Focus week 2: Vector Lattices and Ordered Structures

Fundamental questions about regular operators between Banach lattices

Author: Anthony Wickstead (Queen's University Belfast)

Time: 13th of May, at 10:00 a.m. **Place:** Sala Naranja (ICMAT)

Many old questions concerning regular operators between Banach lattices have received little attention in recent years, even though they remain not being completely answered. In many cases, even what has been established seems not to be as well known as it ought to be. In this talk I will look at three such questions, describe what is known and, more importantly, point out significant gaps in our knowledge that are worthy of further investigation. The three topics are:

- 1. When can lattice operations be calculated using the Riesz-Kantorovich formulae?
- 2. When is it true that $|T^*| = |T|^*$?
- 3. When do spaces of regular operators, under the regular norm, form (isometrically or isomorphically) a classical Banach lattice?

Introduction to order theory

Author: Eugene Bilokopytov (University of Alberta)

Time: 13th, 14th, 16th and 17th of May, at 11:30 a.m. **Place:** Sala Naranja (ICMAT) Order structures appear everywhere in mathematics, but are rarely studied on their own. The abstract order theory is usually studied by logicians and computer scientists. The Positivity community is dedicated to studying order structures which appear in Analysis, but such studies tend to have a relatively narrow focus. In this mini-course I will try to place some familiar motives in a more abstract context. On top of a refresher of the basic definitions of the order theory we will consider the following topics.

- 1. Galois connections. These are abstract versions of the ubiquitous "tension" between elements of a set and possible conditions on these elements, along with various polarities on a set. The term has its origin in the theory of field extensions, but other examples range from orthogonal complement of a subset of a Hilbert space to Legendre transform from convex analysis.
- 2. Hull (aka closure) structures. Given a notion of "niceness" of an element (for example of a subset of a given set or a function) a non-nice element can be "improved" by finding the smallest nice element above it. For example, a closure of a set in a topological space, a subgroup generated by a subset of a group, the concave envelope of a function on a vector space, etc. We will discuss a general theory of such a procedure.
- 3. Special ordered sets. As in any area of mathematics there are numerous properties which a given ordered set may or may not satisfy (e.g. being a lattice, Dedekind completeness, distributivity), and there are classes of ordered sets with additional (algebraic and/or topological) structure. We will briefly consider Boolean algebras and ordered vector spaces including the topic of their Dedekind completion.

Order Topologies – some applications

Author: Emanuel Chetcuti (University of Malta)

Time: 13th of May, at 15:00 **Place:** Sala Naranja (ICMAT)

Associated to any order structure it is possible to consider various 'analytic structures' given by order-convergence and order-topologies. It is no surprise that these analytic structures carry information about the starting order structure. But when the order structure has an additional structure coupled to it, what additional information can be drawn from the analytic structure? This is, perhaps, a too general question, but in the talk we will see some examples where this approach is successful and useful.

Uo-convergence on infinitely distributive lattices

Author: Kevin Abela (University of Malta)

Time: 13th of May, at 16:00 **Place:** Sala Naranja (ICMAT)

Order convergence was studied extensively on partially ordered sets, lattices and vector lattices. Uo-convergence is generally explored in the context of vector lattices due to it being an analogue of pointwise convergence in function spaces. Furthermore, in $L_p(\mu)(1 \le p \le \infty)$, uo-convergence of sequences coincides with almost everywhere convergence. In our work we explore uo-convergence on infinitely distributive lattices. In particular, we investigate several results that hold in the context of vector lattices and extend them on infinitely distributive lattices.

Tensor products of vector lattices

Author: Vladimir Troitsky (University of Alberta)

Time: 14th, 16th and 17th of May, at 10:00 a.m. **Place:** Sala Naranja (ICMAT)

Constructions of tensor products of vector and Banach lattices go back to Fremlin papers in the early 1970s. Since then, these constructions have been revisited many times, and several new approaches have been developed. In my talk, I will go over some of more recent variants of the constructions.

On the existence of a positive approximate identity

Author: Jamel Jaber (University of Carthage)

Time: 14th of May, at 15:00 **Place:** Sala Naranja (ICMAT)

This presentation delves into various aspects of Banach lattice algebras. We explore the presence of positive approximate identity within this class of algebras. Additionally, we investigate the existence of positive approximate identities for the positive projective tensor product of Banach lattice algebras, uncovering several open problems in the process. Moreover, we discussed the representation of approximately unital Banach lattice algebras. Some of these results stem from a collaborative work with Azouzi.

Tensor product of Riesz Sub-spaces

Author: Mohamed Amine Ben Amour (University of Carthage)

Time: 14th of May, at 15:40 **Place:** Sala Naranja (ICMAT)

In this talk, we will explore the Riesz tensor product of certain Riesz subspaces along with their Dedekind completion. We'll delve into the implications for components and how they relate to the theory of weak mixing within the context of ergodic theory.

Inner band projections on Banach lattice algebras

Author: David Muñoz-Lahoz (UAM-ICMAT)

Time: 14th of May, at 16:20 **Place:** Sala Naranja (ICMAT)

A Banach lattice algebra (BLA) is a Banach lattice that also has a Banach algebra structure in which the product of positive elements is positive. Classical examples of BLAs are the C(K) spaces with pointwise product and the space of regular operators Lr(X) over an order (Dedekind) complete Banach lattice X with the product given by composition. In this talk we construct inner band projections, a family of band projections that can be defined on any BLA. We present some of their general properties, focusing our attention on spaces of regular operators. We also use inner band projections to provide a characterization of atomic and order continuous Banach lattices: a Banach lattice X is atomic and order continuous if and only if all band projections on Lr(X) are inner.

Embedded unbounded order convergent sequences in topologically convergent nets in vector lattices

Author: Marcel De Jeu (Leiden University and University of Pretoria)

Time: 16th of May, at 15:00 **Place:** Sala Naranja (ICMAT)

In recent joint work with Yang Deng, it was shown that, for a fairly large class of locally solid topologies on vector lattices, a topologically convergent net has an embedded sequence that is unbounded order convergent to the same limit. Our result implies, and often improves, many of the known results in this vein in the literature. While explaining this, a number of results on metrisability and submetrisability of locally solid topologies on vector lattices will also be covered.

Unbounded convergence and completeness

Author: Youssef Azouzi (University of Carthage)

Time: 16th of May, at 16:00 **Place:** Sala Naranja (ICMAT)

For any type of convergence we can consider its unbounded version. In the first part of our talk we present some results in this direction that unify and generalize several earlier results. In a second part we prove that spaces $L^p(T)$ are strongly complete, $1 \le p \le \infty$, where T is a conditional expectation operator. Other aspects of completeness will be also discussed.