Workshop on Geometry and Dynamics of Foliations

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Abstracts booklet

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INVITED LECTURES

Dynamics of the horocyclic flow for homogeneous and non-homogeneous foliations by hyperbolic surfaces

Fernando Alcalde Cuesta

ECSING Research Group - GeoDynApp Team

The aim of this talk is to present some progress towards the understanding of the dynamics of the horocyclic flow on compact foliated manifolds by hyperbolic surfaces. This is motivated by a question formulated by Matilde Martínez and Alberto Verjovsky on the minimality of this flow when the action of the affine group generated by the combined action of the geodesic and horocyclic flows is minimal too.

Firstly, we shall extend the classical theorem proved by Gustav A. Hedlund in 1936 on the minimality of the horocyclic flow on compact hyperbolic surfaces to homogeneous manifolds for the product of $\mathrm{PSL}(2,\mathbb{R})$ and any connected Lie group G. We shall give an elementary proof that does not use the famous Ratner's Orbit-Closure Theorem. We shall also show that this is always the case for homogeneous Riemannian and Lie foliations. This is a joint work with Françoise Dal'Bo.

Examples and counter-examples will take an important place in our talk. They will serve to illustrate our result, as well as a theorem by Martínez and Verjovsky that characterises the minimality of the affine action. We shall use another classical example to briefly describe some work in progress with Dal'Bo, Martínez and Verjovsky in the non-homogeneous case.

C^1-actions in dimension 1: centralizers and rigidity

Christian Bonatti

with Églantine Farinelli, Nacho Monteverde, Andrés Navas, Cristobal Rivas

CNRS and Université de Bourgogne

A classical consequence of Szekeres' and of Kopell's works is that the C^1-centralizer of a C^2 diffeomorphism of $[0,1]$ is abelian unless the diffeomorphism coincides on the identity map on some non-empty open set. This shows that a purely algebraic assumption on a subgroup of $\mathrm{Diff}^2([0,1])$ allows us to recover topological properties.

Neither Szekeres nor Kopell's work hold for C^1 diffeomorphisms and the C^1 centralizer of C^1 diffeomorphisms may be very huge: for instance, with Farinelli, we proved that there are diffeomorphisms of $[0,1]$ with no fixed point in $(0,1)$ and whose centralizer contains a free group $\mathbb{F}_2$.

Nevertheless, with Monteverde, Navas and Rivas we prove a C^1-rigidity property of the Bauslag-Solitar group $B(1,2) = \langle\, g, f \mid ghg^{-1} = h^2 \,\rangle$ which implies that the centralizer of a diffeomorphism f of $[0,1]$ contains a subgroup isomorphic to $B(1,2)$ if and only if f coincides with the identity map on some non-empty open interval. In other words we recovered a purely algebraic assumption on subgroups of $\mathrm{Diff}^1([0,1])$ which leads to topological properties.

Contact perturbations of Reebless foliations are universally tight

JONATHAN BOWDEN

Universität Augsburg

We will discuss the relationship between tightness for contact structures and Reebless foliations in light of Eliashberg and Thurston's approximation theorem. This leads to a characterisation of Reeblessness in terms of contact topology and suggests an interesting extension of tightness for confoliations.

Smoothability of finite depth foliations

LAWRENCE CONLON

with JOHN CANTWELL

Washington University in St. Louis

We study foliations of finite depth on compact n-manifolds, developing a condition sufficient for C^1-smoothability. The condition is that "all junctures have bounded growth type." This is applied to the finite depth foliations of sutured 3-manifolds constructed by David Gabai from well groomed sutured manifold hierarchies. This construction is not strictly algorithmic and allows choices of junctures that do not have bounded growth, but we show that one can always choose junctures with bounded growth. Gabai's foliations will be C^2-smoothable if and only if the junctures can be chosen to be compact (0 growth). This is typically impossible, but when it happens the construction is algorithmic, the resulting C^2 foliation is actually C^∞ and is uniquely determined by the sutured manifold hierarchy. For depth 2 foliations, a sufficient condition for C^1-smoothability is that the junctures have quasi-polynomial growth.

Centralizers of smooth interval diffeomorphisms

HÉLÈNE EYNARD-BONTEMPS

IMJ – PRG, Université Pierre et Maris Curie (Paris, France)

When studying the space of smooth $\mathbb{Z}^2$-actions on the interval or the circle, one is confronted with the existence of diffeomorphisms with "nasty" centralizers, the first examples being due to Sergeraert in the seventies. More precisely, while the C^1 centralizer of a smooth contraction f of the half-line $[0, +\infty)$ is always a one-parameter group, naturally identified to $\mathbb{R}$, with $f \simeq 1$, the C^∞ centralizer, on the other hand, can assume various shapes. In Sergeraert's construction, for example, it restricts to the infinite cyclic group generated by f. But it can get "worse": the C^∞ centralizer can contain irrational numbers, and thus be dense in $\mathbb{R}$, while not being all of $\mathbb{R}$. It is then natural to wonder, more generally, which subgroups of $\mathbb{R}$ can be realized as the C^∞ centralizer of such a contraction. A first observation, as the construction we will present suggests, is that there might be som e restrictions on the kind of irrational numbers that can arise in this setting.

Knot theory of R-covered Anosov flows: Homotopy versus isotopy of closed orbits

SERGIO R. FENLEY

with THOMAS BARTHELME

Florida State University

Anosov flows have infinitely many closed orbits - a countable number. About 20 years ago we proved the following unexpected and striking result: suppose that Φ is an $\mathbb{R}$-covered Anosov flow in an atoroidal manifold. Then every closed orbit is freely homotopic to infinitely many other closed orbits. $\mathbb{R}$-covered means that the stable foliation lifts to the universal cover as a foliation with leaf space the real numbers $\mathbb{R}$. We also showed that there is an infinite family of examples satisfying the hypothesis (the manifolds are hyperbolic).

In this talk we analyse whether these infinitely many freely homotopic orbits represent the same knot in the manifold. That is, are they isotopic? We show that if the stable foliation is transversely orientable then this is indeed true. This means that there is an infinite family of closed orbits which are isotopic to each other. To analyse this question we use the topological structure of skewed, $\mathbb{R}$-covered Anosov flows, which is very rich. We then use the universal circle of (say) the stable foliation of the flow. This allows us to produce geometric walls in the universal cover, which realize the free homotopies between closed orbits described above. These walls project to a free homotopy between the closed orbits in the manifold. Then we do a very careful analysis of the possible self intersections of the quotient annuli and show they produce an isotopy between the closed orbits, even in the case that the annulus is not embedded.

