Quantum phases of matter from a computer-science perspective

DALL-E: “entangled many-body quantum state”

ICMAT Focus Week:
Quantum many body systems
and quantum information

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INSTITUTO DE CIENCIAS MATEMÁTICAS

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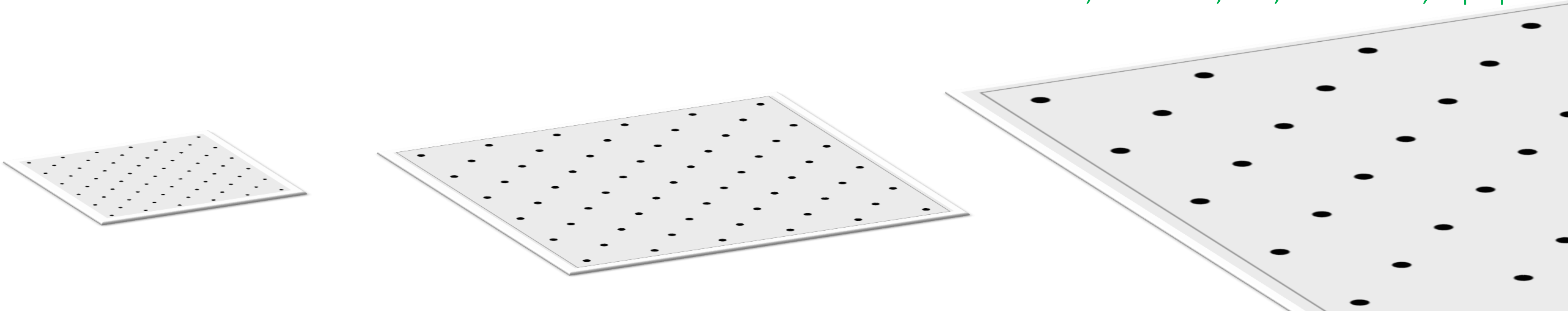
QUANTUM PHASES OF MATTER

- *"For a large collection of similar particles, a **phase** is a region in some parameter space in which the thermal equilibrium states possess some properties in common that can be distinguished from those in other phases."* N. Read, Topological phases and quasiparticle braiding

- Consider a family of n-qubit geometrically local Hamiltonians. Their ground states are said to be in a **gapped phase** if they retain an energy gap as $n \to \infty$ (**thermal limit**).

- Phases of matter are robust to small perturbations throughout the geometry.
  - Stability has been shown for all sorts of phases on **Euclidean lattices** (arXiv:2205.10460).
  - Understanding **QLDPC codes** as phases of matter require other manifolds.

A. Lavasani, M. Gullans, VVA, M. Barkeshli, in prep.



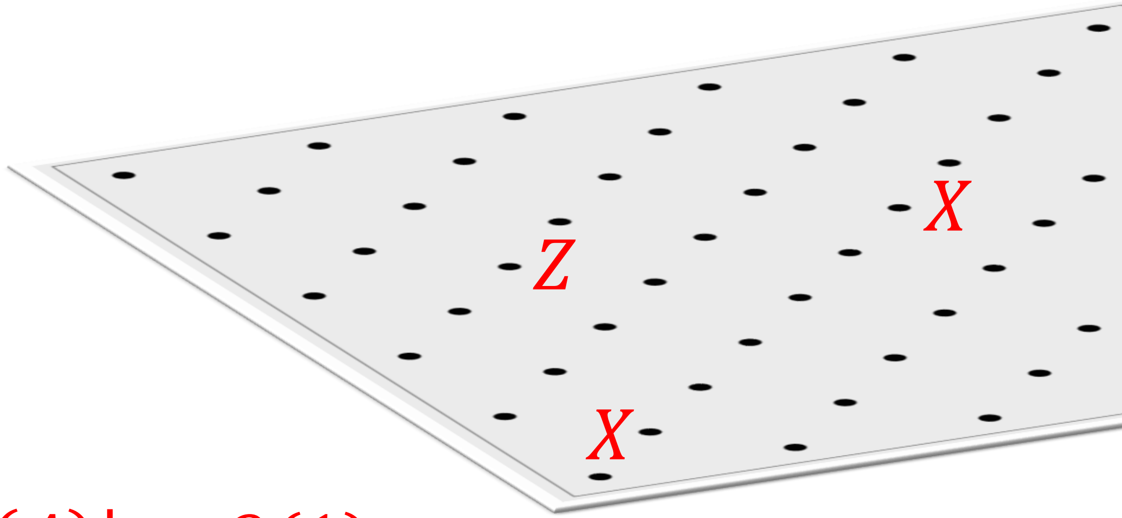
ALL ABOUT LOCALITY

- Locality is defined w.r.t. a **metric** on the underlying qubit manifold, e.g., Euclidean distance. Manifold topology is fixed.

