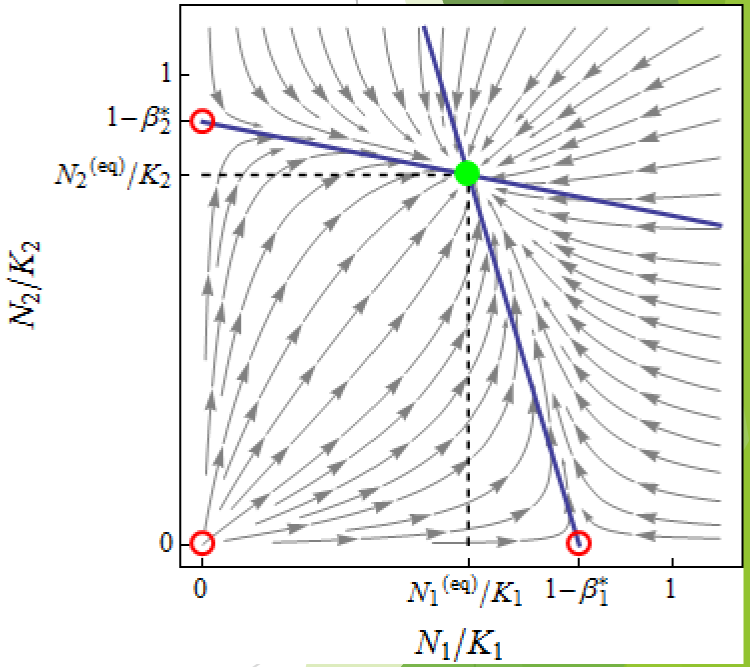
- Evolution equations

$$\frac{dN_1}{dt} = r_1 N_1 \frac{K_1 - N_1 - \alpha_1 N_2}{K_1} - \beta_1 N_1$$
$$\frac{dN_2}{dt} = r_2 N_2 \frac{K_2 - N_2 - \alpha_2 N_1}{K_2} - \beta_2 N_2$$

- Stable coexistence when $1/\left(\alpha_2^*(1-\beta_1^*)\right) > 1 > \alpha_1^*(1-\beta_2^*)$, (with $\alpha_i^* \equiv \alpha_i\, K_j/K_i$ and $\beta_i^* \equiv \beta_i/r_i$), with equilibrium point

$$N_1^{(eq)} = K_1 \frac{1 - \beta_1^* - \alpha_1^*\,(1-\beta_2^*)}{1 - \alpha_1^*\,\alpha_2^*}$$
$$N_2^{(eq)} = K_2 \frac{1 - \beta_2^* - \alpha_2^*(1-\beta_1^*)}{1 - \alpha_1^*\,\alpha_2^*}$$

- Harvesting and competition displace the deterministic equilibrium, reducing the effective carrying capacities.



Two competing harvested species: Population synchrony scales

- Procedures for one species models can be generalized.
- We assume $|\epsilon_i| \ll 1$ (small population fluctuations), $\alpha_i^* \ll 1$ (inter-specific competition weaker than intra-specific competition: competitive exclusion principle), and $\beta_i^* \ll 1$ (harvesting rates smaller than growth rates).

$$l_1^2 = l_{e1}^2 + \frac{m_1\, l_{m1}^2}{r_1}(1 + \alpha_1^*\,\Phi_1 + \beta_1^*) + \mathcal{O}(\alpha_1^{*2}, \beta_1^{*2}, \alpha_1^*\beta_1^*)$$

$$l_2^2 = l_{e2}^2 + \frac{m_2\, l_{m2}^2}{r_2}(1 + \alpha_2^*\,\Phi_2 + \beta_2^*) + \mathcal{O}(\alpha_2^{*2}, \beta_2^{*2}, \alpha_2^*\beta_2^*)$$

(Φ_i: competition sensitivity.)

Harvesting

- Population synchrony scales

$$l_1^2 = l_{e1}^2 + \frac{m_1\, l_{m1}^2}{r_1}(1 + \beta_1^*) + \mathcal{O}(\beta_1^{*2})$$

$$l_2^2 = l_{e2}^2 + \frac{m_2\, l_{m2}^2}{r_2}(1 + \beta_2^*) + \mathcal{O}(\beta_2^{*2})$$

- **Small dispersal contribution** $\Longrightarrow$ harvesting **does not affect** the spatial synchrony scales.
- **Large dispersal contribution** $\Longrightarrow$ harvesting **increases synchrony scale** of both species.