Linearization and geometry around leaves

RUI LOJA FERNANDES

University of Illinois at Urbana Champaign

In this talk I will discuss some recent results concerning the normal forms and the geometry around leaves of geometric structures that have an underlying foliation, due to several people (Crainic, Marcut, Struichner, Weinstein, Zung, etc.). A unifying role is played by a far reaching result on linearization around saturated submanifolds, valid for any Riemannian Lie groupoid, obtained recently in joint work with Matias del Hoyo.

Towards the classification theorem for onedimensional pseudogroups

VICTOR KLEPTSYN

with B. DEROIN, D. FILIMONOV AND A. NAVAS

Université de Rennes

My talk, based on joint works with B. Deroin, D. Filimonov and A. Navas, will be devoted to the recent progress in the understanding of (pseudo)-group actions on the circle, as well as foliations of real codimension one.

One large class of such actions is those that are sufficiently rich: there are local flows in local closure. Roughly speaking, restricting the dynamics on some subinterval J and closing it in $C^1(J)$, one finds a one-parameter subgroup generated by some vector field (to be more precise, a neighborhood of identity in such subgroup: the flow is no longer defined once the points leave J). In this case, it is easy to obtain the Lebesgue-ergodicity of the action as a corollary of the one of such local flow (and there are some other interesting conclusions) – as do Loray, Nakai, Rebelo, Scherbakov. As Ghys' commutator technique shows, an analytic action is guaranteed to fall in this class provided that there is a free subgroup, generated by the elements sufficiently close to the identity.

Another large class consists of the actions admitting a Markov partition. The presence of such a partition is quite restrictive, giving us a good control on the action. An example of such action is the standard action of $PSL(2,\mathbb{Z})$, or (in the non-minimal case) the Schottky group.

Recent results, obtained in a joint project with B. Deroin, D. Filimonov, A. Navas suggest (though do not establish in its full generality) that there is nothing else but these two classes. In other words, the following alternative seems to hold: an action either admits a Markov partition, or has local flows in its local closure.

Compact foliations in Poisson geometry

David Martínez Torres

Pontificia Universidade Catolica, Rio de Janeiro

A Poisson manifold is foliated by symplectic leaves; the behavior of the (possibly singular) foliation can be rather complicated. We introduce 'compact type conditions' for Poisson manifolds so that their leaf spaces become orbifolds (i.e, our conditions can be thought of as analogs of finite holonomy for compact foliatons). We will also see how these orbifolds are endowed with integral affine structures. This is joint work with M. Crainic and R. L. Fernandes.

The space of contact Anosov flows on 3-manifolds

Shigenori Matsumoto

Department of Mathematics, Nihon University, Tokyo

In this talk, we investigate the space of contact Anosov flows on an oriented closed 3-manifold N. An Anosov vector field A on N is said to be *contact* if it is the Reeb vector field of some contact form τ, that is, if $\tau(A) = 1$ and $\iota_A(d\tau) = 0$. If A is contact Anosov, then it leaves the volume form $\tau \wedge d\tau$ invariant: $\mathcal{L}_A(\tau \wedge d\tau) = 0$.

Let Ω be a fixed C^∞ volume form on N, and let $\tau \wedge d\tau = f\Omega$ for some positive valued C^∞ function on N. Then the vector field fA, called a *time change of* A, leaves Ω invariant.

Denote by $\mathcal{A}_\Omega(N)$ the space of the Ω-preserving Anosov vector fields, and by $\mathcal{C}A_\Omega(N)$ the subspace of $\mathcal{A}_\Omega(N)$ consisting of the time changes of contact Anosov flows. It is natural to ask how $\mathcal{C}A_\Omega(N)$ looks like in $\mathcal{A}_\Omega(N)$.

Until before [1], the only known examples of contact Anosov flows are geodesic flows of negatively curved surfaces. Our main result is the following.

Main Theorem The space $\mathcal{C}A_\Omega(N)$ forms an C^1-open subset of an affine subspace of $\mathcal{A}_\Omega(N)$ of codimension equal to the Betti number of N. In particular, if N is a rational homology sphere, the subset $\mathcal{C}A_\Omega(N)$ is C^1-open in $\mathcal{A}_\Omega(N)$.

In [1], plenty of examples of contact Anosov flows are constructed on various manifolds including hyperbolic 3-manifolds. The above theorem can also serve as producing new examples which are C^1-near to classical examples.

[1] P. Foulon and B. Hasselblatt, *Contact Anosov flows on hyperbolic 3-manifolds,* Geometry and Topology **17**(2013), 1225-1252.
[2] S. Matsumoto, *The space of (contact) Anosov flows on 3-manifolds,* J. Math. Sci. Univ. Tokyo **20**(2013), 445-460.

Haefliger structures and giggling as tools in the h-principles

Gaël Meigniez

Université de Bretagne-Sud

Incompressible fluids on foliated manifolds

Yoshihiko Mitsumatsu

Dept. of Math., Chuo University, Tokyo, Japan

Even though analytical foundations for the fluid mechanics is still very hard to establish and not yet enough developed, it is still a tempting idea to look at fluids how they flow on manifolds for understanding the topology and geometry of manifolds. Moreover in the case of foliated manifolds. we even encounter a difficulty in writing down a proper Euler equation, the equation of motion for ideal foliated fluids.

This talk is concerning a more primitive stage than the genuine fluid mechanics, namely, trying to understand, in the case of codimension 1 foliations on closed oriented 3-manifolds, the space of velocity fields of foliated ideal fluids. One of the main tools is the *asymptotic linking.*

For a volume form $dvol$ on a closed oriented manifold M, $\mathcal{X}$ denotes the set of smooth vector fields on M, $\mathcal{X}_d$ the set of divergence free vector fields, and $\mathcal{X}_h$ the kernel of the *asymptotic cycle* : $\mathcal{X}_d \twoheadrightarrow H_1(M)$. The asymptotic linking lk is a symmetric bi-linear form on $\mathcal{X}_h$.

Under the presence of a codimension 1 foliation $\mathcal{F}$ on M $\mathcal{X}(M;\mathcal{F}) = \{X \in \mathcal{X}\ ;\ X/\!\!/\mathcal{F}\}$ and $\mathcal{X}_*(M;\mathcal{F}) = \mathcal{X}(M;\mathcal{F}) \cap \mathcal{X}_*$ for $* = d,\ h$. Also $\mathcal{X}_{loc}(M;\mathcal{F})$ denotes the span of locally supported ones in $\mathcal{X}_d(M;\mathcal{F})$, which is, in a reasonable sense, fairy understandable.

