Workshop Banach spaces and Banach lattices

Banach lattices with disjointness preserving isometries: Linear versus vector lattice structure

Author: Yves Raynaud (Sorbonne University)

Time: 20^{th} of May, at 10:00 a.m.
Place: Sala Naranja (ICMAT)

It is well known that ultrapowers (and more generally ultraproducts) of Banach lattices have a natural structure of Banach lattices. We are interested in kind of a reverse question: if a Banach space has an ultrapower which is linearly isometric to a Banach lattice L, is the original space E linearly isometric to a Banach lattice Λ that has L for ultrapower?

We show that the answer is positive for certain classes of Banach lattices in which the linear isometric embeddings are disjointness preserving. This was before known only for L_p-spaces ($1 \leq p < \infty, p \neq 2$).

Approximate Morse-Sard type results for non-separable Banach spaces

Author: Mar Jimenez-Sevilla (Universidad Complutense de Madrid)

Time: 20^{th} of May, at 11:30 a.m.
Place: Sala Naranja (ICMAT)

In this talk we will review some results on ranges of derivatives and present several results on approximation by smooth functions with no critical points. For E and F Banach spaces, if $g : E \to F$ is a C^1 smooth function, a point $x \in E$ is called critical if $Dg(x)$, the derivative of g at x, is not a surjective operator. The set of critical points is denoted by $\mathcal{C}_g$. The classical Morse-Sard theorem states that if $g : \mathbb{R}^n \to \mathbb{R}^m$ is C^k-smooth with $k > \max\{n-m, 0\}$, then the set of critical values $g(\mathcal{C}_g)$ is of Lebesgue measure zero in $\mathbb{R}^m$. In many cases Morse-Sard theorem fails when $dim(E) = \infty$. In this talk we present sufficient conditions to ensure an approximate strong version of the Morse-Sard theorem for mappings from an infinite-dimensional Banach space E to a Banach space F. Namely, under some appropriate conditions on an infinite-dimensional (not necessarily separable) Banach space E, for every (non-zero) quotient F of E (or a certain class of quotients F of E), every continuous function $f : E \to F$, and for every continuous function $\varepsilon : E \to (0, \infty)$, there exists a C^k smooth function $g : E \to F$ with no critical points such that $\|f(x) - g(x)\| \leq \varepsilon(x)$ for all $x \in E$. Here k depends on the smoothness of E. Joint work with D. Azagra and M. García-Bravo.

Questions and results around James' Theorem

Author: Gonzalo Martínez Cervantes (University of Murcia)

Time: 20$^{\text{th}}$ of May, at 12:15 p.m.
Place: Sala Naranja (ICMAT)

The well-known James' Theorem states that a Banach space is reflexive if and only if every bounded linear functional on it attains its norm. In this talk we will investigate operator and lattice versions of this result.

Representing p-multinormed spaces in Banach lattices

Author: Timur Oikhberg (University of Illinois)

Time: 20$^{\text{th}}$ of May, at 15:00
Place: Sala Naranja (ICMAT)

The theory of p-multnormed spaces has its origins in the early 1990s; the initial goal was to give an "abstract" characterization of subspaces of Banach lattices. More specifically, a p-multinormed space is a normed space E, for which $\ell_p \otimes E$ is equipped with a left tensorial cross-norm (left-tensoriality means that, for any $u \in \ell_p \otimes E$ and $T \in B(\ell_p)$, we have $\|T\|\|u\| \geq \|(T \otimes I_E)(u)\|$); for $p = \infty$, we replace ℓ_p by c_0.

Early investigations (by L.McClaran and J.Marcolino-Nhany) showed that any ∞-multinormed space can be realized as a subspace of a Banach lattice (with the norm on $\ell_p \otimes E$ generated by $\widetilde{E(\ell_p)}$). However, for finite values of p, there are p-multinormed spaces where such a realization is impossible. In this work, we to find criteria for the existence of such realizations, and, more generally, realizations as subspaces of ordered Banach spaces. We also show that, in general, many "different" realizations exist.

Time permitting, I shall survey other recent progress in p-multinormed spaces, and discuss open problems in the area. I shall also mention certain Banach space problems which arise from the theory of p-multinormed spaces, but may be of independent interest.

