ICMAT–China Exploratory Workshop

Madrid November 17-21, 2014

ICMAT
INSTITUTO DE CIENCIAS MATEMÁTICAS

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Titles + Abstracts:

Luis Álvarez-Cónsul (ICMAT-CSIC)

Gravitating vortices and coupled Kähler-Yang-Mills equations

After explaining the relation between vortices on a Riemann surface and instantons on a Kähler surface via dimensional reduction, we go on to study gravitating vortices and their relation to certain coupled equations for Kähler metrics and Yang-Mills connections. Joint work with M. Garcia-Fernandez and O. Garcia-Prada.

José M. Arrieta (ICMAT-UCM)

The biharmonic operator in a domain with an oscillatory boundary

We analyze the spectral behavior of higher-order elliptic operators when the domain is perturbed. We give special attention to the bi-harmonic operator with the so-called intermediate boundary conditions and analyze its behavior when the boundary of the domain has some oscillatory behavior. We will show that there is a critical oscillatory behavior and that the limit problem depends on whether we are above, below or just sitting on this critical value. This is a joint work with Pier D. Lamberti from the University of Padova, Italy.

María Barbero (ICMAT-UC3M)

Applications of Morse families in dynamics

Recently, Lagrangian submanifolds defined by a Morse family have turned out to be very useful to describe physical systems. For instance, the usual necessary conditions for optimality have been recovered with those geometric elements [1]. Moreover, when suitably coupling a Morse family to Dirac structures [2] a new geometric framework arises to interconnect mechanical and control systems [3]. In this talk, we make clear the benefits of using Morse families to handle geometric mechanical systems.

References:

[1] M. Barbero Liñán, D. Iglesias Ponte, D. Martín de Diego, *Morse families in optimal control problems*. Preprint 2012. arXiv:1211.4511

[2] T.J. Courant, *Dirac manifolds*, Trans. Amer. Math. Soc. 319 (2): 631-661, 1990.

[3] M. Dalsmo, A. van der Schaft, *On representations and integrability of mathematical structures in energy-conserving physical systems*. SIAM J. Control Optim., 37(1): 54-91, 1999.

Angel Castro (ICMAT-CSIC)

Shallow waters with vorticity

In this talk we will present a hamiltonian formulation of the water waves problem with vorticity. In the shallow water regime the length of the waves is assumed much larger that the depth of the water. We will show the existence of the shallow water limit and discuss some properties of the models obtained in this approximation.

Fernando Chamizo (ICMAT-UAM)

Lattice points in the 3-dimensional torus

The lattice point discrepancy in $\mathbb{R}^3$ is defined as the difference between the number of points in the lattice $\mathbb{Z}^3$ inside an expanding solid region and its volume. In this talk, we consider the problem for the torus in $\mathbb{R}^3$ generated by the rotation of a circle around the z-axis. We show the underlying ideas leading to prove that the lattice point discrepancy is $O\big(R^\sigma\big)$ for any $\sigma > 4/3$ where R is the scaling parameter. This is a joint work with Dulcinea Raboso.

Felipe Cucker (CityU HK)

On the Computation of Eigenpairs

We describe some recent advances for the problem of computing eigenpairs of complex arbitrary matrices. The nature of these results is mostly theoretical, our algorithms being too slow compared with those currently used, but nonetheless relevant in the measure that they give an answer to the main theoretical question posed for the eigenpair problem.

Hui-Hui Dai (CityU HK)

Series-asymptotic Solutions for Crease Formations in a Gel Layer

An analytical study on crease formations in a swelling gel layer is conducted. By a method of coupled series-asymptotic expansions, we formulate a nonlinear eigenvalue problem of ordinary differential equations (ODEs), which are then solved analytically to obtain solutions for all the post-bifurcation branches. The results provide deep insights on crease formations, including the unveiling of three pathways to crease, determination of the bifurcation type, establishment of a lower bound for mode numbers and two scaling laws. A number of experimental results are captured, whose interpretations are provided. It appears that the present work offers a comprehensive understanding on crease formation, a widely-spread phenomenon. This is a joint work with Xiaoyi Chen.