Our fist observations are the relations between the spaces $\mathcal{X}_d(M;\mathcal{F}) \supset \mathcal{X}_h(M;\mathcal{F}) \supset \mathcal{X}_{loc}(M;\mathcal{F})$ and lk. Especially we see $\mathcal{X}_h(M;\mathcal{F})/\mathcal{X}_{loc}(M;\mathcal{F})$ carries an induced pairing from $(\mathcal{X}_h(M;\mathcal{F}),\ lk)$. The next observation relates the induced pairing on $\mathcal{X}_h(M;\mathcal{F})/\mathcal{X}_{loc}(M;\mathcal{F})$ to a differential invariants on $H^1(M;\mathcal{F})$ (an interpretation of the Godbillon-Vey by Arraut-dos Santos). Our previous results on $H^1(M;\mathcal{F})$ implies the nontriviality of this quotient. Of course the image of $\mathcal{X}_d(M;\mathcal{F})/\mathcal{X}_h(M;\mathcal{F})$ in $H_1(M)$ depends on $\mathcal{F}$. We will introduce a nontrivial example of this image problem.

If the situation allows, not only the Euler equation but also the topics around conservation laws are discussed.

[1] Arnold VI, Khesin BA. Topological Methods in Hydrodynamics. Springer; 1998.
[2] Arraut JL, Dos Santos NM. The characteristic mapping of a foliated bundles, Topology, 31-2 (1998), 545-555.

[3] Matsumoto Sh, Mitsumatsu Y. Leafwise cohomology and rigidity of certain group actions, Ergodic Theory and Dynamical Systems, 23 (2003), 1839-1866.
[4] Mitsumatsu Y. Helicity in Differential Topology and Incompressible Fluids on Foliated 3-Manifolds, Procedia IUTAM Volume 7, (2013), 167-174.

Foliated symplectic topology

Francisco Presas

ICMAT, Madrid

Symplectic Topology has become a deep subarea of the classical Differential Topology. The main reason is twofold:

1. there are unexpected rigidity phenomena making the intersection theory richer (Floer theory) and providing unexpected behaviours in the naturally associated dynamical systems (Arnold's, Conley's and Weinstein's conjectures),
2. the category is flexible enough to capture many examples: existence of several h–principles for the classification problem of contact structures, existence of symplectic h–cobordism theorems, etc.

The goal of this talk is to introduce a class of foliations that allows us to define a "Foliated Symplectic Topology". The so called contact and symplectic foliations allows us to create a dictionary taking Symplectic Topology concepts into the Foliation Theory framework. As an example we will detail the study of the "overtwisted contact foliations" (flexible object) and we will state a foliated Weinstein conjecture that we will partially prove (rigid phenomenon).

CONTRIBUTED TALKS

Isometry flows on orbit spaces and applications to the theory of foliations

Marcos M. Alexandrino

Instituto de Matemática e Estatística, Universidade de São Paulo (USP), Brazil

In this talk, we discuss the following result: Given a proper isometric action $K \times M \to M$ on a complete Riemannian manifold M then each continuous isometric flow on the orbit space M/K is smooth, i.e., it is the projection of a K-equivariant smooth flow on the manifold M. The first application of our result concerns Molino's conjecture, which states that the partition of a Riemannian manifold into the closures of the leaves of a singular Riemannian foliation is still a singular Riemannian foliation. We prove Molino's conjecture for the main class of foliations considered in his book, namely orbit-like foliations. We also discuss smoothness of isometric actions on orbit spaces. This talk is based on a joint work with Dr. Marco Radeschi (wwu- Munster) [1] and is aimed at a broad audience of students, faculties and researchers in Geometry.

[1] Marcos M. Alexandrino, Marco Radeschi, *Smoothness of isometric flows on orbit spaces and Molino's conjecture*. Preprint (2014) arXiv:1301.2735 [math.DG]

Rigidity of certain solvable actions on the tori

Masayuki Asaoka

Department of Mathematics, Kyoto University

For $n \geq 1$ and $k \geq 2$, let $\Gamma_{n,k}$ be the finitely presented group

$$\langle a, b_1, \ldots, b_n \mid ab_ia^{-1} = b_i^k,\, b_ib_j = b_jb_j\ (\forall i, j = 1, \ldots, n)\rangle.$$

This group is solvable and $\Gamma_{1,k}$ is the Baumslag-Solitar group $BS(1,k)$. For a basis $B = (v_1, \ldots, v_n)$ of $\mathbb{R}^n$, we define an affine $\Gamma_{n,k}$-action $\hat{\rho}_B$ on $\mathbb{R}^n$ by $\hat{\rho}_B^a(x) = kx$ and $\hat{\rho}_B^{b_i}(x) = x + v_i$. There are two natural compactifications of $\mathbb{R}^n$. The first is the one-point compactification $S^n = \mathbb{R}^n \cup \{\infty\}$. The second is the product $\mathbb{T}^n = (\mathbb{R} \cup \{\infty\})^n$ of the one-point compactification of $\mathbb{R}$. The affine $\Gamma_{n,k}$-action on $\mathbb{R}^n$ extends to both compactifications. We write $\bar{\rho}_B$ and ρ_B them for the extended actions to S^n and $\mathbb{T}^n$, respectively. Remark that ρ_B coincides with $\bar{\rho}_B$ if $n = 1$.

For the case $n = 1$, Burslem and Wilkinson ([3]) proved the local rigidity of ρ_B. For the case $n \geq 2$ and S^n, the action $\bar{\rho}_B$ is not locally rigid but it exhibits rigidity in the following sense.

Theorem ([1]) *For any given basis B_0 of $\mathbb{R}^n$, there exists a C^2-neighborhood $\mathcal{U}$ of $\bar{\rho}_{B_0}$ in the space of $\Gamma_{n,k}$-actions on S^n such that any C^∞ action ρ in $\mathcal{U}$ is C^∞ conjugate to $\bar{\rho}_B$ for some basis B. In particular, ρ preserves a smooth conformal structure diffeomorphic to the standard conformal structure on S^n.*

The proof is divided into two parts; local rigidity of the local action at the global fixed point ∞ and extension of a local conjugacy to a global one.

In this talk, we apply the method used in the proof of this theorem to another extension ρ_B of $\hat{\rho}_B$ on $\mathbb{T}^n$. Remark that we can see that ρ_B is not locally rigid if $n \geq 2$, like the S^n case.

Main Theorem ([2]) *For any given basis B_0 of $\mathbb{R}^n$, there exists a C^2-neighborhood $\mathcal{U}$ of ρ_{B_0} in the space of $\Gamma_{n,k}$-actions on $\mathbb{T}^n$ such that any C^∞ action ρ in $\mathcal{U}$ is C^∞ conjugate to ρ_B for some basis B.*

Proof of the first part – local rigidity of the local action at the global fixed point – is almost same as the S^n case. Main difficulty for $\mathbb{T}^n$ case appears in the second part – extension of a local conjugacy to a global one.