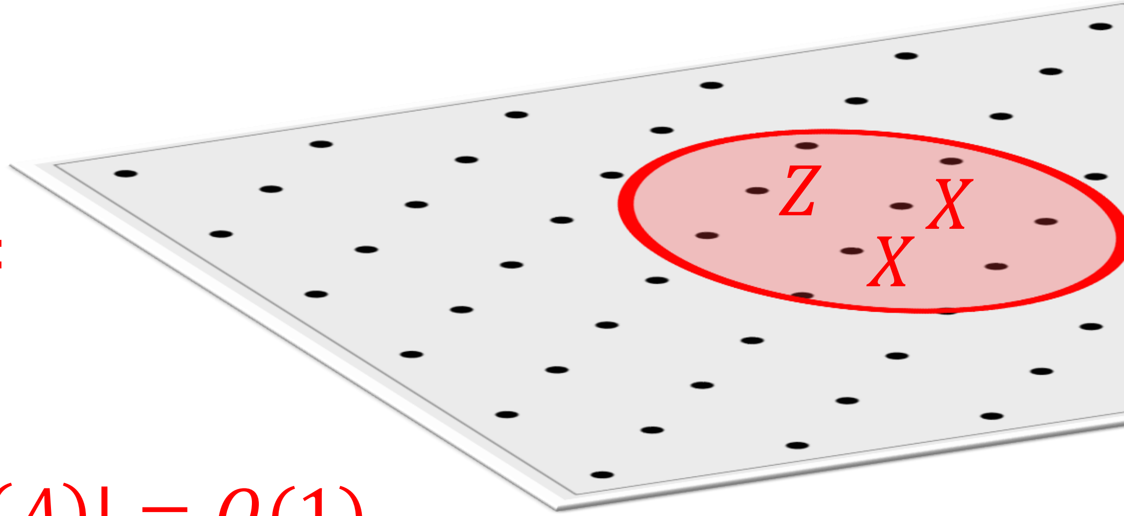
- An operator on a lattice is **local** if
  1. ($n = \infty$) it is finitely supported
  2. ($n \to \infty$) it its support is independent and small w.r.t. n.

- A family of finite-n-local operators is **geometrically local** if their support is contained in a ball of radius indep. of n.

- A quantum operation $\mathcal{O}$ is **causal** (a.k.a. locality-preserving) if it maps local operators to local operators.
  - ✓ local/causal → quasi-local/causal

