Rapid thermalization of spin chain commuting Hamiltonians

Modified logarithmic Sobolev inequalities for quantum many-body systems

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Joint work with **I. Bardet, L. Gao, A. Lucia, D. Pérez-García, C. Rouzé**

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Motivation: Open quantum many-body systems

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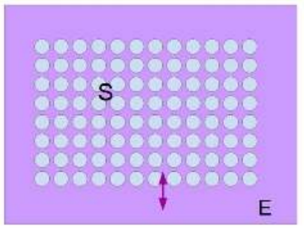

- Finite lattice $\Lambda \subset\subset \mathbb{Z}^d$.
- Hilbert space associated to Λ is $\mathcal{H}_\Lambda = \bigotimes_{x \in \Lambda} \mathcal{H}_x$.
- Density matrices: $\mathcal{S}_\Lambda := \mathcal{S}(\mathcal{H}_\Lambda) = \{\rho_\Lambda \in \mathcal{B}_\Lambda \, : \, \rho_\Lambda \geq 0 \text{ and } \operatorname{tr}[\rho_\Lambda] = 1\}$.

- Dynamics of S is dissipative!
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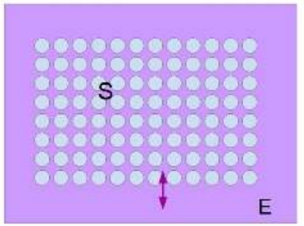

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Quantum Markov semigroup / Dissipative quantum evolution

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A **quantum Markov semigroup** is a 1-parameter continuous semigroup $\{\mathcal{T}_t\}_{t\geq 0}$ of completely positive, trace preserving (CPTP) maps (a.k.a. quantum channels) in $\mathcal{S}_\Lambda$.

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The infinitesimal generator $\mathcal{L}_\Lambda$ of the previous semigroup of quantum channels is usually called **Liouvillian**, or **Lindbladian**.

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For $\rho_\Lambda \in \mathcal{S}_\Lambda$, $\quad \mathcal{L}_\Lambda(\rho_\Lambda) = -i[H_\Lambda, \rho_\Lambda] + \sum_{k\in\Lambda} \widetilde{\mathcal{L}}_k(\rho_\Lambda) \quad$ **GKLS equation**.

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Mixing $\Leftrightarrow$ Convergence

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We also assume that the quantum Markov process studied is **reversible**, i.e., it satisfies the **detailed balance condition**:

$$\langle f, \mathcal{L}_\Lambda^*(g)\rangle_\sigma = \langle \mathcal{L}_\Lambda^*(f), g\rangle_\sigma ,$$

for every $f, g \in \mathcal{B}_\Lambda$ and Hermitian, where

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Mixing time

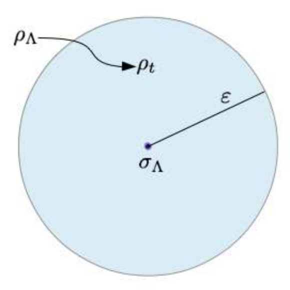
- Under the previous conditions, there is always convergence to σ_Λ.
- How fast does convergence happen?

Note $\mathcal{T}_\infty(\rho) := \sigma_\Lambda$ for every ρ.

Mixing time

We define the **mixing time** of $\{\mathcal{T}_t\}$ by

$$t_{\text{mix}}(\varepsilon) = \min\left\{ t > 0 : \sup_{\rho_\Lambda \in \mathcal{S}_\Lambda} \left\| \mathcal{T}_t(\rho) - \mathcal{T}_\infty(\rho) \right\|_1 \leq \varepsilon \right\}.$$



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We say that $\mathcal{L}_\Lambda$ satisfies **rapid mixing** if

$$\sup_{\rho_\Lambda \in \mathcal{S}_\Lambda} \|\rho_t - \sigma_\Lambda\|_1 \leq \text{poly}(|\Lambda|)e^{-\gamma t}.$$

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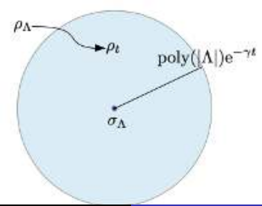

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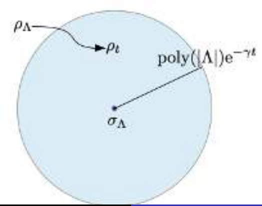


Applications to quantum information/quantum computing

What are the implications of rapid mixing?

Rapid mixing

$$\sup_{\rho \in \mathcal{S}(\mathcal{H}_\Lambda)} \| T_t(\rho) - \sigma \|_1 \leq \text{poly}(|\Lambda|) e^{-\gamma t}$$

Mixing time: $\tau(\varepsilon) = \mathcal{O}(\text{polylog}(|\Lambda|))$

"Negative" point of view:

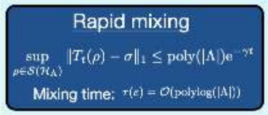
- Quantum properties that hold in the ground state but not in the Gibbs state are **suppressed too fast** for them to be of any reasonable use.