[1] M.Asaoka, Rigidity of certain solvable actions on the sphere. *Geom. and Topology* **16** (2013), 1835–1857 (electronic).
[2] M.Asaoka, Rigidity of certain solvable actions on the tori. preprint.
[3] L.Burslem and A.Wilkinson, Global rigidity of solvable group actions on S^1. *Geom. Topol.* **8** (2004), 877-924 (electronic).

Classification of isoparametric foliations on complex projective spaces

Miguel Domínguez-Vázquez

IMPA, Rio de Janeiro, Brazil

An isoparametric foliation on a Riemannian manifold is a singular Riemannian foliation whose regular leaves have parallel mean curvature and the distribution defined by their normal bundles is integrable. When such a foliation is induced by a Lie group isometric action, we say that the foliation is homogeneous.

The study of these objects goes back to Levi-Civita, Segre and Cartan, who showed that all codimension one examples in Euclidean and hyperbolic spaces are homogeneous. In spheres, however, there are inhomogeneous examples of codimension one [2], but not of higher codimension [3].

Recently, in [1] we obtained the complete classification of irreducible isoparametric foliations of codimension at least two on complex projective spaces, as well as an almost complete classification in codimension one. Surprisingly, there are inhomogeneous examples even in codimension larger than one.

In this talk, I plan to discuss the main ideas of our classification, which include the use of the Hopf fibration and of the so-called extended Vogan diagrams. I will also present an unexpected characterization of the inhomogeneous examples by means of prime numbers.

[1] M. Domínguez-Vázquez, Isoparametric foliations on complex projective spaces, arXiv:1204.3428v3, to appear in *Trans. Amer. Math. Soc.*
[2] D. Ferus, H. Karcher, H. F. Münzner, Cliffordalgebren und neue isoparametrische Hyperflächen, *Math. Z.* **177** (1981), no. 4, 479–502.
[3] G. Thorbergsson, Isoparametric foliations and their buildings, *Ann. of Math.* (2) **133** (1991) 429–446.

On classification of structurally stable diffeomorphisms with 2-dimensional nonwandering sets on 3-manifolds.

VIACHESLAV GRINES

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It was proved by R. Plykin in 1971 that any two-dimensional basic set Ω of a diffeomorphism f satisfying Axiom A by Smale (A-difffeomorphism) given on a 3-manifold M^3 is either an attractor or a repeller. It was proved in [1] that any structurally stable diffeomorphism $f : M^3 \to M^3$ whose nonwandering set contains a 2-dimensional expanding attractor or a contracting repeller is topologically conjugated to a DA-diffeomorphism obtained from Anosov diffeomorphism by Smale surgery operation.

A basic set Ω of f is called a *surface basic set* if it belongs to an invariant surface topologically embedded in M^3. It was proved in [2] that a 2-dimensional surface basic set of an A-diffeomorphism $f : M^3 \to M^3$ is homeomorphic to a torus tamely embedded in M^3 and the restriction of f^k is topologically conjugate to an Anosov automorphism for some $k \geq 1$. It was recently proved by R. Brown that any 2-dimensional basic set of an A-diffeomorphism $f : M^3 \to M^3$ is either an expanding attractor or an attracting repeller or a surface basic set.

The present report is devoted to the complete topological classification of structurally stable diffeomorphisms given on M^3 whose nonwandering sets consist of 2-dimensional surface attractors and repellers. The results were obtained in collaboration with Yu. Levchenko, V. Medvedev and O. Pochinka.

The author thanks grants RFBR 12 -01- 00672 -a, 13 -01- 12452 - ofi-m and RNF 14-11-00446 for partially financial support.

[1] Grines, V. and Zhuzhoma, E. On structurally stable diffeomorphisms with codimension one expanding attractors, Trans. Amer. Math. Soc., 2005 , vol. 357 , no. 2 , pp. 617 - 667.
[2] V.Z. Grines , V.S. Medvedev, E.V. Zhuzhoma. On Surface Attractors and Repellers on 3-manifolds. Mathematical Notes, 2005, 78:6, 757–767.

Transitive dual foliations

LUIS GUIJARRO

Universidad Autónoma de Madrid - ICMAT

A few years ago, Burkhard Wilking introduced the idea of the dual foliation associated to a singular Riemannian foliation in a manifold with a nonnegatively curved metric. Roughly said, the dual leaves are constructed by taking curves that stay orthogonal to the original leaves everywhere. In the same paper, Wilking showed that when the manifold has positive sectional curvature, the dual foliation contains only one leaf. We refer to this phenomena as saying that the dual foliation is transitive.

In this talk we prove that when the original foliation is given by the fibers of a Riemannian submersion, the dual foliation is often transitive even without the positive curvature assumption due to a variety of reasons. First, we give conditions on the long exact homotopy sequence associated to the submersion asssuring transitivity of the dual foliation. We also prove transitivity for torus actions on simply-connected manifolds, and for principal submersions.

Variation formulas for transversally harmonic maps

Seoung Dal Jung

Jeju National University, Korea

Let $(M, \mathcal{F})$ and $(M', \mathcal{F}')$ be two foliated Riemannian manifolds and let $\phi : M \to M'$ be a smooth foliated map, i.e., ϕ is a leaf-preserving map. Then ϕ is a transversally harmonic map if it is a solution of $\tau_b(\phi) = 0$, where $\tau_b(\phi)$ is a transversal tension field, which is given by $\tau_b(\phi) = \mathrm{tr}_Q \tilde{\nabla} d_T \phi$. That is, transversally harmonic maps are considered as harmonic maps between the leaf spaces [1,2]. In this talk, we give variation formulas for transversally harmonic maps and some applications [3].

[1] J. Konderak and R. Wolak, *On transversally harmonic maps between manifolds with Riemannian foliations*, Quart. J. Math. Oxford Ser.(2) 54(2003), 335-354.
[2] J. Konderak and R. Wolak, *Some remarks on transversally harmonic maps*, Glasgow Math. J. 50(2008), 1-16.
[3] S. D. Jung, *Variation formulas for transversally harmonic and biharmonic maps*, J. Geom. Phys. 70(2013), 9-20.

Group actions, Foliations and Genericity

Gilbert Hector

We propose a review of Foliations and Laminations Theory at the junction of different streams of investigations. Precisely consider a lamination $\mathcal{L}$ on a compact space M, a complete transversal Q and the holonomy pseudo-group $\mathcal{P}$ acting on Q. Our approach relies on the following observations and results:

(1) Álvarez-López and Candel (and others) observed in [1] that $\mathcal{P}$ may be considered as a **finitely generated group** of transformations of Q so that any leaf L of $\mathcal{L}$ has the quasi-isometry type of a homogeneous space.

