MAIN TALKS

Aula Naranja (ICMat, Campus de Cantoblanco)

Abstracts (in alphabetical order):

Akman, Murat	**Mitrea,** Dorina
Auscher, Pascal	**Mitrea,** Irina
Brown, Russell	**Mitrea,** Marius
Chen, Li	**Nyström,** Kaj
Dindoš , Martin	**Rios,** Cristian
Hofmann, Steve	**Tolsa,** Xavier
Mayboroda, Svitlana	**Toro,** Tatiana

Hausdorff dimension of p-harmonic measure

Murat Akman, ICMAT, Spain

In this talk we present a recent result in the study of the Hausdorff dimension of p-harmonic measure in space associated to a positive weak solution to p-Laplace equation in an open subset and vanishing on a portion of the boundary of that open set. We show that p-harmonic measure is concentrated on a set of $\sigma-$finite $n-1$ dimensional Hausdorff measure for $p > n$ and the same result holds for $p = n$ with an assumption on the boundary. We also construct an example of a domain in space for which corresponding p-harmonic measure has Hausdorff dimension strictly less than $n-1$ for $p \geq n$.

Representation for weak solutions of BVP

Pascal Auscher, Université Paris-Sud, France

We report on recent works with S. Stahlhut and M. Mourgoglou.

The first order approach of elliptic systems on the upper half-space with time-independent coefficients, developed in earlier works with Axelsson, Hofmann and McIntosh in the context of boundary value problems, allowed to obtain a complete description of weak solutions of such elliptic systems in the natural classes: for the Neumann problem, one requires L^2 control of a non-tangential maximal function of the gradient; for the Dirichlet problem, one requires the square function of Lusin being in L^2. All weak solutions in each class can then be described in terms of a semigroup: it is in fact, the gradient of the solution that has a trace in an appropriate space and enjoys a semigroup representation from its trace. The recent results of Rosen show that it shows that all solutions must have an abstract layer potential representation.

The extension of such statements with L^p controls, $p \neq 2$, is a natural question. Among the difficulties are that these semigroups do not extend to bounded operators on those L^p and even if they do, they do not have kernel representation if we do not assume regularity conditions such as the De Giorgi property on solutions. Nevertheless, this can be overcome by use ot the theory of Hardy spaces associated to an operator and the statement above can be shown to extend to the natural ranges of p for each problem. No regularity theory for solutions is assumed.

We will describe this work and also connect to other developments in this area.

The mixed problem

Russel Brown, University of Kentucky, USA

I will discuss the mixed problem for the Laplacian in a Lipschitz domain Ω. We let D and N be subsets of the boundary with $\partial\Omega = D \cup N$ and $D \cap N = \emptyset$. The mixed problem for the Laplacian is the problem of finding a solution u to the boundary value problem

$$\begin{cases} \Delta u = 0, & \text{in } \Omega \\ u = f_D, & \text{on } D \\ u = f_N, & \text{on } N \end{cases}$$

The data f_D will have gradient in $L^p(D)$ and $f_N \in L^p(N)$. Our goal is to find conditions on Ω, D, N and p so that the mixed problem has a unique solution u and the nontangential maximal function of ∇u is in $L^p(\partial\Omega)$. We will also discuss results for other elliptic operators. This talk will survey progress on this problem including joint work with M. Wright, J. Taylor, K. Ott, M. Mitrea, I. Mitrea, D. Mitrea, L. Lanzani, and L. Capogna.

Quasi Riesz transforms, Hardy spaces and generalized sub-Gaussian heat kernel estimates

Li Chen, ICMAT, Spain

On Riemannian manifolds satisfying the doubling volume property and the sub-Gaussian heat kernel estimate, we establish that quasi Riesz transforms $\nabla e^{-\Delta}\Delta^{-\alpha}$ ($\alpha \in (0, 1/2)$) are L^p bounded for $1 < p < 2$. While for $p > 2$, Riesz transform may not be L^p bounded. We also study the Hardy space theory on metric measure spaces satisfying the doubling volume property and different local and global heat kernel estimates. We define Hardy spaces via molecules and square functions which are adapted to the heat kernel estimates. We show the relation between different H^1 spaces as well as the relation between H^p and L^p. As an application of this Hardy space theory, we obtain the $H^1 - L^1$ boundedness of the quasi Riesz transforms on Riemannian manifolds as above.