$A =$ 

$|\mathrm{supp}(A)| = O(1)$

$A =$ 

$|\mathrm{supp}(A)| = O(1)$

$\mathrm{supp}(A) \subset \mathrm{Ball}_{O(1)}(\text{some center})$

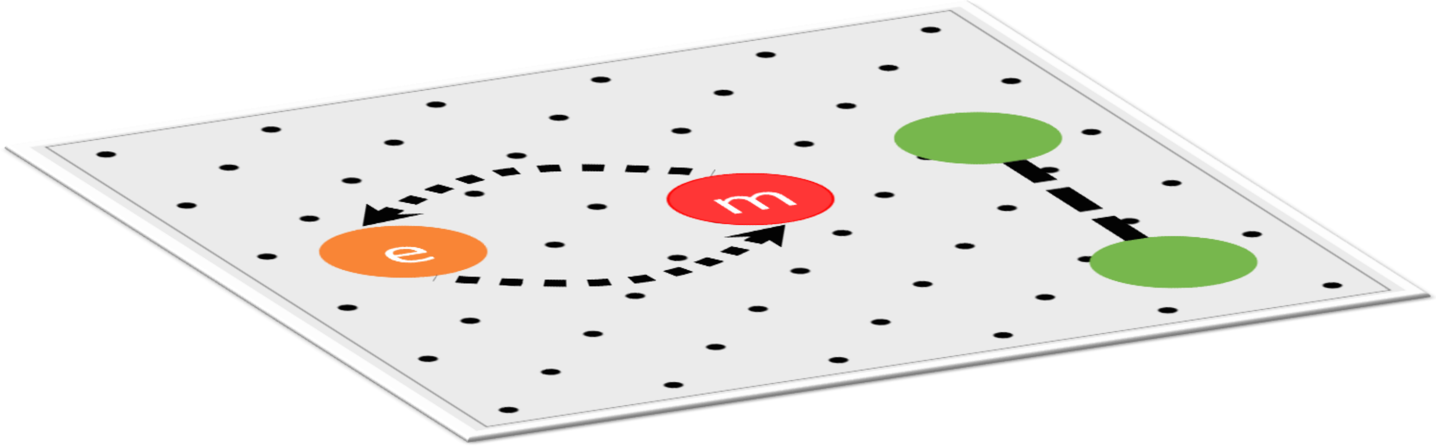
PHASE CLASSIFICATION

Phases are classified by behavior of their **excitations**. Excitations can occur:

…in the bulk of the manifold:

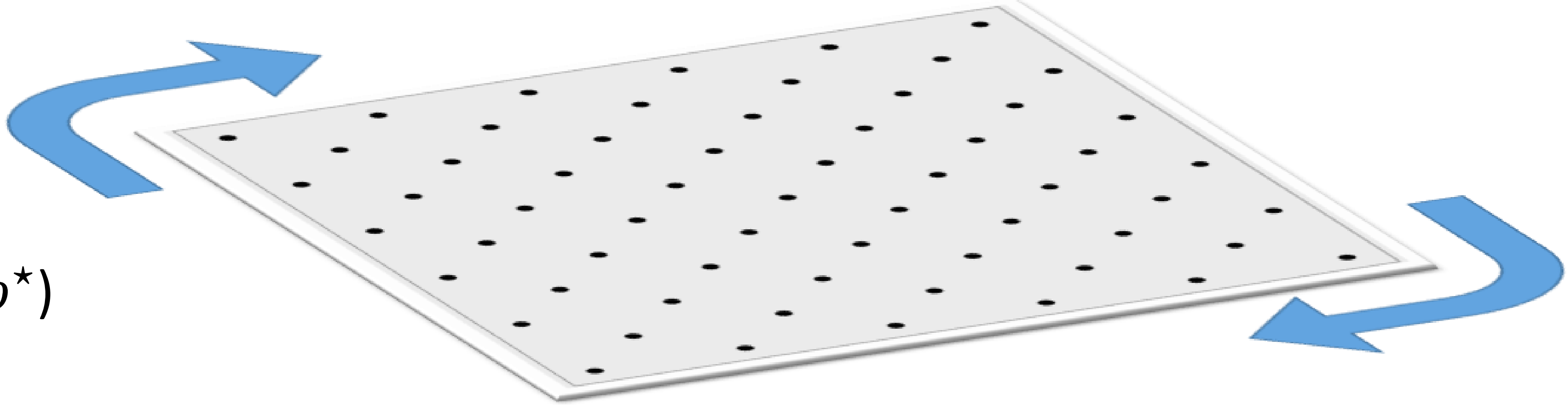
- Anyon theories classified by **braided fusion categories**:
  - ✓ Anyon types a
  - ✓ Exchange statistics $\theta(a)$
  - ✓ Fusion rules

E.g., see arXiv:2211.03798



…or on the boundary:

- **Chiral central charge** c_- counts difference between right & left "movers" ($\rho \neq \rho^\star$)
- "Movers" can carry "fractional" heat current.
- Unifying relation →