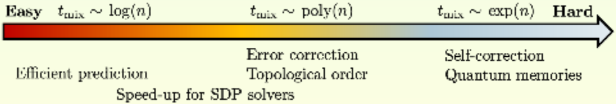
"Positive" point of view:

- Thermal states with short mixing time can be **constructed efficiently** with a quantum device that simulates the effect of the thermal bath.
- This has important implications as a self-studying open problem as well as in optimization problems via simulated annealing type algorithms.

Applications to quantum information/quantum computing

If **rapid mixing**, no **error correction:**





Main applications or consequences:

- Robust and efficient **preparation of topologically ordered phases** of matter via dissipation.
- Design of more efficient **quantum error-correcting codes** optimized for correlated Markovian noise models.
- **Stability** against local perturbations (Cubitt, Lucia, Michalakis, Pérez-García '15)
- **Area law** for mutual information (Brandao, Cubitt, Lucia, Michalakis, Pérez-García '15)
- Gaussian **concentration inequalities** (Lipschitz observables) (C., Rouzé, S. Franca '20)
- Finite blocklength refinement of **quantum Stein lemma** (C., Rouzé, Stilck Franca '20)
- **Quantum annealers:** Output an energy closed to that of the fixed point after short time (C., Rouzé, Stilck Franca '20)
- **Preparation Gibbs states:** Existence of local quantum circuits with logarithmic depth to prepare the Gibbs state (C., Rouzé, Stilck Franca '20)
- Establish the absence of **dissipative phase transitions** (Bardet, C., Gao, Lucia, Pérez-García, Rouzé '21)
- Examples of interacting **SPT phases** with decoherence time growing logarithmically with the system size for thermal noise (Bardet, C., Gao, Lucia, Pérez-García, Rouzé '21)

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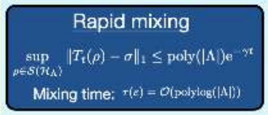

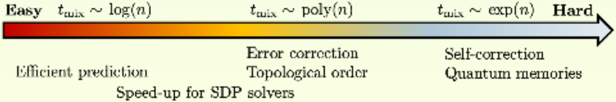

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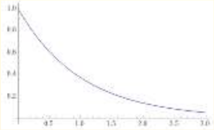

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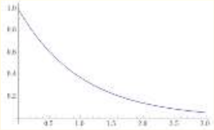

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If $\liminf_{\Lambda \nearrow \mathbb{Z}^d} \alpha(\mathcal{L}_\Lambda) > 0$:

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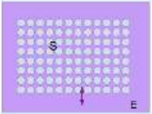
$$e^{t\mathcal{L}}(\rho) \xrightarrow{t \to \infty} \sigma$$



Notation: $\Lambda \subset\subset \mathbb{Z}^d$ lattice

$\{T_t\}_{t \geq 0}$ Quantum Markov semigroup

$\mathcal{L}$ Inf. generator (Lindbladian)

Mixing time of the semigroup $\{T_t\}_{t \geq 0}$

$$\tau(\varepsilon) = \min \left\{ t > 0 : \sup_{\rho \in \mathcal{S}(\mathcal{H}_\Lambda)} \| T_t(\rho) - \sigma \|_1 \leq \varepsilon \right\}$$

Quantum spin systems

Modified Logarithmic Sobolev Inequality (MLSI)

$$D(T_t(\rho)\|\sigma) \leq D(\rho\|\sigma)\, e^{-2\alpha(\mathcal{L})t}$$

Relative entropy: $D(\rho\|\sigma) := \operatorname{tr}[\rho(\log\rho - \log\sigma)]$

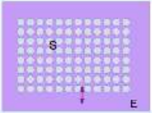
Kastoryano-Temme, '13

Rapid mixing

$$\sup_{\rho \in S(\mathcal{H}_\Lambda)} \|T_t(\rho) - \sigma\|_1 \leq \operatorname{poly}(|\Lambda|)e^{-\gamma t}$$

