# Introduction to order theory

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Order structures appear everywhere in mathematics, but are rarely studied on their own. The abstract order theory is usually studied by logicians and computer scientists. The Positivity community is dedicated to studying order structures which appear in Analysis, but such studies tend to have a relatively narrow focus. In this mini-course I will try to place some familiar motives in a more abstract context. On top of a refresher of the basic definitions of the order theory we will consider the following topics.

(1) Galois connections. These are abstract versions of the ubiquitous "tension" between elements of a set and possible conditions on these elements, along with various polarities on a set. The term has its origin in the theory of field extensions, but other examples range from orthogonal complement of a subset of a Hilbert space to Legendre transform from convex analysis.

(2) Hull (aka closure) structures. Given a notion of "niceness" of an element (for example of a subset of a given set or a function) a non-nice element can be "improved" by finding the smallest nice element above it. For example, a closure of a set in a topological space, a subgroup generated by a subset of a group, the concave envelope of a function on a vector space, etc. We will discuss a general theory of such a procedure.

(3) Special ordered sets. As in any area of mathematics there are numerous properties which a given ordered set may or may not satisfy (e.g. being a lattice, Dedekind completeness, distributivity), and there are classes of ordered sets with additional (algebraic and/or topological) structure. We will briefly consider Boolean algebras and ordered vector spaces including the topic of their Dedekind completion.