Stability of piecewise linear maps and bi-Lipschitz group invariants

Author: Daniel Freeman (Saint Louis University)

Time: 21st of May, at 10:00 a.m.
Place: Sala Naranja (ICMAT)

Let G be a group of isometries on R^d. We consider maps $T_G : R^d/G \to R^n$ which lift to a continuous, positively homogeneous, and piecewise linear map $T : R^d \to R^n$. We will discuss how these maps arise naturally in applications such as real phase retrieval, max filtering, and group invariant representations using coorbits. We prove that T_G is one-to-one if and only if it is a bi-Lipschitz embedding. That is, there exists constants $A, B > 0$ so that for all $x, y \in R^d$,

$$A \min_{g\in G} \|x - gy\| \leq \|Tx - Ty\| \leq B \max_{g\in G} \|x - gy\|.$$

We give bounds on the constants in terms of the singular values for the linear pieces of T and the principle angles between certain cones. This is joint work with Rima Alaifari, Dorsa Ghoreishi, Mitchell Taylor, and Pedro Tradacete.

Super-reflexive twisted sums of G-spaces and automatic continuity

Author: Denis de Assis Pinto Garcia (IME-USP)

Time: 21st of May, at 11:30 a.m.
Place: Sala Naranja (ICMAT)

Let G be a topological group. A G-Banach space is an ordered triple (G, X, u), where X is a Banach space, and $u\colon G \to \mathcal{B}(X)$ is a bounded left action of G on X. We say that a G-Banach space (G, X, u) is a $G_{\mathcal{Top}}$-Banach space if the action u is (τ_G, SOT)-continuous. We will show that, if

$$0 \longrightarrow (G, X, u) \xrightarrow{\iota} (G, Z, \lambda) \xrightarrow{q} (G, Y, v) \longrightarrow 0$$

is an exact sequence of G-Banach spaces such that (G, X, u) and (G, Y, v) are $G_{\mathcal{Top}}$-Banach spaces, and such that Z is super-reflexive, then (G, Z, λ) is also a $G_{\mathcal{Top}}$-Banach space. Afterwards, we will describe the linear derivations associated to a G-quasi-linear map $\Omega\colon Y \curvearrowright X$ in the case where the quasi-norm induced by Ω on $X \oplus_\Omega Y$ is super-reflexive (in the sense that it is equivalent to a uniformly convex norm).

A counterexample to the complemented subspace problem in Banach lattices

Author: David de Hevia Rodríguez (ICMAT)

Time: 21^{st} of May, at 12:15 p.m.
Place: Sala Naranja (ICMAT)

A quite relevant question within Banach lattice theory reads as follows: Must every complemented subspace of a Banach lattice be isomorphic to a Banach lattice? This question is known as the *Complemented Subspace Problem for Banach lattices*.

To begin analyzing this question, it is natural to look at examples of Banach spaces that we know that are not isomorphic to Banach lattices. These are some of the most common criteria that we use to find such examples: every Banach lattice which does not contain isomorphic copies of c_0 or ℓ_1 must be *reflexive* (so James's space cannot be isomorphic to a Banach lattice); every Banach lattice has an *unconditional basic sequence* (this is not the case for arbitrary Banach spaces as shown by T. Gowers and B. Maurey); and every Banach lattice has *local unconditional structure* in the sense of Gordon and Lewis (the space $H^\infty(\mathbb{D})$ fails this property). It is interesting to mention that these properties of Banach lattices also hold for complemented subspaces in Banach lattices, so the results we have mentioned above do not help us to address our question. In particular, the aforementioned examples cannot be isomorphic to complemented subspaces in Banach lattices.

Some authors, including myself, have recently found the first counterexample to the *Complemented Subspace Problem*: the space PS_2 constructed by G. Plebanek and A. Salguero (2021), which is a 1-complemented subspace of a

$C(K)$-space, cannot be isomorphic to any Banach lattice. In this talk, I will try to explain superficially how we have been able to prove that this space cannot be isomorphic to any Banach lattice. Finally, I will point out some connections between this problem and free Banach lattices.

This talk is based on a joint work with G. Martínez Cervantes, A. Salguero Alarcón, and P. Tradacete.

Saturation Recovery and Phase Retrieval

Author: Dorsa Ghoreishi (Saint Louis University)

Time: 21^{st} of May, at 15:00
Place: Sala Naranja (ICMAT)

Frames for a Hilbert space allow for a linear and stable reconstruction of a vector from linear measurements. In many real-world applications, sensors are set up such that any measurement above and below a certain threshold would be clipped as the signal gets saturated. We study the recovery of a vector from such measurements which is called declipping or saturation recovery. Phase retrieval is a similar problem where the goal is to recover a vector where only the intensity of each linear measurement of a signal is available and the phase information is lost. Using a frame theoretic approach to saturation recovery, we characterize when saturation recovery of all vectors in the unit ball is possible and then compare some of the known results in phase retrieval with saturation recovery.
This is joint work with Wedad Alharbi, Daniel Freeman, Brody Johnson, and Nirina Randrianarivony.