Carlos Escudero (ICMAT-UAM)

Kinetic theory of two-species coagulation

We will outline our study on the stochastic process of two-species coagulation. This process consists in the aggregation dynamics taking place in a ring. Particles and clusters of particles are set in this ring and they can move either clockwise or counterclockwise. They have a probability to aggregate forming larger clusters when they collide with another particle or cluster. We study the stochastic process by means of Boltzmann equations that do not conserve momentum and therefore give rise to an interesting dynamics. We determine the long time behavior of such a model, making emphasis in one special case in which it displays self-similar solutions. In particular, these calculations answer the question of how the system gets ordered, with all particles and clusters moving in the same direction, in the long time.

Bohan Fang (BICMR)

Eynard–Orantin recursion and all genus mirror symmetry

The mirror of a toric variety is a Landau–Ginzburg model. In case this mirror model is (or could be reduced to) an affine curve, one could run the Eynard–Orantin recursion and obtain higher genus invariants, which

should predict GW invariants of the original toric variety. When the toric variety is a CY 3-fold, this is the BKMP conjecture. In the talk, I will illustrate this phenomenon through a more basic example – the equivariant projective line.

Marco Fontelos (ICMAT-CSIC)

Phase field modelling of drops in electric fields

The modelling of drops under the action of electric fields is of practical importance in connection with microdevices where small amounts of fluid need to be manipulated. Nevertheless, the associated free boundary problems involve various difficulties: presence of moving contact lines, changes in the drop's topology, instabilities of various types,... Some of these can be overcome by introducing a phase field formulation of the problem. We introduce it in a thermodynamically consistent way and present analysis of existence and uniqueness of solutions for the resulting model as well as numerical implementation for various situations of interest.

Francisco Gancedo (Univ. de Sevilla & ICMAT)

On the absence of singularity formation for SQG sharp front and the Muskat problem

The purpose of this talk is to rule out finite time singularity scenarios for initially smooth interfaces separating two incompressible fluids. We will consider two models from fluid dynamics: surface quasi-geostrophic (SQG) sharp front and the Muskat problem. For both systems we show lack of splash singularities. In Muskat we further prove global existence results for controlled initial slope.

Jian Ge (BICMR)

$1/4$-pinched contact sphere theorem

In this talk I will present a proof of $1/4$-pinched sphere theorem, which says if the curvature of a contact 3-manifold with compatible Riemannian metric is $1/4$-pinched, then the contact structure is universal tight. We also have some results for open manifolds. This is a joint work with Yang Huang.

Shuai Guo (BICMR)

Quantum curve and Enumerative geometry

The problem of counting curves dates back to the 19th century. In 1990s, a number of these classical enumerative problems were solved by physicists using mirror symmetry. In this talk, we will introduce a special type of mirror symmetry, where all the B-model invariants be extracted from certain algebraic curve.

Qing Han (BICMR)

Boundary expansions of minimal surfaces in the hyperbolic space

The minimal surface equation in the hyperbolic space is given by a quasi-linear elliptic equation, which is non-uniformly elliptic and becomes singular on the boundary. The focus of this talk is to use the expansion near the boundary to discuss the regularity of solutions.

Feimin Huang (CAS)

Compressible Euler equations and related problems

In this talk, I will present some progress on the hyperbolic system of conservation laws, in particular, on the compressible Euler equations and related problems.

Libing Huang (Chern)

Curvatures of Left Invariant Finsler Metrics on Lie Groups

By introducing the notion of spray vector and connection operator on the Lie algebra, we obtain elegant formulas for the curvatures of a left invariant Finsler metric on a Lie group. The applications of these formulas allow several classical theorems in Riemannian geometry to be generalized to Finsler case. For example, it is proved that any left invariant Finsler metric on a noncommutative nilpotent Lie group cannot have constant Ricci curvature. Besides, we construct a few new examples with constant flag curvature or constant Ricci curvature. All these examples have vanishing S curvature.

Alberto Ibort (ICMAT-UC3M)

On the geometrical foundations of Field Theories on manifolds with boundary: Palatinis formulation of gravity

Multisymplectic geometry provides a nice framework to describe the geometrical setting of classical field theories. This formalism will be extended to incorporate manifolds with boundaries to provide the foundations for the formulation of Quantum Field theories. The example of Palatini's gravity will be analyzed.

Andrei Jaikin (ICMAT-UAM)

Pro-p methods in the solution of some problems about finite p-groups

I will explain how pro-p groups can help to answer some questions about finite p-groups. The origin of pro-p methods lies in the Leedham–Green's solution of the coclass conjectures. In my talk I will give an answer on the following two questions.

