Focus week 3: Interactions with descriptive set theory

Applications of descriptive set theory to basis problems in Banach spaces and lattices

Author: Christian Rosendal (University of Maryland)

Time: 27^{th} and 29^{th} of May, at 10:00 a.m.
Place: Sala Naranja (ICMAT)

In Banach spaces and lattices, the concept of a Schauder basis is occasionally too restrictive or alternatively too weak and thus other concepts of a basis may be needed. One way of doing this is by replacing the norm-convergence of the sequence of partial sums with respect to the basis with an alternative notion of convergence, for example, formulated in terms of the order structure of a lattice or convergence along a filter. However, this leads to the question of whether the corresponding set of biorthogonal functionals are continuous, which has been raised in different contexts by Gumenchuk-Karlova-Popov, Taylor-Troitsky and Ganichev-Kadets. We address these problems using techniques from descriptive set theory and note that their solutions are related to the question of the descriptive complexity of different notions of order convergence in separable Banach lattices. This reports on joint work with A. Avilés, M.A. Taylor and P. Tradacete.

Projectively universal objects and Polish spaces of spaces

Author: Tomasz Kania (Jagiellonian University)

Time: 27^{th}, 28^{th} of May, at 11:30 a.m. and 30^{th} of May at 10:00 a.m.
Place: Sala Naranja (ICMAT)

We will outline a novel approach to building Polish spaces (not only Borel spaces) of classes of separable objects such as C*-algebras, Banach algebras, Banach lattices etc. based on the existence of projective objects that are quotient onto every objects from a given category and demonstrate how to use this framework for getting upper Borel complexity bounds for various kinds of properties of interest.

Descriptive set theoretic applications in Banach lattices: a survey of results and open questions

Author: Mary Angelica Tursi (Independent scholar)

Time: 28th of May, at 10:00 a.m., and 29th and 30th of May at 11:30 a.m.
Place: Sala Naranja (ICMAT)

In this series of talks, we apply descriptive set theoretic tools to Banach lattices in three major areas. First, we begin with preliminaries of techniques used to encode various classes of separable Banach lattices in descriptive terms and evaluate the descriptive complexity of these classes. Second, we explore initial results on the isomorphic and isometric theory of Banach lattices in terms of complexity of isomorphism or isometry relations. Finally, we explore some results on Polish group dynamics in the Banach lattice setting. For each area, an added emphasis will be placed on open questions and directions for future research.

Polish spaces of Banach lattices

Author: Mariusz Niwiński (Jagiellonian University)

Time: 31st of May, at 10:00 a.m.
Place: Sala Naranja (ICMAT)

In the article *Descriptive complexity of some isomorphism classes of Banach spaces*, from 2018, Gilles Godefroy and Jean Saint-Raymond proposed a construction of the Polish space classifying all separable Banach spaces. During my talk I would like to discuss whether a similar approach could be used for classifying separable Banach lattices. For this purpose I will also refer to the

notion of a free Banach lattice and consider if its properties can be useful to achieve such construction.
