Stochastic Systems Simulation and Control

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Reading material (Mar 5, 2015)

John Gough[1] (Aberystwyth University, UK)

Quantum filtering & control

It is a surprising fact that one may formulate a theory of state estimation (filtering) in the framework of quantum mechanics. That is, while quantum measurement is notoriously difficult to interpret, it has a mathematical structure that allows a treatment remarkably similar to classical filtering theories. One may then consider measurement-based feedback, and even specify control problems for quantum systems. We will review the basic theory, and show some recent examples of the actual quantum control. We will also discuss issues such as filter stability, non-classical noise sources, separation theorems, and risk-sensitive problems.

Martin Hairer[2] (University of Warwick, UK)

Regularity structures[3]

These are short notes from a series of lectures given at the University of Rennes in June 2013, at the University of Bonn in July 2013, at the XVIIth Brazilian School of Probability in Mambucaba in August 2013, and at ETH Zurich in September 2013. They give a concise overview of the theory of regularity structures as exposed in the article [Hai14]. In order to allow to focus on the conceptual aspects of the theory, many proofs are omitted and statements are simplified. We focus on applying the theory to the problem of giving a solution theory to the stochastic quantisation equations for the Euclidean Φ^4_3 quantum field theory.

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1 http://www.aqstic.net/people/john-gough

2 http://www.hairer.org/

3 http://www.hairer.org/notes/Regularity.pdf

Terry Lyons[4] (University of Oxford, UK)

Rough paths, Signatures and the modelling of functions on streams[5]

Rough path theory is focused on capturing and making precise the interactions between highly oscillatory and non-linear systems. The techniques draw particularly on the analysis of LC Young and the geometric algebra of KT Chen. The concepts and theorems, and the uniform estimates, have found widespread application; the first applications gave simplified proofs of basic questions from large deviation theory and substantially extending Ito's theory of SDEs; the recent applications contribute to (Graham) automated recognition of Chinese handwriting and (Hairer) formulation of appropriate SPDEs to model randomly evolving interfaces. At the heart of the mathematics is the challenge of describing a smooth but potentially highly oscillatory and vector valued path x_t parsimoniously so as to effectively predict the response of a nonlinear system such as $dy_t = f(y_t)dx_t$, $y_0 = a$. The Signature is a homomorphism from the monoid of paths into the group like elements of a closed tensor algebra. It provides a graduated summary of the path x. Hambly and Lyons have shown that this non-commutative transform is faithful for paths of bounded variation up to appropriate null modifications. Among paths of bounded variation with given Signature there is always a unique shortest representative. These graduated summaries or features of a path are at the heart of the definition of a rough path; locally they remove the need to look at the fine structure of the path. Taylor's theorem explains how any smooth function can, locally, be expressed as a linear combination of certain special functions (monomials based at that point). Coordinate iterated integrals form a more subtle algebra of features that can describe a stream or path in an analogous way; they allow a definition of rough path and a natural linear "basis" for functions on streams that can be used for machine learning.

Dominique Manchon[6] (CNRS, Clermont-Ferrand, France)

The Hopf algebra of finite topologies and mould composition

In recent works by L. Foissy, Cl. Malvenuto and F. Patras[7], a Hopf algebra structure is constructed on the linear span of all topologies on the finite sets [n]={1,...,n}. We will describe a new "internal" coproduct, which interacts with the abovementioned Hopf structure in a precise way. We exhibit a surjective Hopf algebra morphism L onto WQSym as well as a new "internal" coproduct

4 http://people.maths.ox.ac.uk/tlyons/

5 http://dmle.cindoc.csic.es/pdf/MATEMATICAIBEROAMERICANA_1998_14_02_01.pdf

6 http://math.univ-bpclermont.fr/~manchon/

7 http://lanl.arxiv.org/abs/1403.7488 and http://fr.arxiv.org/abs/1407.0476

on WQSym such that L respect both internal coproducts. The picture is most conveniently outlined in the species formalism, which we will briefly explain. Characters of the Hopf algebra of finite topologies can be called "quasi-ormoulds" in J. Ecalle's terminology: convolution with respect to the external coproduct, resp. the internal coproduct, decribes quasi-ormould product, resp. quasi-ormould composition. This is joint work with Frédéric Fauvet (Université de Strasbourg, France), Loïc Foissy[8] (Université du Littoral Côte d'Opale, France).

[8] http://www-lmpa.univ-littoral.fr/fr/fiche.php?personne=Loic%20Foissy