1. Let G be a finite non-abelian p-group. Is it true that $|G|$ divides $|Aut(G)|$?
2. Let k be an odd natural number. Is there an infinite number of finite 2-groups with exactly k real irreducible characters?

Some results of the talk are obtained in collaboration with Jon González-Sánchez and Joan Tent.

Min Ji (CAS)

An Integral Identity and Measure Estimates for Stationary Fokker–Planck Equations

We consider a Fokker-Planck equation in a general domain in $\mathbb{R}^n$ with L^p_{loc} drift term and $W^{1,p}_{loc}$ diffusion term for any $p > n$. It is a partial differential equation of parabolic type and comes from the stochastic differential equation. By deriving an integral identity, we give several measure estimates for stationary solutions in an exterior domain with respect to the diffusion and Lyapunov-like or anti-Lyapunov-like functions. These estimates will be useful to problems such as the existence

and non-existence of stationary solutions in a general domain as well as the concentration and limit behaviors of stationary solutions as diffusion tends to zero.

Xiangdong Li (CAS)

W-entropy formula and optimal transportation on Riemannian manifolds

In 2002, G. Perelman introduced the W-entropy and proved its monotonicity along the Ricci flow. As its application, Perelman proved the noncollapsing theorem for Ricci flow. On the other hand, J. Lott, K.-T. Sturm and C. Villani proved the convexity of the Boltzmann entropy along the geodesics on the Wasserstein space over Riemannian manifolds with non-negative Bakry–Emery Ricci curvature. In this talk, we present some new results in the study of the W-entropy and optimal transport problem on Riemannian manifolds with fixed or time dependent metrics and potentials. We prove the W-entropy formula for the heat equation of the Witten Laplacian on manifolds with fixed and time dependent Riemannian metrics and potentials, and give a probabilistic interpretation of the W-entropy. Moreover, we introduce a family of geometric flows on the Wasserstein space over manifolds, which interpolate the heat equation of the Witten Laplacian and the geodesic flow on the Wasserstein space, and prove the convexity of the Boltzmann entropy along these flows.

Ruochuan Liu (BICMR)

The eigencurve is proper

The Coleman–Mazur eigencurve is a p-adic analytic space that parameterizes (in an appropriate sense) overconvergent p-adic analytic families of systems of Hecke eigenvalues arising from classical eigenforms on GL_2. This geometric object and its generalizations for higher-rank groups have been extremely useful in the investigation of arithmetic properties of special values of L-functions, geometrization and generalizations of "Hida theory", and beyond.

A very basic geometric question for a long time has been whether the eigencurve is "proper" in the sense that analytic maps into it from a

punctured disc extend across the puncture (reminiscent of the valuative criterion for properness in algebraic geometry). In the first half of the talk we will discuss the eigencurve and the properness question. Then we relate this problem to p-adic interpolation properties arising from modular forms, and finally we present the proof of properness using techniques in p-adic Hodge theory. This is joint work with Hansheng Diao.

Fernando Lledó (ICMAT-UC3M)

Modular Theory and its applications to Quantum Physics

In the first part of this talk I will review basic facts on Modular Theory for von Neumann algebras. In particular, I will present the so-called modular conjugation and modular group. I will illustrate these objects with several elementary examples. In the second part of the talk I will mention several applications of this theory to quantum physics.

- F. Lledó, *Modular Theory by example*, Cont. Math. 534 (2011) 73-96

Shiguang Ma (Chern)

On the radius pinching estimates and the uniqueness of constant mean curvature surfaces in asymptotically flat 3 manifolds

The existence and uniqueness of CMC spheres in asymptotically flat manifolds is an interesting problem in mathematical relativity and differential geometry. In my talk, I will describe the recent progress in this area and my work on the uniqueness of CMC spheres.

Vicente Muñoz (ICMAT-UCM)

$Spin(7)$-instantons on complex 4-tori

Using gauge theory for $Spin(7)$ manifolds of dimension 8, we develop a procedure, called Spin-rotation, which transforms (stable) holomorphic structures on a vector bundle over a complex torus of dimension 4 into a new holomorphic structure over a different complex torus. We show non-trivial examples of this procedure by rotating a decomposable Weil abelian variety into a non-decomposable one. As a byproduct, we obtain a Bogomolov type inequality, which gives restrictions for the existence of stable bundles on an abelian variety of dimension 4.