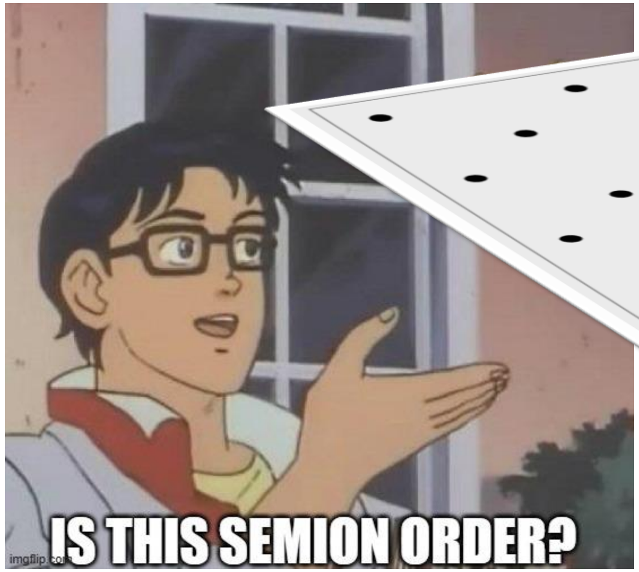


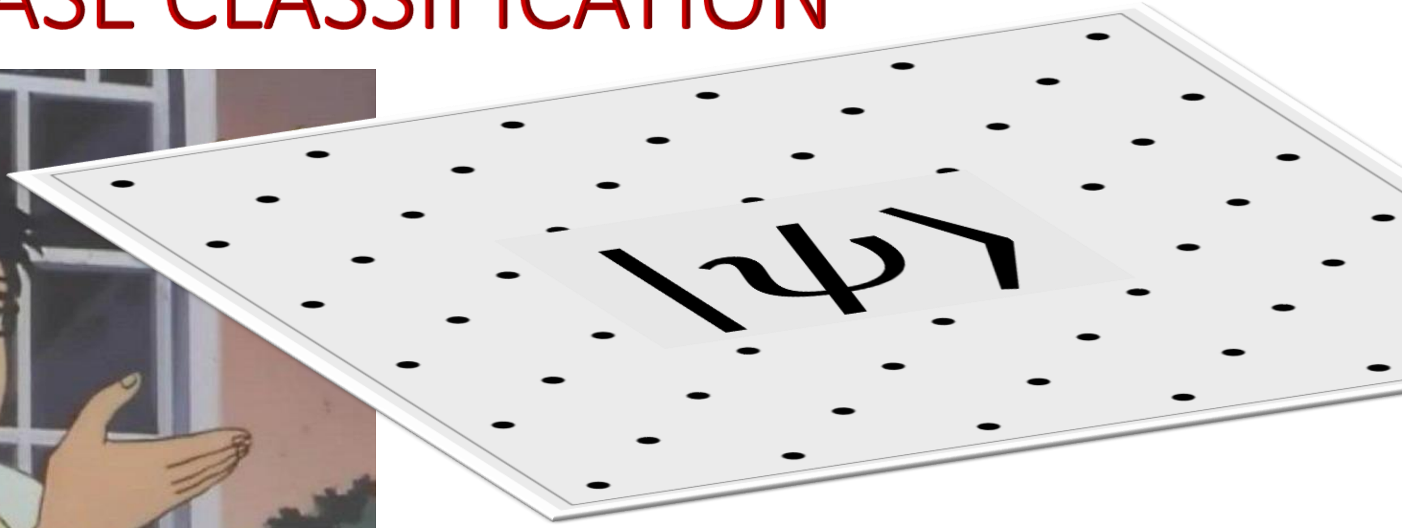
$$\exp\left(i\frac{2\pi}{8}c_-\right) = \frac{1}{\sqrt{\#_{\text{anyons}}}}\sum_a \theta(a)$$

Kitaev, arXiv:cond-mat/0506438

CERTIFIED PHASE CLASSIFICATION

Problem: Given access to copies to a 2D gapped phase $|\psi\rangle$, determine the phase.



