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Characterizing Lyapunov Domains via Riesz Transforms on Hölder Spaces

Abstract: Under mild geometric measure theoretic assumptions on an open set $\Omega \subset \mathbb{R}^n$, we show that the Riesz transforms on its boundary are continuous mappings on the Hölder space $\mathscr{C}^{\alpha}(\partial\Omega)$ if and only if Ω is a Lyapunov domain of order α (i.e., a domain of class $\mathscr{C}^{1+\alpha}$). In the category of Lyapunov domains we also establish the boundedness on Hölder spaces of singular integral operators with kernels of the form $P(x-y)/|x-y|^{n-1+l}$, where P is any odd homogeneous polynomial of degree l in \mathbb{R}^n . This family of singular integral operators, which may be thought of as generalized Riesz transforms, includes the boundary layer potentials associated with basic PDE's of mathematical physics, such as the Laplacian, the Lamé system, and the Stokes system. This is joint work with Marius Mitrea and Joan Verdera.