Homological Algebra Workshop

02-03-15/06-03-15

Monday:

9:45–10:00	Registration
10:00–11:15	Conchita Martínez (I)
11:15–11:45	Coffee Break
11:45-13:00	Antonio Diaz (I)
13:00-15:00	Lunch
15:00–16:00	Antonio Viruel (I)
16:05–16:50	Urban Jezernik

Tuesday:

10:00–11:15	Conchita Martínez (II)
11:15–11:45	Coffee Break
11:45-13:00	Antonio Diaz (II)
13:00-15:00	Lunch
15:00-16:00	Antonio Viruel (II)
16:05-16:50	Joan Tent
16:55-17:40	Mima Stanojkovski

Wednesday:

10:00–11:15	Conchita Martínez (III)
11:15–11:45	Coffee Break
11:45-13:00	Antonio Diaz (III)
13:00-15:00	Lunch

Thursday:

10:00–11:15	Conchita Martínez (IV)
11:15–11:45	Coffee Break
11:45-13:00	Antonio Diaz (IV)
13:00-15:00	Lunch
15:00-16:00	Ramón Flores
16:05-16:45	Oihana Garaialde

Friday:

10:00–11:15	Conchita Martínez (V)
11:15–11:45	Coffee Break
11:45-13:00	Antonio Diaz (V)

Introduction to homological algebra

Conchita Martínez, Universidad de Zaragoza

1. Chain complexes: Homology, maps between chain complexes. Simplicial and CW-complexes. Homotopy equivalences and weak equivalences.
2. Projective, injective and flat resolutions. Free, projective, injective and flat modules. Resolutions. Comparison Lemma.
3. Ext and Tor. Ext using projectives. Examples. Derived funtors. Examples in abelian groups and group (co)homology.
4. Snake Lemma. Long exact sequences. Mapping cone. Ext using injectives.
5. Tools. Mayer-vietoris. Universal coefficients theorem. Shapiro. Limits and colimits.
6. Group cohomology and finiteness properties. Milnor construction. Cayley graphs. Finite presentability, properties F, FP, FP_n, cohomological dimension. Examples.
7. Cohomology with bounded support. Coxeter groups.

Spectral sequences

Antonio Díaz, Universidad de Málaga

This course is an introduction to spectral sequences. Proofs will be given only for the essential constructions. There will be plenty of examples to get used to spectral sequences and to learn "how to compute" with them. The intended program is as follows:

Day 1: Spectral sequences as a black box.

- (a) Spectral sequences of differential modules. Convergence. Extension problems.
- (b) Spectral sequences of algebras. Lifting problems.
- (c) Exploiting the gaps. Working backwards.

Day 2: Inside the black box.

- (a) The spectral sequence of a filtered differential module. Exact couples.
- (b) The spectral sequence of a double differential complex.
- (c) Comparison theorems.

Day 3: Spectral sequences in group cohomology.

- (a) The Lyndon-Hochschild-Serre spectral sequence.
- (b) Central extensions. Wreath products. LHS stops.
- (c) Cohomology of finite abelian groups.

Day 4: Spectral sequences in algebraic topology.

- (a) Basics about algebraic topology.
- (b) Leray-Serre spectral sequence.
- (c) Applications.

Day 5: Other spectral sequences in algebra and topology.

Bibliography

For spectral sequences:
J. McCleary, A User's Guide to Spectral Sequences (Cambridge Studies in Advanced Mathematics 58), Cambridge University Press, 2000.
C.A. Weibel, An Introduction to Homological Algebra (Cambridge Studies in Advanced Mathematics 38), Cambridge University Press, 1995.
S. MacLane, Homology (Classics in Mathematics), Springer, 2013.
A. Hatcher, Spectral sequences in algebraic topology, http://www.math.cornell.edu/ hatcher/SSAT/SSATpage.html

For group cohomology:
K.S. Brown, Cohomology of Groups (Graduate Texts in Mathematics, No. 87), Springer, 1994.
L. Evens, The Cohomology of Groups (Oxford Mathematical Monographs), OUP Oxford, 1991.
A. Adem, R.J. Milgram, Cohomology of Finite Groups (Grundlehren der mathematischen Wissenschaften), Springer, 2004.

For algebraic topology:
A. Hatcher, Algebraic Topology, http://www.math.cornell.edu/ hatcher/AT/ATpage.html
R.M. Switzer, Algebraic topology–homotopy and homology (Classics in Mathematics), Springer-Verlag, 2013.

Further references:
R. Bieri, *Homological dimension of discrete groups. Queen Mary College Mathematical Notes.* Queen Mary College Department of Pure Mathematics, London, second edition, 1981.
M. Davis. *The geometry and topology of Coxeter groups.* London Math. Soc. Monographs Series, 32. Princeton Un. Press, Princeton, NJ, 2008.
R. Geoghegan. *Topological Methods in Group Theory. Graduate Texts in Mathematics.* Springer-Verlag, 2008.

A modified version of the Bousfield-Kan tower

Ramón Flores

In this talk we present a modified version of the Bousfield-Kan tower for topological spaces, and explain its relation with the classical tower. We will describe some relations of these functors with the profinite group completion.

Cohomology of p-groups and Carlson's conjecture

Oihana Garaialde Ocaña

Let G be a p-group of order p^n. If c denotes the class of G, then the coclass of G is $n-c$. Leedham-Green and Newman suggested to classify p-groups by coclass. In fact, Leedham-Green has shown that every p-group of fixed coclass "comes from" a finite number of p-adic uniserial space groups. Using the result for $p = 2$, Jon F. Carlson proved that there are only finitely many algebras for the cohomology $H^*(G; \mathbb{F}_2)$ with trivial coefficients in the finite field of 2 elements. In the same paper, he formulates a conjecture stating that the same result should hold for p odd. In this talk, we give a method to show that certain groups have isomorphic cohomology rings and we will give an outline of the steps to answer Carlson's conjecture.

The Schur multiplier

Urban Jezernik

The Schur multiplier is a homological invariant of groups. Basic by nature, yet quite erratic in behaviour. I will present a very much elementary way of looking at this group and illustrate some advantages that such an approach offers.

Evolving groups

Stanojkovski, Mima

Evolving groups arise as a solution to a question that is inspired from phenomena occurring in Galois cohomology. We will formalize the problem and give a characterization.

A cohomological criterion for p-solvability

Joan Tent Jorques

We shall present a criterion for a finite group to be p-solvable based on Tate's p-nilpotency criterion. Related to this, we shall also prove some results relating the number of generators of a Sylow p-subgroup of G with other invariants of the group. This is joint work with Jon González.

Cohomological uniqueness of groups

Antonio Viruel

The Isomorphism Problem in a category $\mathcal{C}$ consists on providing a procedure that determines whether two objects in $\mathcal{C}$ are isomorphic or not. If such a procedure exists and the techniques fit into some theory $\mathcal{T}$, it is said that $\mathcal{T}$ tells objects in $\mathcal{C}$ apart. In this talk we address the question of whether (finite) groups are told apart if we consider mod p cohomology. We shall present the so far only known general strategy to answer this question, and show how different flavors of mod p cohomology lead to cohomological uniqueness of different families of groups.