Mixing time: $\tau(\varepsilon) = O(\operatorname{polylog}(|\Lambda|))$

$$e^{t\mathcal{L}}(\rho) \overset{t\to\infty}{\longrightarrow} \sigma$$



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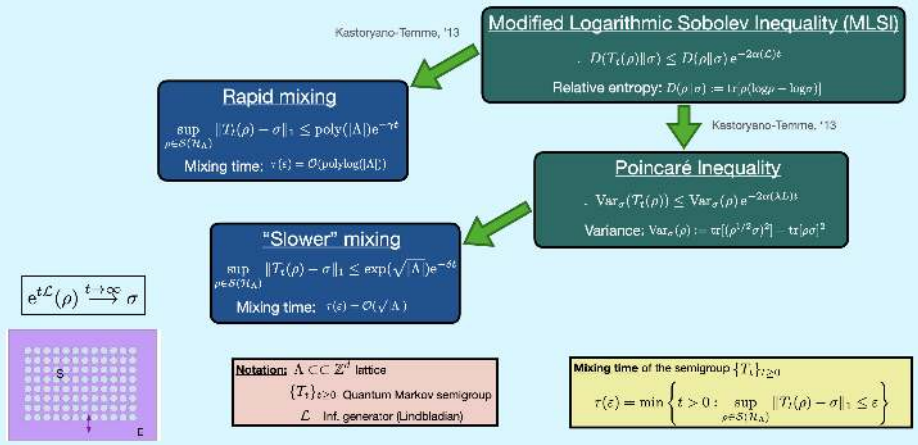
Mixing time of the semigroup $\{T_t\}_{t>0}$

$$\tau(\varepsilon) = \min\left\{ t > 0 : \sup_{\rho \in S(\mathcal{H}_\Lambda)} \|T_t(\rho) - \sigma\|_1 \leq \varepsilon \right\}$$

Quantum spin systems



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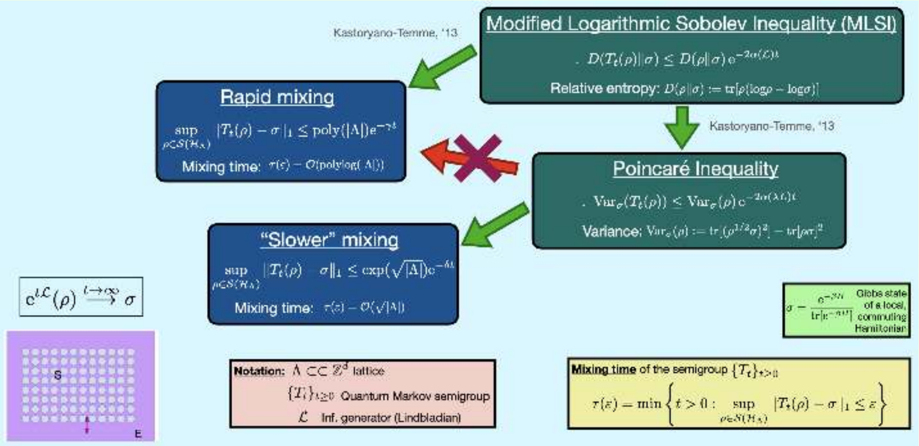

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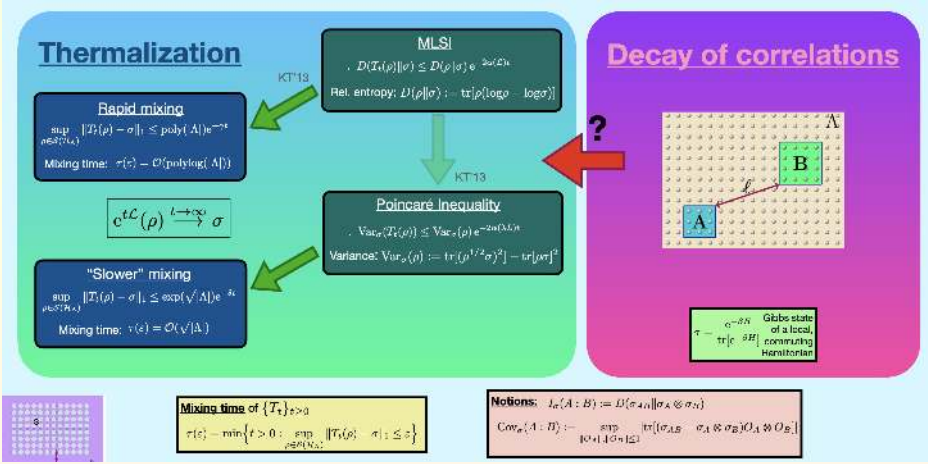

Decay of correlations on Gibbs state

Motivation

Describe the **correlation properties** of **Gibbs states** of local Hamiltonians.

- **Hamiltonian:** $H_\Lambda = H_A + H_B + H_{(A\cup B)^c} + H_{\partial A} + H_{\partial B}$,
- **Gibbs state:** $\sigma_\Lambda(\beta) = \mathrm{e}^{-\beta H_\Lambda} \, / \, \mathrm{Tr}[\mathrm{e}^{-\beta H_\Lambda}]$.

