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The Dirichlet boundary problem for second order parabolic operators satisfying Carleson condition

Abstract: This is a joint work with Sukjung Hwang. We establish L^p, $2 \leq p \leq \infty$ solvability of the Dirichlet boundary value problem for a parabolic equation $u_t - \mathrm{div}(A\nabla u) - B \cdot \nabla u = 0$ on time-varying domains with coefficient matrices $A = [a_{ij}]$ and $B = [b_i]$ that satisfy a small Carleson condition. The result is motivated by similar results for the elliptic equation $\mathrm{div}(A\nabla u) + B \cdot \nabla u = 0$ that were established in the papers Kenig, Pipher, Petermichl and myself. The result complements the papers of Hofmann and Rivera-Noriega where solvability of parabolic L^p (for some large p) Dirichlet boundary value problem for coefficients that satisfy large Carleson condition was established.

The main result says that given $p \in (2, \infty)$ there exists $C(p) > 0$ such that if the Carleson norm of coefficients, namely

$$\delta(X,t)^{-1}\left(\mathrm{osc}_{B_{\delta(X,t)/2}(X,t)} a_{ij}\right)^2 + \delta(X,t)\left(\sup\nolimits_{B_{\delta(X,t)/2}(X,t)} b_i\right)^2$$

is less then $C(p)$ then the Dirichlet boundary value problem for a parabolic equation is solvable in L^p.

I shall also discuss a second result (with J. Pipher and S. Petermich) that shows that $C(p) \to \infty$ as $p \to \infty$ which is a quantitative result complementing the previous results on L^p solvability under large Carleson condition (where only existence of $p < \infty$ is established, but not how it might depend on the Carleson norm).