Stable metric spaces and Kalton's property Q.

Author: András Zsák (University of Cambridge)

Time: 22^{nd} of May, at 10:00 a.m.
Place: Sala Naranja (ICMAT)

Kalton proved that every stable metric space embeds coarsely and uniformly into a reflexive Banach space. He also introduced an invariant called property

Q to provide the first examples of metric spaces that do not coarsely embed into a reflexive space. He asked if every reflexive Banach space coarsely embeds into a stable metric space. Baudier introduced a new invariant called upper stability in an attempt to answer Kalton's question in the negative. In this talk we discuss Kalton's results and related results by Raynaud and by Braga and Swift. We also prove that every metric space with property Q is upper stable which shows that upper stability is not sufficient to distinguish between stable metric spaces and reflexive space in the coarse category. This is joint work with Florent Baudier and Thomas Schlumprecht.

Recent advances in free Banach lattices with upper p-estimates

Author: Enrique García-Sánchez (ICMAT)

Time: 22nd of May, at 11:30 a.m.
Place: Sala Naranja (ICMAT)

The free Banach lattice generated by a Banach space is a free object that associates to every Banach space a Banach lattice satisfying the following universal property: every bounded operator from the Banach space to an arbitrary Banach lattice extends uniquely to a lattice homomorphism defined on the free Banach lattice, with uniform control of the norm. This definition can be generalized in order to construct free Banach lattices associated to more constrained classes of Banach lattices, such as p-convex Banach lattices or Banach lattices with upper p-estimates. In this talk we present an explicit functional representation of the norm of the free Banach lattice with upper p-estimates and some related results. This is a joint work with D. Leung, M. A. Taylor and P. Tradacete.

On the weak-fragmentability index of some Lipschitz-free spaces

Author: Estelle Basset (Laboratoire de mathématiques de Besançon, France)

Time: 22nd of May, at 12:15 p.m.
Place: Sala Naranja (ICMAT)

We show the existence of Lipschitz-free spaces verifying the Point of Continuity Property as badly as possible. For this purpose, we use a generalized construction of the countably branching diamond graphs and get a lower bound of their weak-fragmentability index. As a consequence, we deduce a necessary condition for a complete separable metric space to be universal for countable complete metric spaces. Another corollary is the existence of an uncountable family of pairwise non-isomorphic Lipschitz-free spaces over purely 1-unrectifiable metric spaces. Some results of compact reduction are also obtained.

Polyhedral spaces, with a twist

Author: Alberto Salguero-Alarcón (Universidad Complutense de Madrid)

Time: 22^{nd} of May, at 15:00
Place: Sala Naranja (ICMAT)

A Banach space is said to be polyhedral if all of its finite-dimensional subspaces have a unit ball which is the convex combination of finitely many points. Polyhedral spaces are regarded as a somewhat exotic class of Banach spaces with surprising properties. Following the seminal works of Fonf (and collaborators), we will explore the mysteries surrounding the relation between polyhedral renormings and twisted sums of Banach spaces. In particular, we will show that every space X fitting in a short exact sequence of the form $0 \to c_0(I) \to X \to c_0(J) \to 0$ admits a polyhedral renorming. However, the plan is to focus on what we do *not* know rather than on what we know.

Inclusions of variable Lebesgue spaces: weak compactness and disjoint strict singularity

Author: Francisco L. Hernández (Universidad Complutense de Madrid)

Time: 23^{rd} of May, at 10:00 a.m.
Place: Sala Naranja (ICMAT)

Variable Lebesgue spaces (or Nakano spaces) $L^{p(\cdot)}(\mu)$ are classical non-symmetric function spaces having a renewed relevance due to their applications. We present

suitable characterizations in term of the exponent functions for an inclusion $L^{p(\cdot)}(\mu) \hookrightarrow L^{q(\cdot)}(\mu)$ be disjointly strictly singular or be weakly compact. The strict singularity of the extreme inclusion $L^{\infty}(\mu) \hookrightarrow L^{p(\cdot)}(\mu)$ is also characterized. Decreasing rearrangements of associated exponents play a role for that (in spite of these spaces are non-symmetric).
Joint work with Cesar Ruiz and Mauro Sanchiz.