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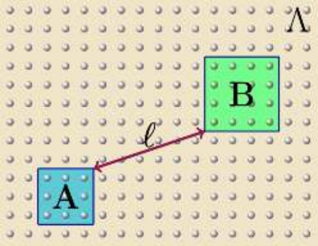

$\ell := \mathrm{dist}(A, B)$

Questions:

For non-commuting Hamiltonians:

$$\mathrm{e}^{-\beta H_{A\cup B}} \approx \mathrm{e}^{-\beta H_A}\, \mathrm{e}^{-\beta H_B} \ ?$$

$$\mathrm{tr}_{A^c}[\sigma_\Lambda] \otimes \mathrm{tr}_{B^c}[\sigma_\Lambda] := (\sigma_\Lambda)_A \otimes (\sigma_\Lambda)_B \approx \mathrm{tr}_{(A\cup B)^c}[\sigma_\Lambda] := (\sigma_\Lambda)_{A\cup B} \ ?$$

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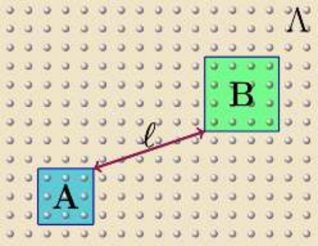

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Decay of correlations on Gibbs state

3 different forms of **decay of correlations**.

Operator correlation

$$\text{Cov}_\sigma(A:B) := \sup_{\|O_A\|=\|O_B\|=1} \left|\text{tr}[O_A \otimes O_B(\sigma_{AB} - \sigma_A \otimes \sigma_B)]\right|$$

Mutual information

$$I_\sigma(A:B) := D(\sigma_{AB}||\sigma_A \otimes \sigma_B)$$

for $D(\rho\|\sigma) = \text{Tr}[\rho(\log\rho - \log\sigma)]$

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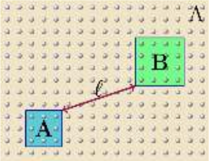
$$\|h(\sigma_{AB})\|_\infty = \left\| \sigma_A^{-1/2} \otimes \sigma_B^{-1/2} \sigma_{AB} \sigma_A^{-1/2} \otimes \sigma_B^{-1/2} - \mathbb{1}_{AB} \right\|_\infty$$



Relation:

$$\frac{1}{2}\mathrm{Cov}_\sigma(A:B)^2 \le I_\sigma(A:B)$$
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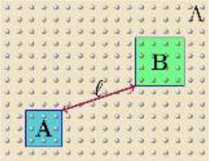

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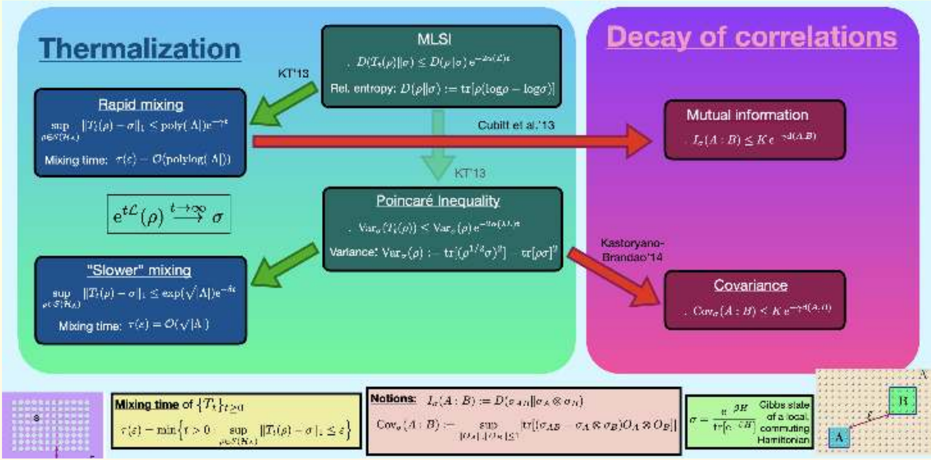
Quantum spin systems



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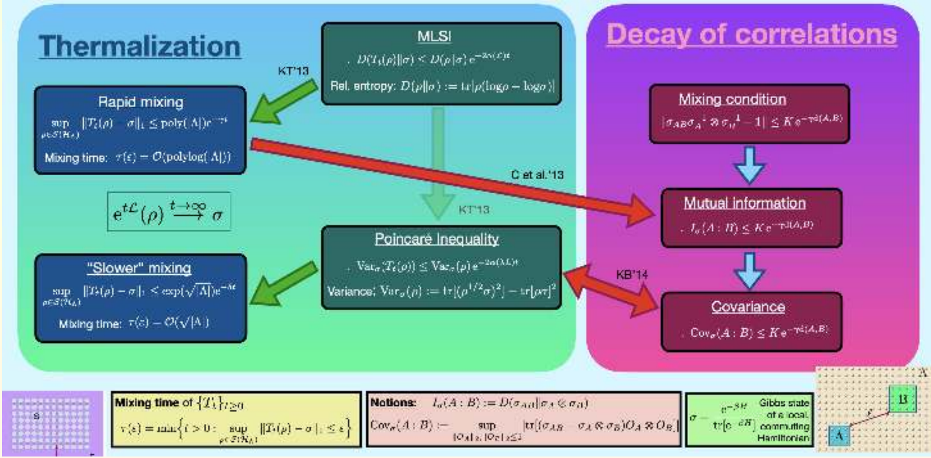