(2) Genericity properties were observed by E. Ghys in [4] in the measure theoretic setting using the harmonic measures of L. Garnett, later extended to minimal laminations by Cantwell-Conlon in [3]. They noticed that there exists a "residual" set of leaves having all either one, two or a Cantor set of ends; a situation which is similar to that of finitely generated groups.

(3) Finally following Bermúdez-Hector in [4] or F. Paulin in [5], one observes that several classical descriptions and theories extend to a very general setting of "foliations" (or group actions) obtained by dropping transverse continuity thus defining for example the so-called Borel-topological (BT for short) category of laminations or measured group actions.

Now our goal is threefold:

(A) For a given lamination $(M, \mathcal{L})$ the group $(Q, \mathcal{P})$ is not uniquely defined but all its representatives are Kakutani equivalent and therefore share common invariants like growth types, endsets, amenability, being or not HNN-extensions, ... Moreover one can may consider them as groups of

i) homeomorphisms if the lamination is transversely modeled on the Cantor set,

ii) Borel isomorphisms if one is interested in measure theoretical aspects,

iii) **Baire or residual homeomorphisms** that is bijective transformations which restrict to homeomorphisms of some residual subset of Q.

We translate to those different settings classical group theoretical notions and results. For example, we investigate the notions of amenability, hyperfiniteness, affability...

(B) In particular, we present a simplified and unified treatment covering all the classical genericity results and show that our approach gives rise to new results for the existence of residual subsets of leaves all of the same topological type.

(C) Also Ghys shows that when the leaves have "generically" more than one end, the holonomy pseudo-group $\mathcal{P}$ is a so-called "HNN-extension" of pseudo-groups. This latter notion is inspired by the corresponding notion for groups. Now considering $\mathcal{P}$ as a group, it becomes essential to compare these two notions.
Indeed we will show that they don't agree and describe the precise relations between them.

[1] Álvarez-López J. and Candel A., *Generic coarse Geometry of leaves*, preprint, Santiago de Compostela (2003).
[2] Bermúdez M. and Hector G., *Laminations hyperfinies et revêtements*, Ergodic Th. and Dynam. Systems, 26 (2006), (2), 305-339.
[3] Cantwell J. and Conlon L., *Generic leaves*, Comment. Math. Helv., 73 (2)(1998), 306-336.
[4] Ghys E., *Topologie des feuilles génériques*, Ann. of Math., 141 (2)(1995), 387-422.
[5] Paulin F., *Analyse harmonique des relations d'equivalence mesurées discretes*, Markov Proc. Rel. Fields, 5 (1999), 163-200.

Riemannian foliations of bounded geometry

Yuri A. Kordyukov

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The notion of bounded geometry for Riemannian foliations was first introduced by Sanguiao in [1]. The definition given in [1] involves certain normal foliation charts. Following [2], we give an equivalent chart-free definition of this notion. We also discuss some properties of Riemannian foliations of bounded geometry and describe applications to a trace formula for foliated flows.

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Anosov diffeomorphisms: one unstable-leafwise curve with globally defined holonomy

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with Victor Kleptsyn

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A famous conjecture due to Smale [3] and Newhouse [2] says that each Anosov diffeomorphism is topologically conjugated to a diffeomorphism given by some standard algebraic procedure. In particular, Smale's conjecture implies that the universal cover of the phase space of an Anosov diffeomorphism is $\mathbb{R}^n$.

Consider the holonomy map along a curve in an unstable leaf. Generally speaking, this map is defined in some subdomain of the stable leaf passing through the initial point of the curve. If the holonomy map is defined in the whole stable leaf, we shall say that it is *globally defined*. A folklore argument says that if a holonomy map along each unstable-leafwise curve is globally defined, then the universal cover of the phase space is $\mathbb{R}^n$. Recently Victor and me proved [1] that there exists *one* unstable-leafwise curve of given diameter with globally defined holonomy map.

The main idea behind the proof is to measure small leafwise distances by their rate of growth under iterations of the Anosov diffeomorphism. I shall discuss the proof and possible ways to generalize our result. I shall also briefly describe some homological obstructions to being a phase space of an Anosov diffeomorphism.

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An exotic $S^3 \times \mathbb{R}$ which is a non-leaf

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We show that there exists an exotic $S^3 \times \mathbb{R}$ which cannot be a leaf of a C^1 codimension 1 foliation in a compact manifold. It would be, as far as we know, the first topologically end periodic non-leaf (of course, for the smooth case) and the first exotic non-leaf.

The point for the construction relies in the so called *complexity* [1] of exotic $\mathbb{R}^4$. It is known that one of the main differences between exotic $\mathbb{R}^4$'s and the standard $\mathbb{R}^4$ is the fact that big compact sets cannot be separated from the end by a smooth sphere. Complexity measures this difference; it is defined by

$$c(X) = \sup_{K \subset X} \left\{ \inf_{\Sigma | K} \beta_1(\Sigma) \right\} .$$

Where K runs over the compact sets of X and Σ runs over the smooth 3-submanifolds disconnecting K from the end of X (this separation condition is denoted by $\Sigma|K$).

In [1], an exotic $\mathbb{R}^4$ with infinite complexity is described. Thus we can find an exhaustion by compact sets $K_1 \subset K_2 \subset \cdots \subset K_n \subset \cdots$ and a increasing function $b : \mathbb{N} \to \mathbb{N} \cup \{0\}$ such that K_1 is a standard 4-ball and K_n cannot be separated from the end by a smooth 3-submanifold with first Betti number lower than $b(n)$. Now, the sets $B_n = K_n \setminus K_{n-1}$ for $n \geq 2$ will be called *blocks*. We can easily puncture this exotic $\mathbb{R}^4$ and copy the same end structure in the new end obtaining an exotic $S^3 \times \mathbb{R}$ which we shall denote by X. This will be our non-leaf.

For each separating block B_n there exists a compact set C_n such that every open set in $X \setminus C_n$ separating X is not diffeomorphic to the interior of B_n. This key property forbids recurrences on X which is the ingredient used to follow the Ghys procedure [3] for codimension 1 foliations. For arbitrary codimension, the study of the Brownian motion in this manifold in the sense of [4] is enough to show that X cannot be a generic leaf in the sense of L. Garnett [2], i.e., relative to a harmonic measure.