Compact inclusions between variable order Hölder spaces

Author: Mauro Sanchiz Alonso (CEU San Pablo)

Time: 23$^{\text{th}}$ of May, at 11:30 a.m.
Place: Sala Naranja (ICMAT)

The Hölder spaces $C^{\alpha}(X)$ or $H^{\alpha}(X)$ for a metric space (X,d) and an order $0 < \alpha < 1$ are a family of Banach function spaces between the continuous functions $C(X)$ and the Lipschitz functions $Lip(X)$. The inclusion between two Hölder spaces $C^{\alpha}(X) \hookrightarrow C^{\beta}(X)$ is continuous if $\alpha \geq \beta$ and it is compact if $\alpha > \beta$.

In this talk, we study a generalization of Hölder spaces, which are the variable order Hölder spaces for a one-variable (or two-variable order functions) $\alpha(\cdot)$ (or $\phi(\cdot,\cdot)$), and try to find a criterion for the inclusion between two variable order spaces be compact. In this generalizated spaces, the inclusion $C^{\alpha(\cdot)}(X) \hookrightarrow C^{\beta(\cdot)}(X)$ is also continuous if $\alpha(\cdot) \geq \beta(\cdot)$, but the question of the compactness of this inclusion is far from trivial.

We give a candidate property of criterion for the inclusion be compact. We show that this property is sufficient for the compactness of the inclusion. Also, in the case that the order $\alpha(\cdot)$ is log-Hölder continuous (and no contitions on $\beta(\cdot)$) we show that this property is also necessary and hence a criterion. Other criteria is found under different hypothesis. The novelty of considering two variable order functions $\phi(\cdot,\cdot)$ widens the view of the problem and might be necessary to find a precise criterion.

The content of this is based in the recent paper (joint work with Przemysław Górka and César Ruiz):
Compact inclusions between variable Hölder spaces. JMAA (2024), in press (pre-proof), doi.org/10.1016/j.jmaa.2024.128328.

Countable unions of operator ranges and spaceability

Author: Miguel Ángel Ruiz Risueño (Universidad de Castilla-La Mancha)

Time: 23$^{\text{rd}}$ of May, at 12:15 p.m.
Place: Sala Naranja (ICMAT)

In this talk, we will present several results on spaceability in Banach spaces. We will show that if $\{R_n\}_n$ is a sequence of infinite-codimensional operator ranges in a Banach space E, then there is a closed infinite-dimensional subspace $X \subset E$ such that $X \cap R_n = \{0\}$ for all n. We shall also prove that if E is separable, then there exist two closed isomorphic quasicomplemented subspaces $X, Y \subset E$ such that $(X + Y) \cap R_n = \{0\}$ for each n. In the case that E is a dual space, we will provide several sufficient conditions to ensure the existence of weak*-closed subspaces with the previous properties. The talk is based in a joint work with Mar Jiménez-Sevilla and Sebastián Lajara.

Guarded Fraisse Banach spaces

Author: Michal Doucha (Institute of Mathematics, Czech Academy of Sciences)

Time: 23$^{\text{rd}}$ of May, at 15:00
Place: Sala Naranja (ICMAT)

We introduce a new class of (separable) Banach spaces, hereafter called guarded Fraisse Banach spaces. Their motivation comes from two different directions: (1) They generalize the class of Fraisse Banach spaces recently introduced by Ferenczi, Lopez-Abad, Mbombo and Todorcevic which itself is motivated by both the notion of a Fraisse structure from model theory as well as by the Banach spaces satisfying a relaxed version of the Mazur rotation problem. (2) From a descriptive set theoretic point of view, they have the simplest definition up to isometry; namely, they are exactly the spaces whose isometry class is G_δ in a certain Polish space. The advantage of this class, in contrast to Fraisse Banach spaces, is that it admits a potentially large number of examples. Besides all the L_p's and the Gurarii space we show how using the recent advances on the lattice structure of $L_p(L_q)$ spaces by Raynaud and Tursi we can completely characterize for which values of p and q $L_p(L_q)$ is a guarded Fraisse Banach space. This is joint work with Marek Cuth and Noe de Rancourt.

Solution to a question of J. Lindenstrauss

Author: José Orihuela (Murcia University)

Time: 24^{th} of May, at 10:00 a.m.
Place: Sala Naranja (ICMAT)

For strictly convex renormings we solve a recent question of R. Smith giving a final answer to a long standing Lindenstauss question. We prove that a Banach space admits an equivalent strictly convex norm if, and only if, it has another one with separable faces. A purely topological characterization follows for dual spaces and dual norms.