"Symmetries of symplectic manifolds with singularities"

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and

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Abstract:

Hamiltonian actions are a central object of study in Symplectic geometry. A special attention has been devoted to the toric case for which the moment map takes values in $\mbox{mathbb } R^n$. Toric symplectic manifolds provide natural examples of integrable systems and every integrable system on a symplectic manifold is a toric manifold in a neighbourhood of a compact fiber (Arnold-Liouville). The classification of toric symplectic manifolds is given by Delzant's theorem in terms of the image of the moment map (Delzant polytope).

The goal of this minicourse is to present a comprehensive tour for the study of Hamiltonian actions by tori for a special class of Poisson manifolds called b-Poisson manifolds/b-symplectic manifold. This is a simple class of Poisson manifolds which is close to the symplectic realm.

We will prove a Delzant theorem for toric b-symplectic manifolds starting with the case of surfaces. These Poisson manifolds admit an adaptation of Moser's path method and the tools are close to the symplectic case. We also plan to comment on applications to quantization and further avenues of study (generalization of b-symplectic manifolds).

The minicourse is organized in three sessions:

Session 1 (by Eva Miranda):

- 1. Motivation and outline of the minicourse
- 2. Classification of circle actions on surfaces
- 3. Classification of symplectic surfaces with singularities of type ax+b.
- 4. Classification of circle actions on b-symplectic surfaces.
- 5. Discussion on applications to quantization.

Session 2 (by Geoffrey Scott)

- 1. Introduction to b-calculus: b-tangent and b-cotangent bundle
- 2. The path method for b-symplectic manifolds.
- 3. A Delzant theorem for b-symplectic manifolds. Applications.

Session 3 (by Geoffrey Scott)

Further avenues of study. Generalizations.

References and further reading:

- 1. M. Audin, Torus actions on symplectic manifolds, second revised edition, Progress in Mathematics, Volume 93 (2004)
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- 3. V. Guillemin, E. Miranda and A.R. Pires, Symplectic and Poisson geometry on bmanifolds, arXiv 1206.2020.
- 4. V. Guillemin, E. Miranda, A.R. Pires and G. Scott, Toric actions on b-symplectic manifolds, arXiv:1309.1897.
- 5. R. Melrose, Atiyah-Patodi-Singer Index Theorem (book), Research Notices in Mathematics, A.K. Peters, Wellesley, 1993.
- 6. R. Nest and B. Tsygan, Formal deformations of symplectic manifolds with boundary, J. Reine Angew. Math. 481, 1996, pp. 27-54.
- 7. O. Radko, A classification of topologically stable Poisson structures on a compact oriented surface, Journal of Symplectic Geometry, 1, no. 3, 2002, pp. 523--542.
- 8. G. Scott, The Geometry of bk Manifolds, http://arxiv.org/abs/1304.3821