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The Calabi invariant and extensions of Diffeomorphism groups

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Let D be a disc in $\mathbb{R}^2$ of radius $r > 0$ with a symplectic form $dx \wedge dy$ and G denote the group of symplectomorphisms on D. There exists a short exact sequence of groups involving G:

$$1 \longrightarrow G_{\mathrm{rel}} \longrightarrow G \longrightarrow G_{\partial} \longrightarrow 1,$$

where $G_\partial = \mathrm{Diff}_+(\partial D)$ is the group of orientation preserving diffeomorphisms on the boundary ∂D and $G_{\mathrm{rel}} = \{g \in G : g|_{\partial D} = \mathrm{id}\}$ the group of relative symplectomorphisms on the disc. On the normal subgroup G_{rel} one has a celebrated homomorphism $\mathrm{Cal} : G_{\mathrm{rel}} \to \mathbb{R}$ due to Calabi. Thus, dividing the exact sequence by the kernel of Cal, one obtains another exact sequence:

$$0 \longrightarrow \mathbb{R} \longrightarrow G/\ker(\mathrm{Cal}) \longrightarrow G_\partial \longrightarrow 1,$$

which turned out to be a central extension of G_∂. Then Tsuboi's theorem [2] can be rephrased as follows: the resulting class of the central extension in $H^2(G_\partial;\mathbb{R})$ is a constant multiptle of the Euler extension of G_∂, namely, the central extension given by the universal covering space of G_∂ and the constant is equal to a symplectic volume of the disc. In this talk I will exhibit another proof of Tsuboi's theorem. A novelty of the proof is twofold; we employ a double complex due to Bott [1], which is a simplicial de Rham model of foliated disc bundle and introduce a notion, so called 'connection cochain', to clarify the relation between the Calabi homomorphism and the Euler class. In fact, we finally obtain a transgression formula connecting the Euler class to the Calabi homomorphism.

We also talk about several applications of the transgression formula. First we relate the Calabi homomorphism to the first Miller-Morita-Mumford class after generalizing the exact sequence to the case of the Hamiltonian diffeomorphisms on a punctured surface with genus more than 1. The second application is to derive an integration formula for the Euler class of a flat circle bundle. By using the formula one can obtain a cyclic cocycle and a longitudinal index theorem on a flat circle bundle that involves the power of the Euler class. Third, we mention the case of a higher dimensional disc with a symplectic form. The Calabi homomorphism also induces a central extension of a certain diffeomorphism group on the boundary. We clarify the relation to the Euler cocycle of the central extension.

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Connected but not locally connected minimal sets of codimension two foliations

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The classification of the minimal sets plays an important role in the study of codimension one foliations. In case of codimension two foliations, there are many kinds of minimal sets and it is impossible to classify all the minimal sets. Thus we fix a set as a target, and consider constructing and classifying codimension two foliations with such a minimal set. For example, we obtained several results when the minimal set is the Siepiński set (joint works with A. Biś and P. Walczak).

In this talk, we consider the case when the minimal set is the multiple Warsaw circle. The graph of $\sin 1/x$ is a well-known example of connected but not locally connected sets. The set obtained by inserting infinitely many such components all over the circle is called a multiple Warsaw circle.

In 1955, Gottschalk and Hedlund constructed a minimal homeomorphism of the multiple Warsaw circle. However this homeomorphism is defined only on this set. In 1982, Handel constructed a homeomorphism of a surface whose minimal set is the multiple Warsaw circle.

The author recently constructed a diffeomorphism of a surface with the multiple Warsaw circle as a minimal set by using the method of circle inverse limits. In this talk, we discuss about codimension two foliations whose minimal sets are the product of the multiple Warsaw circle and the circle.

Rigidity of Riemannian foliations with locally symmetric leaves

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The relation between the geometry of leaves and the global structure of foliations has been studied by many authors. An example is Zimmer's work [1] on Lie foliations. He proved that the holonomy group of a minimal Lie foliation is arithmetic if the leaves are isometric to a symmetric space of noncompact type. In this talk, we will present a rigidity result on Riemannian foliations [2], which may be regarded as a generalization of Zimmer's theorem.

The typical examples of Riemannian foliations whose leaves are locally isometric to a symmetric space of noncompact type are the following:

Example (homogeneous Riemannian foliations)**.** Let G be a connected Lie group and S a compact Lie subgroup of G. Let H be a connected semisimple Lie group and K a maximal compact subgroup of H. For a torsion-free cocompact lattice Γ of $H \times G$, we have a Riemannian foliation on $\big((K\backslash H) \times (S\backslash G)\big)/\Gamma$, which is induced from the product foliation $(K\backslash H) \times (S\backslash G) = \bigsqcup_{g \in G} K\backslash H \times Sg$.

The main result of this talk is as follows:

Theorem. Let $(M, \mathcal{F})$ be a compact manifold with a minimal Riemannian foliation. Assume that M admits a Riemannian metric such that every leaf of $\mathcal{F}$ is locally isometric to a symmetric space $X = \prod X_i$, where X_i is an irreducible Riemannian symmetric space of noncompact type of dimension greater than two. Then $(M, \mathcal{F})$ is diffeomorphic to a homogeneous Riemannian foliation.

In the case where $(M, \mathcal{F})$ is a Lie foliation, the theorem is proved by using conformal structures on the leafwise boundary of infinity and by establishing a variant of strong Mostow rigidity. The general case follows from the Molino theory and classification of leafwise isometries of homogeneous Lie foliations.

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[2] G. Meigniez and H. Nozawa, Rigidity of Riemannian foliations with locally symmetric leaves, in preparation.

Ergodicity of embedded singular laminations

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It was proven by Fornæss and Sibony [1] that any minimal lamination by Riemann surfaces without singularities in the complex projective plane supports a unique directed harmonic positive current of mass one which can be understood as a global attractor of the leaves via an averaging process à la Ahlfors.

This situation is not special of the projective plane and a similar result can be stated for every minimal lamination without closed currents embedded in a homogeneous compact Kähler surface [4]. These results also hold if we allow hyperbolic singularities in the lamination [2], [3].

The proof of these results relies in the intersection theory introduced in [1] and a careful local study of the behaviour of the laminations with respect to certain suitable family of automorphisms.

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The mixed Yamabe problem for harmonic foliations

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The mixed scalar curvature of a foliation $\mathcal{F}$ with normal distribution $\mathcal{D}$ on a Riemannian manifold (M, g) is the averaged mixed sectional curvature. It is just a function and belongs to the extrinsic geometry of a pair $(\mathcal{F}, \mathcal{D})$. We apply the biconformal deformation for prescribing the (leafwise) constant mixed scalar curvature of a harmonic foliation. The problem is solvable when the leaves are compact and M is fibered instead of being foliated. For $\dim \mathcal{F} > 1$, we use the nonlinear heat type equation and spectral parameters of the leafwise Schrödinger operator. For $\dim \mathcal{F} \leq 3$, we apply perturbations of the ground state.