The Dirichlet boundary problem for second order parabolic operators satisfying Carleson condition

Martin Dindoš, University of Edinburgh, UK

This is a joint work with Sukjung Hwang. We establish L^p, $2 \le p \le \infty$ solvability of the Dirichlet boundary value problem for a parabolic equation $u_t - \text{div}(A\nabla u) - B \cdot \nabla u = 0$ on time-varying domains with coefficient matrices $A = [a_{ij}]$ and $B = [b_i]$ that satisfy a small Carleson condition. The result is motivated by similar results for the elliptic equation $\text{div}(A\nabla u) + B \cdot \nabla u = 0$ that were established in the papers Kenig, Pipher, Petermichl and myself. The result complements the papers of Hofmann and Rivera-Noriega where solvability of parabolic L^p (for some large p) Dirichlet boundary value problem for coefficients that satisfy large Carleson condition was established.

The main result says that given $p \in (2, \infty)$ there exists $C(p) > 0$ such that if the Carleson norm of coefficients, namely

$$\delta(X,t)^{-1} \left(\text{osc}_{B_{\delta(X,t)/2}(X,t)} a_{ij}\right)^2 + \delta(X,t) \left(\text{sup}_{B_{\delta(X,t)/2}(X,t)} b_i\right)^2$$

is less then $C(p)$ then the Dirichlet boundary value problem for a parabolic equation is solvable in L^p.

I shall also discuss a second result (with J. Pipher and S. Petermich) that shows that $C(p) \to \infty$ as $p \to \infty$ which is a quantitative result complementing the previous results on L^p solvability under large Carleson condition (where only existence of $p < \infty$ is established, but not how it might depend on the Carleson norm).

Quantitative Rectifiability and boundary behavior of harmonic functions

Steve Hoffmann, University of Missouri, USA

A classical theorem of F. and M. Riesz states that for a simply connected domain in the complex plane with a rectifiable boundary, harmonic measure and arc length measure on the boundary are mutually absolutely continuous. On the other hand, an example of C. Bishop and P. Jones shows that the latter conclusion may fail, in the absence of some sort of connectivity hypothesis.

In this talk, we present a survey of recent developments in an ongoing program to find scale-invariant, higher dimensional versions of the F. and M. Riesz Theorem, as well as converses. In particular, we discuss substitute results that continue to hold in the absence of any connectivity hypothesis. We also discuss related theory for solutions of more general elliptic operators than the Laplacian.

Boundary value problems for elliptic operators with real non-symmetric coefficients

Svitlana Mayboroda, University of Minnesota, USA

Sharp estimates for the solutions to elliptic PDEs in L^∞ in terms of the corresponding norm of the boundary data follow directly from the maximum principle. It holds on arbitrary domains for all (real) second order divergence form elliptic operators $-divA\nabla$. The well-posedness of boundary problems in L^p, $p < \infty$, is a far more intricate and challenging question, even in a half-space. In particular, it is known that some smoothness of A in t, the transversal direction to the boundary, is needed.

In the present work we establish the well-posedness in L^p of the Dirichlet problem for all divergence form elliptic equations with real (possibly non-symmetric) coefficients independent on the transversal direction to the boundary. Equivalently, we show that for all such operators the L-harmonic measure is A^∞ with respect to the Lebesgue measure on the boundary. Previous results in this direction were restricted to the symmetric or to the lower dimensional case.

This is joint work with S. Hofmann, C. Kenig, and J. Pipher.

Characterizing Lyapunov Domains via Riesz Transforms on Hölder Spaces

Dorina Mitrea, University of Missouri, USA

Under mild geometric measure theoretic assumptions on an open set $\Omega \subset \mathbb{R}^n$, we show that the Riesz transforms on its boundary are continuous mappings on the Hölder space $C^\alpha(\partial\Omega)$ if and only if Ω is a Lyapunov domain of order α (i.e., a domain of class $C^{1+\alpha}$). In the category of Lyapunov domains we also establish the boundedness on Hölder spaces of singular integral operators with kernels of the form $P(x-y)/|x-y|^{n-1+l}$, where P is any odd homogeneous polynomial of degree l in $\mathbb{R}^n$. This family of singular integral operators, which may be thought of as generalized Riesz transforms, includes the boundary layer potentials associated with basic PDE's of mathematical physics, such as the Laplacian, the Lamé system, and the Stokes system. This is joint work with Marius Mitrea and Joan Verdera.

