## LSAA: Seminars

## Complemented Subspaces of Banach Lattices

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Let X be a Banach lattice and P a linear projection defined on it, with dim  $P(X) = \infty$ . What can we say about P(X)? We know that:

- P(X) is reflexive if and only if it does not contain isomorphic copies of  $c_0$  or  $\ell_1$ ;
- P(X) has an unconditional basic sequence;
- P(X) has local unconditional structure (in the sense of Gordon-Lewis).

These properties are not true in general for Banach spaces, so they allow us to give examples of Banach spaces which are not isomorphic to complemented subspaces of Banach lattices: the James space, the Kalton-Peck space, the Gowers-Maurey space without any unconditional basic sequence,  $\mathcal{H}^{\infty}(\mathbb{D})$ ...

However, we cannot use the above criteria to distinguish Banach lattices from subspaces complemented on them. And this is one of the reasons why the following important question had remained open for decades:

**Question (The Complemented Subspace Problem):** Is every complemented subspace of a Banach lattice isomorphic to a Banach lattice?

Some authors, including myself, have recently found a counterexample to this problem: the space  $PS_2$  constructed by G. Plebanek and A. Salguero (2021), which is a 1-complemented subspace of a C(K)-space, cannot be isomorphic to any Banach lattice.

In this talk we will discuss some advances on this topic (the Complemented Subspace Problem) and its connections with free Banach lattices.