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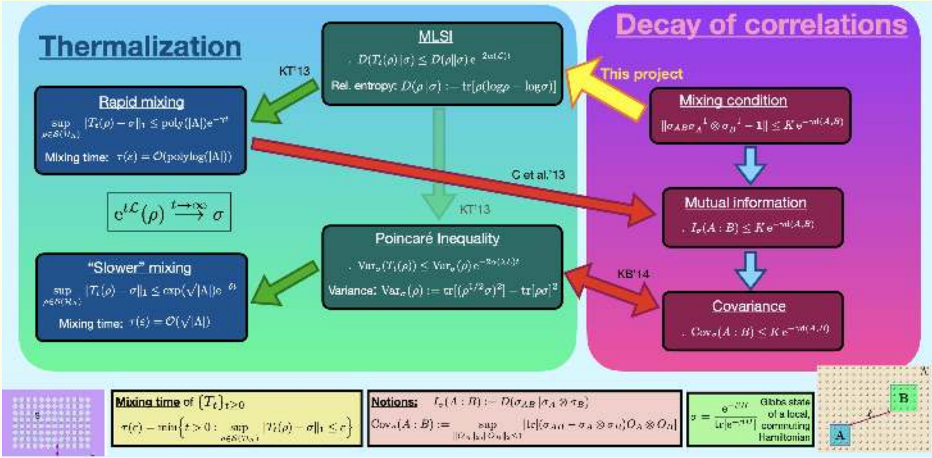


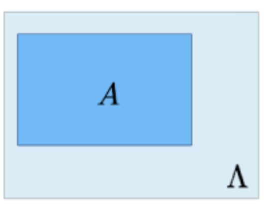
Objective

MLSI constant

$$\alpha(\mathcal{L}_\Lambda) := \inf_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_\Lambda(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2D(\rho_\Lambda || \sigma_\Lambda)}$$

What do we want to prove?

$$\liminf_{\Lambda \nearrow \mathbb{Z}^d} \alpha(\mathcal{L}_\Lambda) \geq \Psi(|\Lambda|) > 0.$$



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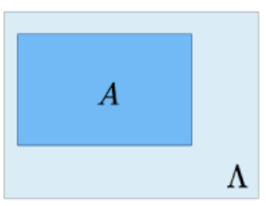

Can we prove something like

$$\alpha(\mathcal{L}_\Lambda) \geq \Psi(|A|)\, \alpha(\mathcal{L}_A) > 0 \, ?$$

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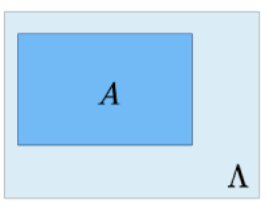

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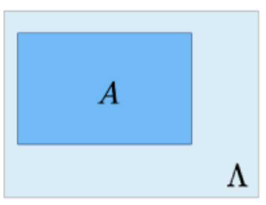

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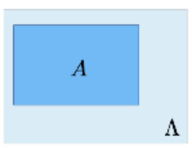

MLSI constant

The **MLSI constant** of $\mathcal{L}_\Lambda = \sum\limits_{k \in \Lambda} \mathcal{L}_k$ is defined by

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The **conditional MLSI constant** of $\mathcal{L}_\Lambda$ on $A \subset \Lambda$ is defined by

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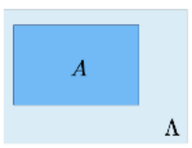
Conditional MLSI constant



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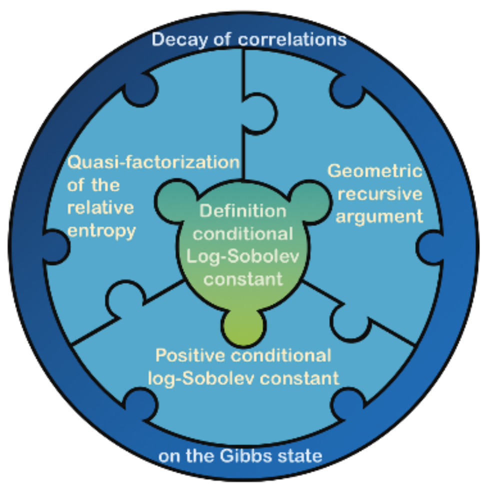
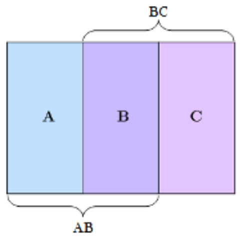
Strategy

Used in (C.-Lucia-Pérez García '18) and (Bardet-C.-Lucia-Pérez García-Rouzé, '19).



