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Quasi Riesz transforms, Hardy spaces and generalized sub-Gaussian heat kernel estimates

Abstract: On Riemannian manifolds satisfying the doubling volume property and the sub-Gaussian heat kernel estimate, we establish that quasi Riesz transforms $\nabla e^{-\Delta}\Delta^{-\alpha}$ ($\alpha \in (0, 1/2)$) are L^p bounded for $1 < p < 2$. While for $p > 2$, Riesz transform may not be L^p bounded. We also study the Hardy space theory on metric measure spaces satisfying the doubling volume property and different local and global heat kernel estimates. We define Hardy spaces via molecules and square functions which are adapted to the heat kernel estimates. We show the relation between different H^1 spaces as well as the relation between H^p and L^p. As an application of this Hardy space theory, we obtain the $H^1 - L^1$ boundedness of the quasi Riesz transforms on Riemannian manifolds as above.