Competition

- Population synchrony scales

$$l_1^2 = l_{e1}^2 + \frac{m_1\, l_{m1}^2}{r_1}\left(1 + \alpha_1^* \, \Phi_1\right) + \mathcal{O}({\alpha_1^*}^2)$$

$$l_2^2 = l_{e2}^2 + \frac{m_2\, l_{m2}^2}{r_2}\left(1 + \alpha_2^* \, \Phi_2\right) + \mathcal{O}({\alpha_2^*}^2)$$

- **Small dispersal contribution** $\Longrightarrow$ competition **does not affect** the synchrony scale.
- Large dispersal contribution $\Longrightarrow$ effect of competition depends on sign of the competition sensitivity.
- We distinguish two cases: uncorrelated or correlated environmental noises.

Competition: uncorrelated environmental noises

- Uncorrelated environmental noises: $\rho_{12}(y) = 0$.
- Competition sensitivities are $\Phi_1 = \Phi_2 = 1$, i.e.,

$$l_i^2 = l_{ei}^2 + \frac{m_i\, l_{mi}^2}{r_i}(1 + \alpha_i^*) + \mathcal{O}\left({\alpha_i^*}^2\right)$$

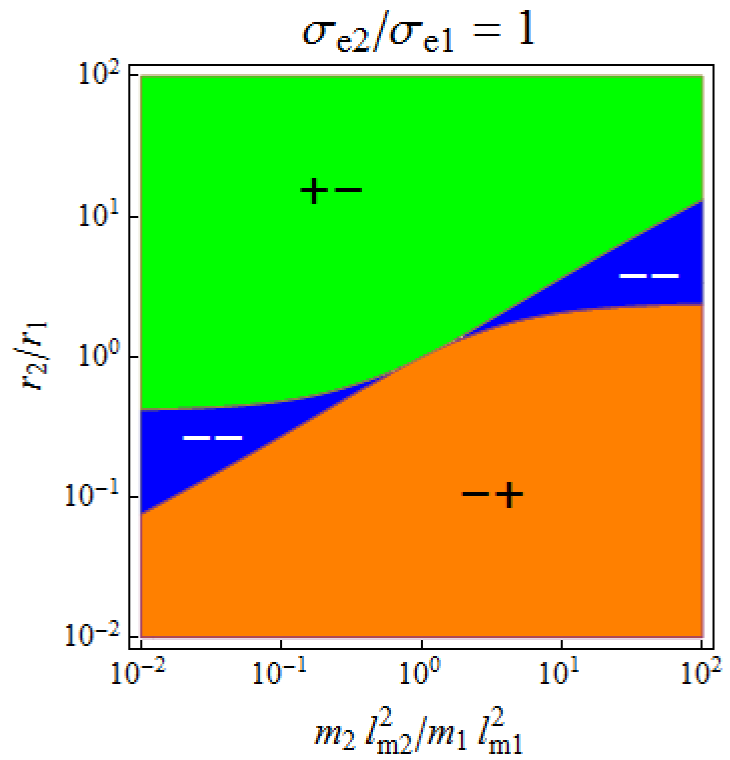
- Competition between species with uncorrelated environmental noises always **increases the synchrony scale of both species**.