Quasi-factorization of the relative entropy



Quasi-factorization of the relative entropy

Given $\Lambda = ABC$, it is an inequality of the form:

$$D(\rho_\Lambda \| \sigma_\Lambda) \leq \xi(\sigma_{ABC}) \left[D_{AB}(\rho_\Lambda \| \sigma_\Lambda) + D_{BC}(\rho_\Lambda \| \sigma_\Lambda) \right] ,$$

for $\rho_\Lambda, \sigma_\Lambda \in \mathcal{S}(\mathcal{H}_{ABC})$, where $\xi(\sigma_{ABC})$ depends only on σ_{ABC} and measures how far σ_{AC} is from $\sigma_A \otimes \sigma_C$.

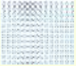
Example: Tensor product fixed point

(C.-Lucia-Pérez García '18)

(Beigi-Datta-Rouzé '18)

$$\mathcal{L}_\Lambda(\rho_\Lambda) = \sum_{x\in\Lambda} (\sigma_x \otimes \rho_{x^c} - \rho_\Lambda) \qquad \textbf{heat-bath}$$

$$D_x(\rho_\Lambda \| \sigma_\Lambda) := D(\rho_\Lambda \| \sigma_\Lambda) - D(\rho_{x^c} \| \sigma_{x^c})$$



$$\sigma_\Lambda = \bigotimes_{x\in\Lambda} \sigma_x ,$$

$$D(\rho_\Lambda || \sigma_\Lambda) \leq$$

$$\leq \sum_{x\in\Lambda} D_x(\rho_\Lambda || \sigma_\Lambda)$$

$$\leq \sum_{x\in\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_x(\rho_\Lambda)(\log\rho_\Lambda - \log\sigma_\Lambda)]}{2\alpha_\Lambda(\mathcal{L}_x)}$$

$$\leq \frac{1}{2\inf\limits_{x\in\Lambda} \alpha_\Lambda(\mathcal{L}_x)} \sum_{x\in\Lambda} -\operatorname{tr}[\mathcal{L}_x(\rho_\Lambda)(\log\rho_\Lambda - \log\sigma_\Lambda)]$$

$$= \frac{1}{2\inf\limits_{x\in\Lambda} \alpha_\Lambda(\mathcal{L}_x)} \left(-\operatorname{tr}[\mathcal{L}_\Lambda(\rho_\Lambda)(\log\rho_\Lambda - \log\sigma_\Lambda)]\right)$$

$$\leq \left(-\operatorname{tr}[\mathcal{L}_\Lambda(\rho_\Lambda)(\log\rho_\Lambda - \log\sigma_\Lambda)]\right).$$

Dynamics

Let $\sigma_\Lambda = \frac{e^{-\beta H_\Lambda}}{\operatorname{tr}\left[e^{-\beta H_\Lambda}\right]}$ be the Gibbs state of finite-range, commuting Hamiltonian.

Heat-bath generator

The **heat-bath generator** is defined as:

$$\mathcal{L}_\Lambda^H(\rho_\Lambda) := \sum_{x \in \Lambda} \left(\sigma_\Lambda^{1/2} \sigma_{x^c}^{-1/2} \rho_{x^c} \sigma_{x^c}^{-1/2} \sigma_\Lambda^{1/2} - \rho_\Lambda \right)$$

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Davies generator

The **Davies generator** is given by:

$$\mathcal{L}_\Lambda^{D;*}(X) := i[H_\Lambda, X] + \sum_{x\in\Lambda} \widetilde{\mathcal{L}}_x^D(X)\,,$$

where the $\mathcal{L}_x^D$ are defined in terms of the Fourier coefficients of the correlation functions in the bath and the ones of the system couplings.

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Previous results

Let us recall: For $\alpha(\mathcal{L}_\Lambda)$ a MLSI constant,

$$\left\| \rho_t - \sigma_\Lambda \right\|_1 \leq \sqrt{2 \log(1/\sigma_{\min})}\, e^{-\alpha(\mathcal{L}_\Lambda)\, t}.$$

Using the spectral gap $\lambda(\mathcal{L}_\Lambda)$:

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Let $\mathcal{L}_\Lambda^{H,D}$ be the **heat-bath** or **Davies** generator in 1D. Then, $\mathcal{L}_\Lambda^{H,D}$ has a positive spectral gap that is independent of the system size, for every temperature.

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Let $\mathcal{L}_\Lambda^{H}$ be the **heat-bath** generator with tensor product fixed point. Then, it has a positive MLSI constant.