Javier Parcet (ICMAT-CSIC)

The Riesz transform for the Poisson process

C. Fefferman observed in 1970 that

$$g_\lambda^*(f)(x) = \Big(\int_{\mathbb{R}_+} \int_{\mathbb{R}^n} \Big[\frac{t}{|x-y|+t} \Big]^{n\lambda} t^{1-n} |\nabla P_t f(y)|^2 \, dy \, dt \Big)^{\frac{1}{2}}$$

is not in $L_p(\mathbb{R}^n)$ when $\lambda \leq 2/p$. As a consequence, one can see that Meyer's 1984 formulation for Riesz transforms associated to diffusion semigroups fails for the Poisson process and $p \leq 2n/n+1$. In this talk, we shall reformulate Meyer's problem to incorporate any fractional Laplacian and solve it with dimension free estimates. It turns out that all Hörmander–Mihlin multipliers are Littlewood–Paley averages of our Riesz transforms. This is surprising and also useful since it yields new Sobolev/Besov type endpoint conditions. The Sobolev type condition we give refines the classical one and yields dimension free constants. The novelty of these "classical" estimates is explained from the appearance of (intrinsic) noncommutative phenomena in the problem, even for the Poisson process in the Euclidean space. Our approach holds in fact for arbitrary Markov convolution semigroups on group von Neumann algebras associated to any locally compact unimodular group. Other new estimates also include the word length of free groups. Lust–Piquard's recent work for discrete laplacians on LCA groups is also generalized in several ways.

Daniel Peralta (ICMAT-CSIC)

Eigenfunctions with prescribed nodal sets

I will show that given any separating hypersurface of a closed manifold, there exists a Riemannian metric such that the nodal set of its first nontrivial Laplace–Beltrami eigenfunction is the aforementioned hypersurface. Applications to critical points of low energy eigenfunctions and nodal sets of Dirichlet eigenfunctions will be provided. This is based on joint work with A. Enciso.

Tania Pernas Castaño (ICMAT-CSIC)
Non-splat singularity for the one-phase Muskat problem

The Muskat problem models the evolution of the interface between two fluids of different characteristics in porous media. After a brief introduction of the problem in the framework of the finite-time singularities, the aim of this talk is to prove the absence of splat type singularities formation.

David Ríos (ICMAT, AXA Chair)
AXA Chair Colloquium

A framework for risk management in aviation safety at state level

Any state participating at ICAO (International Civil Aviation Organisation) must develop a State Plan for Operational Security. Despite how technologically advanced is the aviation industry, it is surprising how simplified are the risk management methods adopted in it worldwide. Here I shall describe a project for developing a state-of-the-art framework and tool for risk management in aviation safety in Spain. I shall focus on mathematical and computational challenges in relation with issues like non-homogeneous Poisson processes, underreporting, large data (but scarce data for some purposes!!), extreme events, expert judgement modelling, multi-attribute consequences and the presence of both private and public agents in the sector.

Anibal RODRÍGUEZ (ICMAT-UCM)

A Research Group in PDEs and Infinite Dimensional Dynamical Systems at ICMAT: a prospective overview

We review several research lines developed in the area of PDEs/Infinite dimensional dynamical systems carried out in the last years by a research group integrated in ICMAT. We will discuss the main results and contributions as well as some future developments.

Keith Rogers (ICMAT-CSIC)

Directional maximal operators and lacunarity in higher dimensions

The Kakeya maximal problem considers the directional maximal operator with uniformly distributed directions (the problem being to determine how fast the operator norm blows up as the number of directions is increased). On the other hand, Strömberg, Córdoba and Fefferman proved that certain infinite sets of directions give rise to bounded maximal operators provided the directions are lacunarily distributed inside a circle. More recently, Bateman characterised completely the sets of directions for which the maximal operator is bounded in two dimensions. The most elegant way to prove the positive part of his theorem is via a localisation principle due to Alfonseca, Soria and Vargas. In this talk, I will recall these results in more detail and describe how they can/could be extended to higher dimensions. This is joint work with Javier Parcet.

Jesús M. Sanz Serna (ICMAT-UC3M & Univ. de Valladolid)

Building on Feng Kang's foundations

Prof. Feng Kang (1920-1993) was a strong supporter of the idea that numerical methods have to reflect essential qualitative features of the system being simulated; errors, invariable present in numerical analysis, must not only be quantitatively small but also qualitatively acceptable. I shall discuss a number of problems, deterministic and stochastic, from a point of view that may be traced back to Feng Kang.