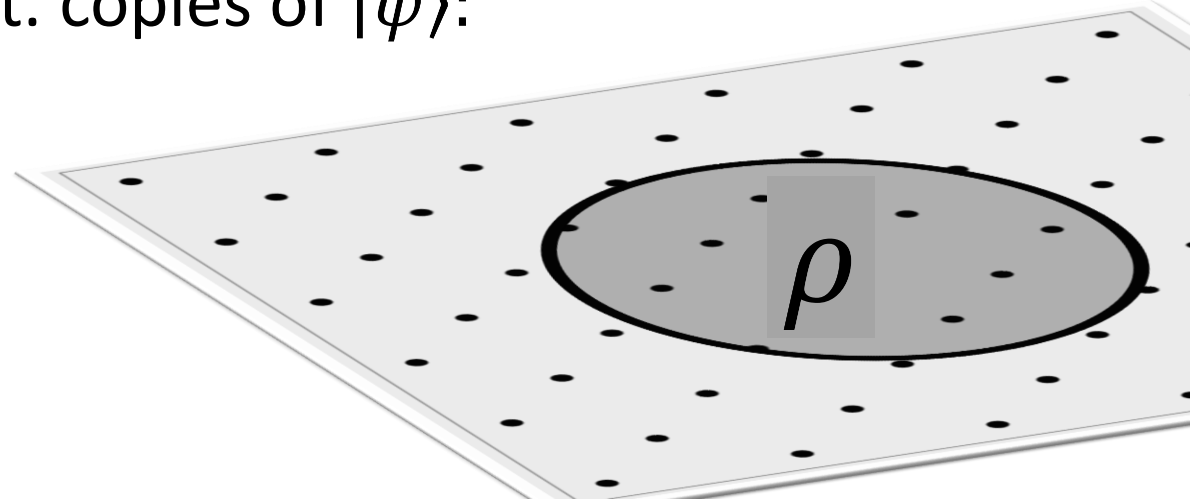


Idea (preliminary): Extract phase data from (**preferably polynomial**) functions of (**preferably local**) density matrices.

- These correspond to **observables** w.r.t. copies of $|\psi\rangle$:

$$\mathrm{Tr}\left(A\rho\right) \leftrightarrow \mathrm{poly}\left(\rho, \rho^{\star}\right) \text{ of deg } \leq 1$$

$$\mathrm{Tr}\left(A\rho^{\otimes 2}\right) \leftrightarrow \mathrm{poly}\left(\rho, \rho^{\star}\right) \text{ of deg } \leq 2$$



OBSERVABLES *A* FROM *BULK* OF *BOSONIC* PHASES

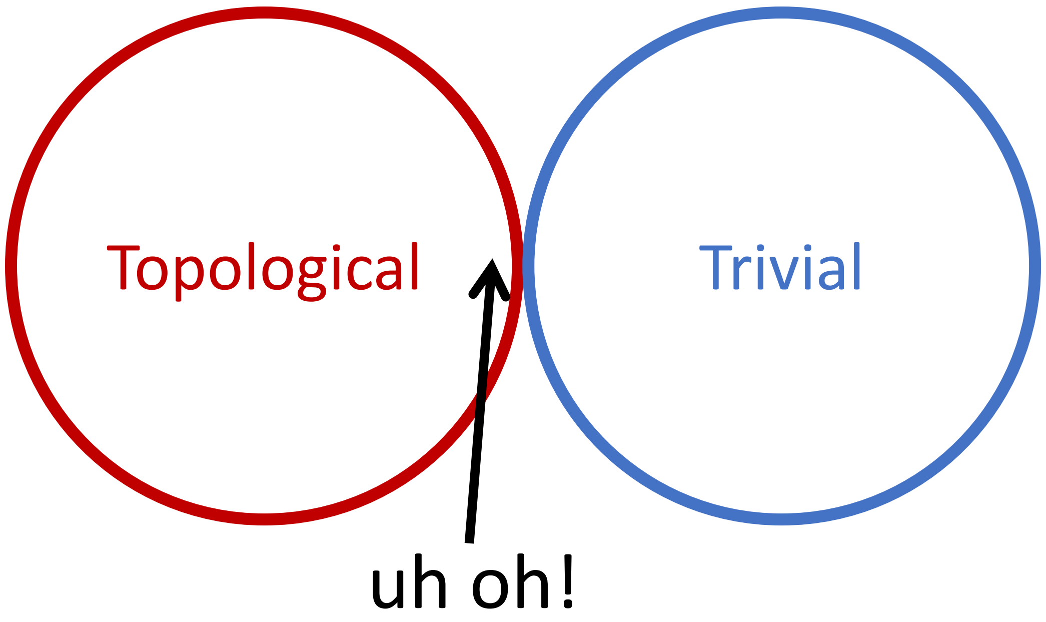
system	symmetry	copies	operator	local	proven?	references
topological	none	1	none work	×	✓	2106.12627

SINGLE-COPY OBSERVABLES DON'T WORK

BWOC, **assume** that there exists an observable A whose expectation value is close to $+1$ (-1) in all representatives of the trivial (topological) phase.