Szegö Projections and Kerzman-Stein Formulas

Irina Mitrea, Temple University, USA

Hardy spaces constitute a classical topic at the interface between Complex Analysis and Harmonic Analysis and progress in a deeper understanding of their geometric and functional analytic properties can have a fundamental impact on related issues. For example, the direct topological sum decomposition of $L^2(\Sigma)$ into $\mathcal{H}^2_\pm(\Sigma)$ (traces on Σ of holomorphic functions on either side of Σ) in the case when Σ is a Lipschitz curve in the plane is equivalent to the boundedness of the principal value version of the Cauchy Singular Integral Operator on $L^2(\Sigma)$ (a famous result due to A. P. Calderón for small Lipschitz constants, and to R. Coifman, A. McIntosh and Y. Meyer in full generality). In this talk I will address a closely related issue, namely the question whether the orthogonal projection P of the Hilbert space $L^2(\Sigma)$ onto the closed subspace $\mathcal{H}^2_+(\Sigma)$ (or $\mathcal{H}^2_-(\Sigma)$) has a bounded extension as an operator on $L^p(\Sigma)$ with $p \neq 2$. This is a rather delicate issue, which interfaces tightly with the geometric character of Σ.The main tools are a new generation of commutator estimates and a far-reaching extension of the so-called Kerzman-Stein formula from Complex Analysis. This is based on joint work with Marius Mitrea and Michael Taylor.

Calderón-Zygmund theory and boundary problems on Riemannian manifolds

Marius Mitrea, University of Missouri, USA

In this talk I will explore the manner in which Global Analysis, ψDo's techniques, Singular Integral Operators, and Geometric Measure Theory can provide effective tools in the treatment of boundary value problems on Riemannian manifolds. Special emphasis is placed on the L^p-Dirichlet problem for the Hodge-Laplacian, as well as more general strongly elliptic systems, in a class of uniformly rectifiable domains with unit conormal sufficiently close to VMO. This is joint work with Dorina Mitrea, Irina Mitrea, and Michael Taylor.

Quasi-linear PDEs and low-dimensional sets

Kaj Nyström, Uppsala universitet, Sweden

In this talk I will discuss new results concerning boundary Harnack inequalities and the Martin boundary problem, for non-negative solutions to equations of p-Laplace type with variable coefficients. The key novelty is that we consider solutions which vanish only on a low-dimensional set Σ in $\mathbb{R}^n$ and this is different compared to the more traditional setting of boundary value problems set in the geometrical situation of a bounded domain in $\mathbb{R}^n$ having a boundary with (Hausdorff) dimension in the range $[n-1, n)$. We establish our quantitative and scale-invariant estimates in the context of low-dimensional Reifenberg flat sets. This is joint work with John Lewis.

Off-diagonal estimates and weighted elliptic operators

Cristian Rios, University of Calgary, Canada

We apply Kato square root bounds and full off-diagonal estimates available for weighted elliptic operators, together with a framework developed by Auscher and Martell to obtain several results, including a functional calculus, weighted L^p Kato estimates, and Riesz transform bounds. As an interesting application, we obtain sufficient conditions on the weights so that the corresponding weighted elliptic operators satisfy unweighted Kato bounds. This is a joint project with David Cruz-Uribe and José María Martell.

Rectifiability, the Jones' β coefficients, and densities

Xavier Tolsa, ICREA – Universitat Autònoma de Barcelona /
Universidad Autónoma de Barcelona, Spain

In this talk I will review some recent results regarding the characterization of n-rectifiable sets in $\mathbb{R}^d$ in terms of different square functions involving the so called β coefficients of Jones, David and Semmes and other coefficients which involve differences of densities. These results are valid for sets $E \subset \mathbb{R}^d$ with finite Hausdorff measure $\mathcal{H}^n$ without any doubling assumption.

The arguments are based on a corona type decomposition which can be applied to study the L^2 boundedness of Calderón-Zygmund operators such as Riesz transforms.

Wasserstein Distance and the Rectifiability of Doubling Measures

Tatiana Toro, University of Washington (Seattle), USA

In joint work with J. Azzam and G. David we study the structure of the support of certain classes of doubling measures. We are interested in two different types of doubling measures: those that are locally well approximated by flat measures and those that infinitesimally self similar. In both settings we prove that the support can be decomposed into rectifiable pieces of different dimensions. In this talk we will present of an overview of these results.