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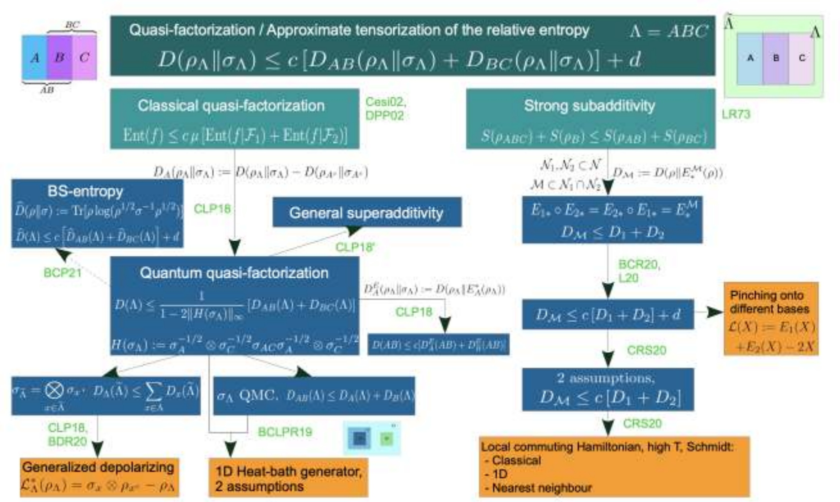
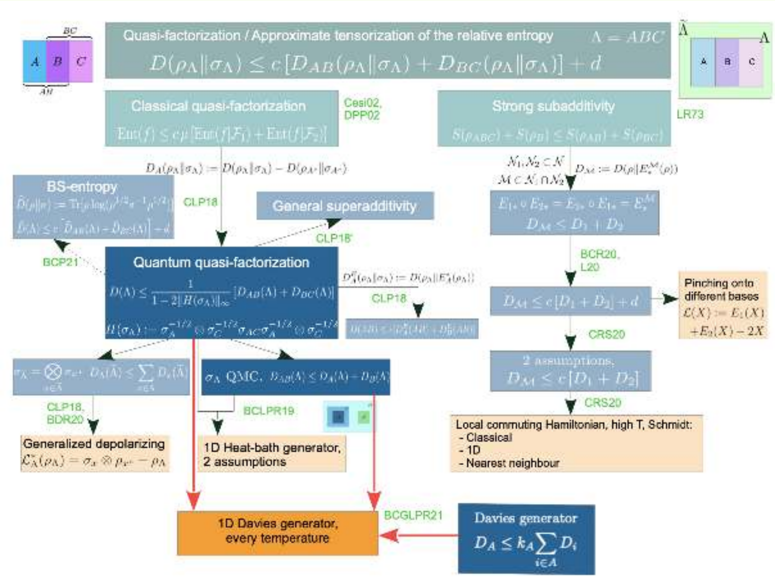
Quasi-factorization of the relative entropy

Results of **Quasi-Factorization** or **Approximate Tensorization**

Results of **Modified Logarithmic Sobolev Inequality**



Quasi-factorization of the relative entropy



MLSI for Davies generators in 1D

MLSI for 1D Davies generators, (Bardet-C.-Gao-Lucia-Pérez García-Rouzé, '22)

Let $\mathcal{L}_\Lambda^D$ be a **Davies** generator with unique fixed point σ_Λ given by the Gibbs state of a commuting, finite-range, translation-invariant Hamiltonian at any temperature in 1D. Then, $\mathcal{L}_\Lambda^D$ satisfies a positive MLSI $\alpha(\mathcal{L}_\Lambda^D) = \Omega(\ln(|\Lambda|)^{-1})$.

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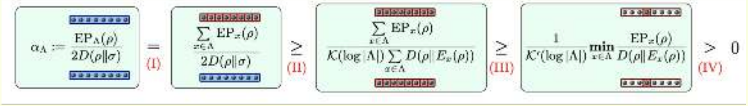
Sketch of the proof: Quasi-factorization

$$\alpha(\mathcal{L}_\Lambda) := \inf_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{-\operatorname{tr}[\mathcal{L}_\Lambda(\rho_\Lambda)(\log \rho_\Lambda - \log \sigma_\Lambda)]}{2D(\rho_\Lambda || \sigma_\Lambda)} = \inf_{\rho_\Lambda \in \mathcal{S}_\Lambda} \frac{\mathrm{EP}_\Lambda(\rho_\Lambda)}{2D(\rho_\Lambda || \sigma_\Lambda)}$$



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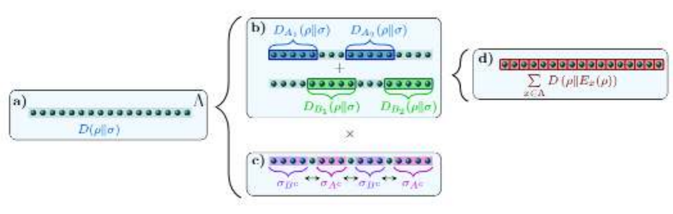

Last step: Spectral gap $\overset{\mathcal{O}(\log n)}{\mapsto}$ MLSI.