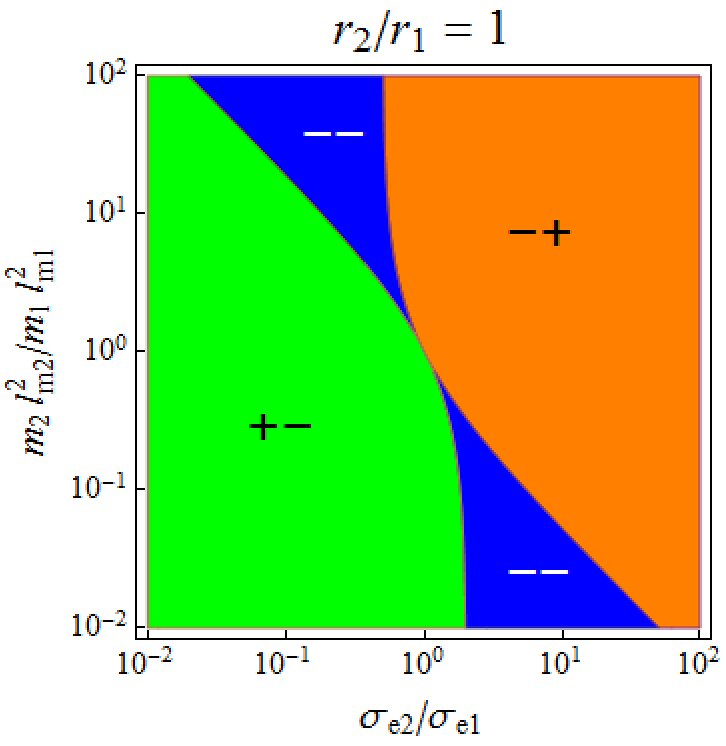
Competition: correlated environmental noises

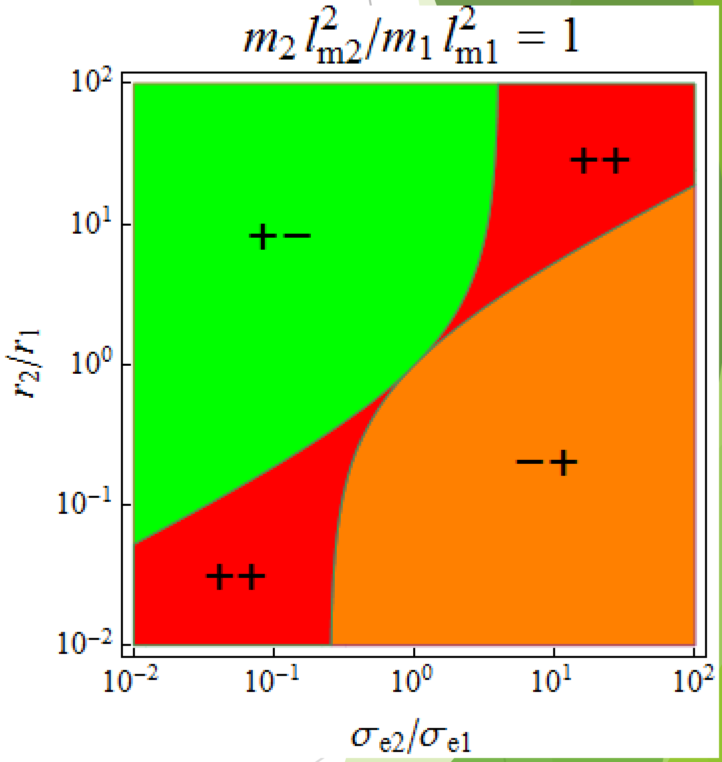
- Correlated environmental noises: $\rho_{12}(y) \neq 0$.
- **<u>A richer situation</u>**: Φ_1 and Φ_2 may be positive or negative.
- For completely correlated environmental noises ($\rho_{12}(y) = \rho_1(y) = \rho_2(y)$), competition increases the spatial scale of the species with
  - the larger environmental variance σ_e,
  - the larger dispersal capacity $m\, l_m^2$,
  - and the smaller growth rate r,

while it decreases the spatial scale of the other one. However, when these effects compete the result is not straightforward.