Binyong Sun (CAS)

p-adic Rankin–Selberg L-functions for $GL(n) \times GL(n-1)$

Kazhdan–Mazur–Schmidt and Schmidt construct p-adic Rankin–Selberg L-functions for $GL(n) \times GL(n-1)$ in the ordinary case. We will explain a representation theoretic reformulation of this construction.

Xiaotao Sun (CAS)

Frobenius split type of moduli spaces of parabolic bundles

A variety over a field of characteristic zero is called of Frobenius split type if there is a dense set of primes p such that its modulo p reduction is Frobenius split. A nice vanishing theorem of cohomology holds for such kind of varieties. In this talk, I will show that moduli spaces of parabolic bundles over a curve is of Frobenius split type.

Piergiulio Tempesta (ICMAT-UCM)

The Lazard formal group, Universal Congruences and L-series

Formal group theory plays a central role in modern algebraic topology and mathematical physics. It will be shown that the notion of universal Bernoulli polynomials and a class of L-functions can be associated with the Lazard universal formal group. General Almkvist–Meurman-type congruences are constructed for the universal Bernoulli polynomials.

Short Bibliography:

• M. Hazewinkel, Formal Groups and Applications, Academic Press, New York, 1978

• P. Tempesta, *The Lazard formal group, universal congruences and special values of zeta functions*, Transactions of the American Mathematical Society, to appear

• P. Tempesta, *L-series and Hurwitz zeta functions associated with the universal formal group*, Annali Scuola Normale Superiore, Classe di Scienze, IX (2010), 1-12

Gang Tian (BICMR)

Analytic Minimal Model Program through Ricci flow

TBA

Hong Wang (Chern)

Symmetric Reduction of Regular Controlled Hamiltonian System and Its Applications

The theory of controlled mechanical systems is a very important subject, following the theoretical development of geometric mechanics, a lot of important problems about this subject are being explored and studied. In this talk, I shall introduce briefly some recent developments of regular symplectic reduction and Hamilton–Jacobi theory of regular controlled Hamiltonian (RCH) systems with symmetry, as well as their applications for the rigid spacecraft with an internal rotor and underwater vehicle with two internal rotors.

Yuefei Wang (CAS)

On Dynamical Systems over Berkovich Space

In recent years there is a considerable interest in non-Archimedean dynamics. The non-Archimedean fields behave topologically very badly, which are totally disconnected and not locally compact. In 1990 Berkovich introduced a new space, which is compact, path-connected and contains the non-Archimedean fields as dense subspaces in the sense of Gel'fand topology. We will talk about the dynamical systems over Berkovich projective space and properties of the Berkovich Fatou and Julia sets, with applications to p-adic dynamics.

Roderick Wong (CityU HK)

Asymptotics of Linear Recurrences

While asymptotic theory for second-order linear differential equations is now well known, the same cannot be said about the second-order linear difference equations. In this talk, I will mention some of the difficulties we have encountered in the development of such a theory for difference equations, and present some of the results that we have obtained in the last 25 years.

Tong Yang (CityU HK)

A Well-posedness Theory of the 3D Prandtl Layer Equations

The well-posedness of the 3D Prandtl layer equations is studied both locally and globally in time under some constraint on its flow structure. It reveals that the classical Burgers equation plays an important role in the analysis. And the monotonicity condition on the velocity and the favorable conidtion on pressure are illustrated in the 3D setting. Moreover, the linear stability of this structured flow is justified. This is a joint work with Chengjie Liu and Yaguang Wang.

Zhifei Zhang (BICMR)

Global well-posedness results for the Navier–Stokes equations

In this talk, I will talk about some recent results on the global well-posedness of the Navier–Stokes equations for some classes of large initial data. The proof of these results mainly relies on Harmonic analysis tools (especially, Littlewood–Paley theory).

Xiang Zhou (CityU HK)

Calculation of Transition State: numerical analysis and algorithmic development

Calculation of transition states for rare events on energy landscape is very important in computational chemistry, material sciences, etc. These transition states are index-1 saddle points. I will talk about the Gentlest Ascent Dynamics and Iterative Minimization Formulation to calculate the transition states. These two methods are based on the min-mode (eigenvector-following) methodology. Their theoretical convergence issues as well as applications will be discussed and presented.