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Coarse homology of leaves of foliations

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In my talk, I will investigate which restrictions are imposed on non-compact Riemannian manifolds that are quasi-isometric to a leaf in a foliation of a compact manifold. It was shown by Paul Schweitzer [1] that every non-compact manifold carries a metric such that the resulting Riemannian manifold cannot be quasi-isometric to a leaf in a codimension 1 foliation of a compact manifold. We show that the coarse homology of these non-leaves constructed by Schweitzer is always non-finitely generated. This motivates the question whether every Riemannian manifold which is quasi-isometric to a leaf in a compact manifold has finitely generated coarse homology.

We give a fully negative answer to this question: Firstly, we show that there exists a large class of two-dimensional leaves in codimension one foliations that have non-finitely generated coarse homology. Moreover, we improve Schweitzer's construction by showing that every Riemannian metric can be deformed to a codimension one non-leaf without affecting the coarse homology. In particular, we find non-leaves with trivial coarse homology.

[1] P. Schweitzer, S.J., *Riemannian manifolds not quasi-isometric to leaves in codimension one foliations*, Ann. Inst. Fourier **61** (2011), no. 4, 1599–1631.

A GENERALIZATION OF NOVIKOV'S THEOREM ON THE EXISTENCE OF REEB COMPONENTS IN CODIMENSION ONE FOLIATIONS

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with Fernando Alcalde Cuesta and Gilbert Hector

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We study the structure and existence of generalized Reeb components in codimension one foliations. We show that every connected homological $(m-2)$-dimensional vanishing cycle in a C^2 transversely oriented codimension one foliation $\mathcal{F}$ of a closed m-manifold M lies on the boundary of a homological Reeb component. This extends Novikov's famous theorem to higher dimensions. The homological vanishing cycle is given as an immersion $\phi : B\times[0,1] \to M$, where B is a connected oriented $(m-2)$-manifold, such that $B_t = \phi(B \times \{t\})$ lies on a leaf L_t for every t, $0 \neq [B_0] \in H_{m-2}(L_0)$, and $[B_t] = 0 \in H_{m-2}(L_t)$ for every $t > 0$. A generalized Reeb component with connected boundary is a compact foliated manifold whose interior fibers over the circle with the leaves as fibers and whose boundary is a single compact leaf.

The proof begins by showing the existence of a transverse invariant measure for $\mathcal{F}$. Sacksteder's Theorem and the C^2 hypothesis show that the support of the measure is a finite union of compact leaves, in fact, a single compact leaf. Using Dippolito and Hector's octopus decomposition of an open saturated set, we show that union of the leaves L_t for $t > 0$ is in fact the interior of a generalized Reeb component with the leaf L_0 as the boundary.

This is a partial result of twenty years of research efforts. We hope in the future to complete the proof in the general case, when the vanishing cycle is not connected and consequently L_0 may be a finite union of leaves.

Existence of foliations of two entropy types

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It has been proved by Likorish (1965), Wood (1969) and Thurston (1976) that all the closed manifolds with zero Euler characteristic admit foliations of codimension one. In 1988, Ghys, Langevin and the speaker introduced the notion of geometric entropy for foliations of closed Riemannian manifolds. From the definition it follows easily that the conditions "zero entropy" or "positive entropy" do not depend on Riemannian structures. So, without referring to Riemannian structures, one has just two types of foliations: these with zero entropy and those with positive entropy.

In the talk, we shall discuss the problem of existence of codimension-one foliations of these two types.

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[2] W. Likorish, *A foliation for 3-manifolds*, Ann. of Math. **82** (1965), 414 – 420.
[3] W. Thurston, *Existence of codimension-one foliations*, Ann. of Math. **104** (1976), 249 – 268.
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[5] J. Wood, *Foliations on 3-manifolds*, Ann. of Math. **89** (1969), 336 – 358.

Maps between foliated Riemannian manifolds

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10 years ago with Jerzy Konderak we published our first paper on maps between Riemannian foliated manifolds, Transversally harmonic maps between manifolds with Riemannian foliation, Quart. J. Math. (Oxford) 54 (2003), 335–354. Jerzy Konderak's death in 2005 terminated our potentially fruitful cooperation. The investigation of transversally harmonic maps has been taken up by several mathematicians. I will review the results on these maps obtained by various authors in the last ten years.

POSTERS

Directional approach to dynamics of foliations

ANDRZEJ BIŚ

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Geometric entropy in a sense of Ghys-Langevin-Walczak [1] describes global dynamics of a foliation. More subtle approach to foliation dynamics we obtain considering paths in Cayley graph of holonomy pseudogroups. We will describe a generalization of the notion of local measure entropy, introduced by Brin and Katok [2] for a single map, to paths. It provides measure entropy description of a foliated manifold and indicates areas where entropy focuses. Finally, we apply the theory of dimensional type characteristics of a dynamical system, elaborated by Pesin [3], to obtain relation between topological entropy of a path and its local measure entropies.

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Canonical foliation on Weil bundle

BASILE GUY RICHARD BOSSOTO

Marien Ngouabi University

Let be M a smooth manifold, A a local algebra and M^A the associated Weil bundle. We construct the canonical foliation on M^A and we show that the canonical foliation on the tangent bundle TM is the foliation defined by his canonical vector field.

Manifolds with G_2 structures and (almost) contact structures

HYUNJOO CHO

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I will first give a brief introduction to seven-dimensional Riemannian manifolds with G_2-structures and G_2-manifolds which behave very similarly to Calabi-Yau manifolds. I will show that any seven-dimensional manifold with G_2-structures has an almost contact structure, and then discuss relations between contact structures and G_2-structures on it.

Lifting transverse symplectic structures.

Andrzej Czarnecki

Jagiellonian University

The canonical lift of a foliation to the bundle of transverse orthonormal frames is one of the most fundamental procedures in Riemannian foliations. It can be, of course, performed for any holonomy invariant transverse geometric structure. We will present some facts about lifts of transversally symplectic foliations.

Contracting boundary of Hadamard laminations

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An Hadamard lamination is a lamination of $CAT(0)$ space by Hadamard manifolds. Leaves as well as a foliated space have natural ideal boundaries and it is natural to study a way of embedding leaf boundaries into boundary of the space. In case of Hadamard foliations this problem was solved for hyperbolic space foliated by leaves of short second fundamental form (in fact. for hyperbolic leaves). New results of Sultan and Charney allow to reject the main difficulty i.e. different behaviour in almost Euclidean case. They defined contracting boundary of $CAT(0)$ space eliminating ends of non–hyperbolic geodesics. In particular, quasi–isometries of $CAT(0)$ spaces extend to their contracting boundaries.