Competition:
completely correlated environmental noises



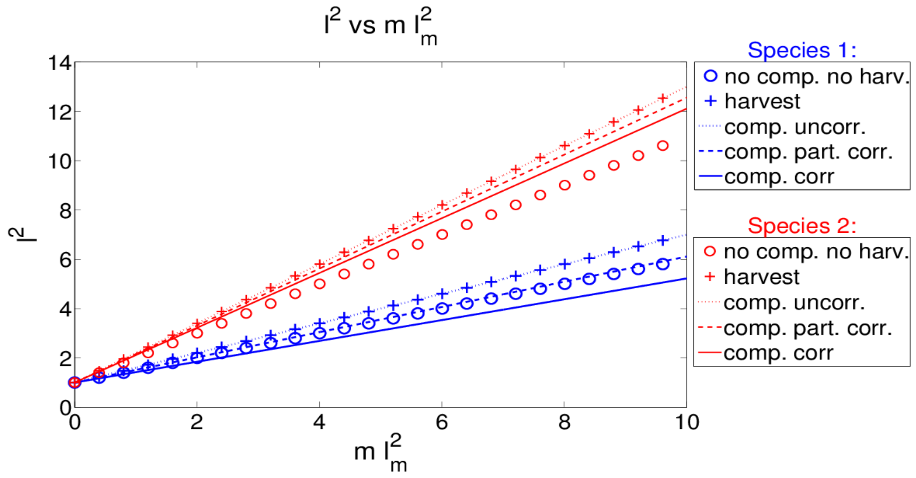




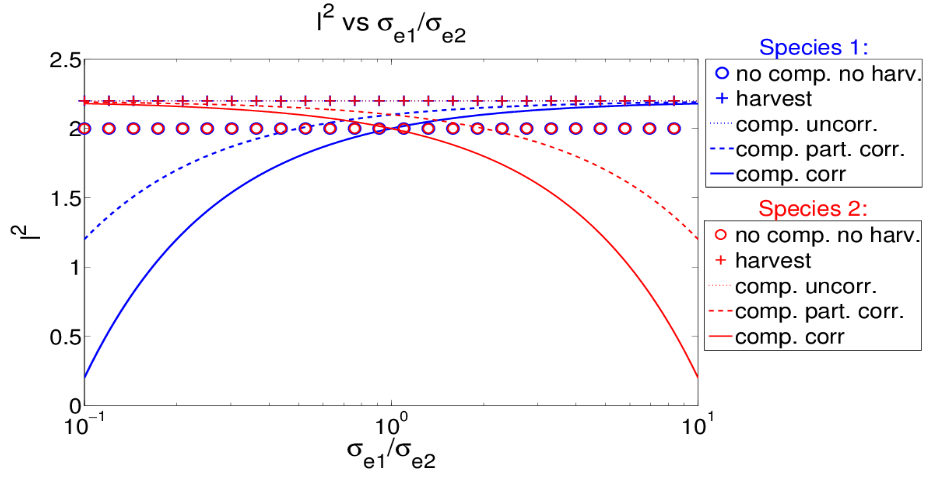
For $\sigma_{e1} = \sigma_{e2}$ or $r_1 = r_2$, never both scales are simultaneously increased
(opposite to uncorrelated noises)

For $m_1 l_{m1}^2 = m_2 l_{m2}^2$, for some sets of values competition increases both synchrony scales
(as in the uncorrelated scenarios)

Example:
effects of harvesting and competition



Synchrony scales as function of the migration capacity, for species with $r_1 = 2$, $r_2 = 1$, $\sigma_{e1} = \sigma_{e2} = 1$, $\alpha_1^* = \alpha_2^* = 0.2$ (for competition cases), and $\beta_1^* = \beta_2^* = 0.2$ (for harvesting cases).



Synchrony scales as function of the ratio of environmental variances, for species with $r_1 = r_2 = 1$, $m_1 l_{m1}^2 = m_2 l_{m2}^2 = 1$, $\alpha_1^* = \alpha_2^* = 0.2$ (for competition cases), and $\beta_1^* = \beta_2^* = 0.2$ (for harvesting cases).

Conclusions

Harvesting and inter-species competition affect the spatial synchrony scales of species when the dispersal contribution is relevant. In this case:

- Harvesting increases the synchrony scales of both species.
- The effect of competition is different for species with uncorrelated environmental noises and for species with correlated environmental noises.
  - For competing species with uncorrelated environmental noises, competition increases the spatial population synchrony scale of both species.
  - For competing species with correlated environmental noises, competition increases the synchrony scale of the species with larger environmental variance, larger migration capacity, and smaller growth rate, while decreases the synchrony scale of the other. However, when these effects compete the result is not straightforward, and competition might increase or decrease the synchrony scales of both species.

These effects are relevant for the sustainable exploitation of natural resources and are useful for the study of global extinction risk.

Thanks for your attention!