Construction of flows without periodic orbits on 3-manifolds

Ana Rechtman Université de Strasbourg

May 6, 2014

Abstract

The problem of determining when the flow of a non-singular vector field on a closed 3-manifold has a periodic orbit has a long history. On one side, there are examples of vector fields whose flow has no periodic orbits on any closed 3-manifold. These were first constructed by P. A. Schweitzer for C^1 vector fields [6], and then by K. Kuperberg in the C^{∞} category, and even real analytic [4]. Schweitzer's construction was then achieved in the volume preserving category by G. Kuperberg, giving C^1 volume preserving vector fields without periodic orbits [3].

On the other side, results by H. Hofer and C. H. Taubes guarantee the existence of a periodic orbit for the flow of a Reeb vector field on any 3-manifold [1, 7]. Recently, the existence of a periodic orbit was stated for a larger class of vector fields [2, 5].

In this course, I will present Schweitzer and K. Kuperberg's constructions, both are based on the use of plugs. A plug is a manifold with boundary of the form $D \times [0, 1]$, with D a disc, endowed with a flow that is inward transverse to $D \times \{0\}$ and outward transverse to $D \times \{1\}$, and has the property that there are orbits entering the plug and trapped. As an introduction to these constructions, I will explain F. W. Wilson's construction of a plug with periodic orbits. The latter was the first construction of a plug and gives the existence of vector field with exactly two periodic orbits on any closed 3-manifold [8].

References

- H. Hofer. Pseudoholomorphic curves in symplectizations with applications to the Weinstein conjecture in dimension three. *Invent. Math.*, 114(3):515–563, 1993.
- M. Hutchings and C. H. Taubes. The weinstein conjecture for stable hamiltonian structures. http://arxiv.org/abs/0809.0140, 31 august, 2008.
- [3] G. Kuperberg. A volume-preserving counterexample to the Seifert conjecture. Comment. Math. Helv., 71(1):70–97, 1996.
- [4] K. Kuperberg. A smooth counterexample to the Seifert conjecture. Ann. of Math. (2), 140(3):723-732, 1994.

- [5] A. Rechtman. Existence of periodic orbits for geodesible vector fields on closed 3-manifolds. Ergodic Theory and Dynamical Systems, 30(6):1817–1841, 2010.
- [6] P. A. Schweitzer. Counterexamples to the Seifert conjecture and opening closed leaves of foliations. Ann. of Math. (2), 100:386–400, 1974.
- [7] C. H. Taubes. The seiberg-witten equations and the weinstein conjecture ii: More closed integral curves of the reeb vector field. arxiv:math/0702366, 2007.
- [8] F. W. Wilson. On the minimal sets of non-singular vector fields. Ann. of Math. (2), 84:529–536, 1966.