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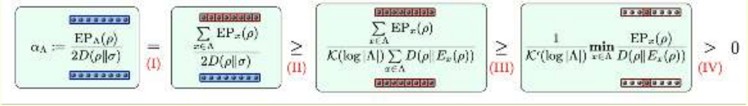

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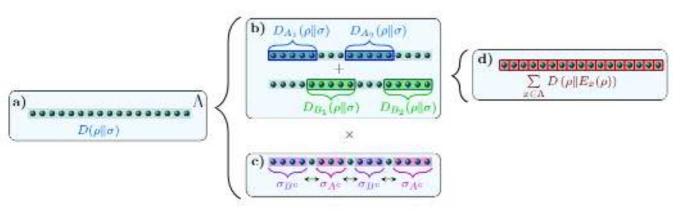

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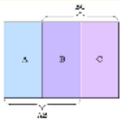
Proof: Conditional relative entropies + Quasi-factorization



Conditional relative entropies: $D_A(\rho_\Lambda \| \sigma_\Lambda) := D(\rho_\Lambda \| \sigma_\Lambda) - D(\rho_{A^c} \| \sigma_{A^c})$,
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Heat-bath cond. expectation: $E_A(\cdot) := \lim_{n\to\infty} \left(\sigma_\Lambda^{1/2} \sigma_{A^c}^{-1/2} \operatorname{tr}_A[\,\cdot\,]\, \sigma_{A^c}^{-1/2} \sigma_\Lambda^{1/2} \right)^n$.

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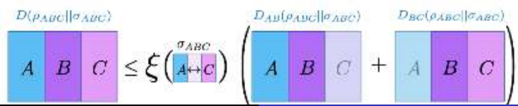
$$\xi(\sigma_{AC}) = \frac{1}{1 - 2\left\| \sigma_A^{-1/2} \otimes \sigma_C^{-1/2}\, \sigma_{AC}\, \sigma_A^{-1/2} \otimes \sigma_C^{-1/2} - \mathbb{1}_{AC} \right\|_\infty}.$$



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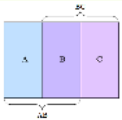

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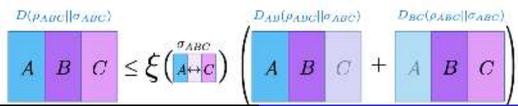

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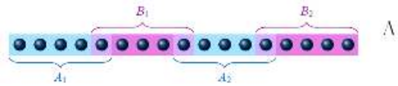

$\sigma_\Lambda = \frac{\mathrm{e}^{-\beta H_\Lambda}}{\mathrm{tr}\left(\mathrm{e}^{-\beta H_\Lambda}\right)}$ is the Gibbs state of a k-local, commuting Hamiltonian H_Λ.

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Quasi-factorization for quantum Markov chains (Bardet-C.-Lucia-Pérez García-Rouzé'19)

Since σ_Λ is a QMC between $A_i \leftrightarrow \partial(A_i) \leftrightarrow (A_i \cup \partial A_i)^c$, then:

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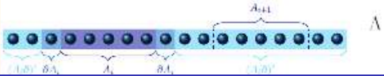

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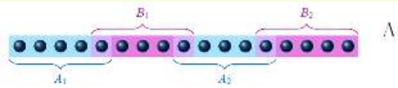

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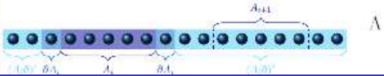
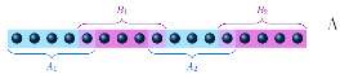


Proof: Decay of correlations



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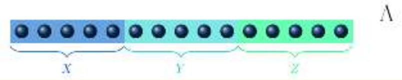

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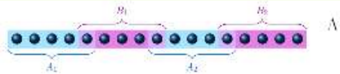

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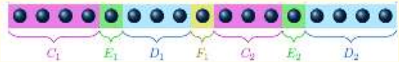

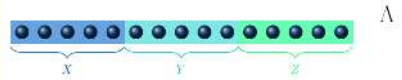

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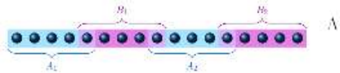

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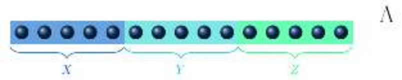

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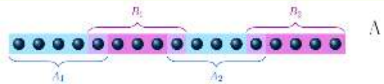

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and thus

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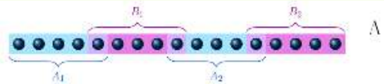

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Proof: Positive CMLSI

Reduction of cond. relative entropies (Gao-Rouzé, '21)

$$D(\rho_\Lambda \| E_{A_i}(\rho_\Lambda)) \leq 4 k_{A_i} \sum_{j \in A_i} D(\rho_\Lambda \| E_j(\rho_\Lambda))$$

Reduction from CMLSI to Gap

$$k_{A_i} \propto \frac{1}{\ln \lambda} ,$$

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The CMLSI of the local generators is positive:

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Thank you for your attention!

Do you have any questions?

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