We shall see geometric properties of an Hadamard lamination with $CAT(0)$ transversal and prolong this lamination into contracting boundary.

[1] R. Charney, H. Sultan, *Contracting boundary of $CAT(0)$ space*, arXiv: 1308:6615
[2] M. Czarnecki, *Hadamard foliations of H^n*, Diff. Geom. Appl. **20** (2004), 357–365

Liouville type theorem for transversally biharmonic maps

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We study Liouville type theorem for transversally biharmonic maps as follows: Let $(M, g, \mathcal{F})$ be a complete foliated Riemannian manifold and all leaves be compact. Let $(M', g', \mathcal{F}')$ be a foliated Riemannian manifold of non-positive transversal sectional curvature. Assume that $Vol(M)$ is infinite, then every transversally biharmonic map $\phi : (M, g, \mathcal{F}) \to (M', g', \mathcal{F}')$ of $\int_M (|\tau_b(\phi)|^4 + |\tau_b(\phi)|^2 + |d_T\phi|^2)\mu_M < \infty$ is transversally constant.

Geometry of non-algebraic leaves of polynomial foliations in $\mathbb{C}^2$

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Consider the set $\mathcal{A}_n$ of all polynomial vector fields of degree n in $\mathbb{C}^2$,

$$\dot{x} = P(x,y)\dot{y} = Q(x,y),$$

where $\deg P = \deg Q = n$. The phase portrait of such vector field (with complex time) is a holomorphic singular foliation. Its leaves are Riemann surfaces; in a generic case, all leaves are dense in $\mathbb{C}^2$.

The topological type of (some or generic) leaf of a generic foliation is still unknown. We obtained some results in this direction.

We proved that in a *dense* subset of $\mathcal{A}_n$, any foliation has a leaf with at least $\frac{(n+1)(n+2)}{2} - 4$ handles.

We also proved that for a generic vector field with a symmetry $P(x,y) = P(-x,y), Q(x,y) = -Q(-x,y)$, almost all leaves have infinite number of handles.

Our last result concerns limit cycles — loops on the leaves with non-identity holonomy maps. A classical theorem due to Yu. Ilyashenko says that a generic vector field from $\mathcal{A}_n$ has a countable set of homologically independent limit cycles. The limit cycles constructed by Ilyashenko converge to the infinite line in $\mathbb{C}P^2$, and their multipliers (derivatives of their holonomy maps) tend to 1.

We prove that a generic foliation from $\mathcal{A}_n$ has a countable set of homologically independent limit cycles which are uniformly bounded in $\mathbb{C}^2$, and their multipliers tend to infinity.

On embedding of Morse-Smale Diffeomorphisms in Flows

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A C^r-diffeomorphism ($r \geq 1$) $f : M^n \to M^n$ on a smooth connected closed manifold of dimension n embeds in a C^l-flow if f is the time-one map of such a flow. J. Palis showed that the set of C^r-diffeomorphisms that embed in C^1-flows is a set of first category in $\mathrm{Diff}^r(M^n)$. In the same time, the structural stability of Morse-Smale diffeomorphisms leads to existence of open sets of diffeomorphisms that embed in a topological flow.

In [1] were stated the following necessary conditions of embedding of a Morse-Smale cascade f in a topological flow (*Palis conditions*): (1) the non-wandering set of f consists only of fixed points; (2) f restricted to each invariant manifold of its fixed points is orientation preserving; (3) for any fixed points p, q having non-empty intersection of invariant manifolds, the intersection does not contain compact components. It was also shown in [1] that conditions (1)-(3) are sufficient in case $n = 2$.

For case $n = 3$ in [2] it was discovered that there is an additional obstacle for the Morse-Smale cascade to embed in a flow and obtained the criteria of embedding such diffeomorphisms in topological flows.

In contrast with case $n = 3$, the following result holds for greater dimension.

Theorem. *If a Morse-Smale diffeomorphism $f : S^n \to S^n$ has no heteroclinical intersection and satisfies Palis conditions then f embeds in a topological flow.*

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[2] V. Grines, E. Gurevich, V. Medevedev, O. Pochinka *On embedding of Morse-Smale diffeomorphisms on a 3-manifold in a topological flow* Sbornik: Mathematics (2012), 203(12):1761.

Notice concerning cohomology of Lie groupoids over foliations

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Every Lie groupoid G over a smooth manifold M determines, in a canonical way, a foliation $\mathcal{F}$ on M. The de Rham cohomology of G is, by definition, the cohomology of left invariants forms on $\mathcal{F}$. When the foliation $\mathcal{F}$ is transversally complete, the Lie groupoid G is locally trivial. Under these hypothesis, we define the notion of piecewise de Rham cohomology of G over a smooth triangulation of M and show that the piecewise de Rham and the de Rham cohomology of G are isomorphic.

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Energy function for structurally stable cascades with expanding attractors and contracting repellers of codimension 1

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The results were obtained together with V. Grines.

We consider the class G of structurally stable diffeomorphisms on 3-manifolds each nontrivial basic set of which has codimension 1 and it is either an expanding attractor or a contracting repeller. According to C. Conley [1], a *Lyapunov function* for a structurally stable diffeomorphism is a continuous function that decreases along wandering trajectories and constant on basic sets. A smooth Lyapunov function is called an *energy function* if the set of critical points coincides with the nonwandering set of the diffeomorphism.

The main result of this paper is the following theorem, which is based on [2], [3].

Theorem. For any diffeomorphism from class G there is an energy function, which is a Morse function out of nontrivial basic sets.

Acknowledgments. This work was financially supported by grants 12-01-00672 , 13 -01- 12452 - ofi-m RFBR .

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[3] V. Grines, F. Laudenbach, O. Pochinka. Self-indexing function for Morse-Smale diffeomorphisms on 3-manifolds. Moscow Math. Journal, 2009, No 4, 801-821.

On the lifted foliation on the transverse vector bundle

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In this work, the transverse vector bundle Q, on a foliated manifold $(M, \mathcal{F})$ is described. We study its structure as a smooth manifold $Q(M, \mathcal{F})$. The lifted foliation $\mathcal{F}^*$ in $Q(M, \mathcal{F})$ is defined and we find a relation between bundle-like metrics on M and Riemannian foliations on $Q(M, \mathcal{F})$. We try to use this foliated model to make a contact structure in $Q(M, \mathcal{F})$ and through this work, we obtain some other results.