States closest to 0 **on both sides**, by assumption, satisfy:

$$\sup_{\psi \in \text{top.}} \langle \psi | A | \psi \rangle < 0 \leq \inf_{\psi \in \text{triv.}} \langle \psi | A | \psi \rangle$$



By averaging over single-qubit unitaries to get the trace, we see that averages are **the same** for both phases:

$$\mathop{\mathbb{E}}_{U_1, \cdots, U_n \in \mathsf{U}_2} \langle \psi | A | \psi \rangle = \frac{\operatorname{Tr} A}{2^n} \quad \xrightarrow{\text{contradiction}} \quad \frac{\operatorname{Tr} A}{2^n} < 0 \leq \frac{\operatorname{Tr} A}{2^n}$$

arXiv:2106.12627, Appx. I

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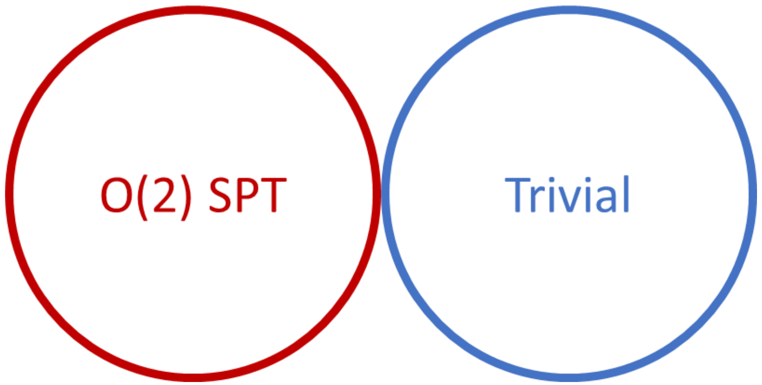
AFFLECK-LIEB TWIST OPERATOR

Local twist acts on $\ell \ll n$ sites and imparts phase on site k that **depends** on k (cf. polarization):

$$A_\ell = \bigotimes_{k\,,\,\left|k-\frac{1}{2}\right|\leq \ell+\frac{1}{2}} \exp\left(-\mathrm{i}2\pi \frac{k+\ell}{2\ell+1} S_k^{(z)}\right)$$



Its expectation value is close to $+1$ (-1) in all states in the trivial (SPT) phase:

- ✓ Twist imparts a **global phase** on GND state in thermal limit (tedious part):

$$\langle\psi|A_\ell H A_\ell^\dagger|\psi\rangle - \langle\psi|H|\psi\rangle \leq \frac{\text{const.}}{\ell}$$

- ✓ **Overlap** between twisted and original GND states is near unity:

$$|\langle\psi|A_\ell|\psi\rangle| \geq 1 - \frac{\text{const.}}{\text{gap}\times\ell}$$

- ✓ Symmetry tells us expectation of twist is real, can only be $+1$ or -1:

$$\langle\psi|A_\ell|\psi\rangle = \langle\psi|A_\ell|\psi\rangle^\star$$

- ✓ Why does (sufficiently large but) finite ℓ work? Expectation remains **the same** as ℓ increases because twist is **continuous** in ℓ!

arXiv:2106.12627, Appx. K; reformulation of arXiv:1804.04337 + book by Tasaki

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HALL CONDUCTIVITY (NEEDS HAMILTONIAN)

$$H = \sum_{\mathsf{j}} H_{\mathsf{j}}$$

$$Q = \sum_{\mathsf{j}} Q_{\mathsf{j}}$$

$$[Q, H_{\mathsf{j}}] = 0$$

Given Hamiltonian H and $U(1)$-symmetry generator ("charge") Q, define **currents** J:

$$J_{\mathsf{jk}} \equiv i\,[H_{\mathsf{j}}, Q_{\mathsf{k}}] - i\,[H_{\mathsf{k}}, Q_{\mathsf{j}}]$$

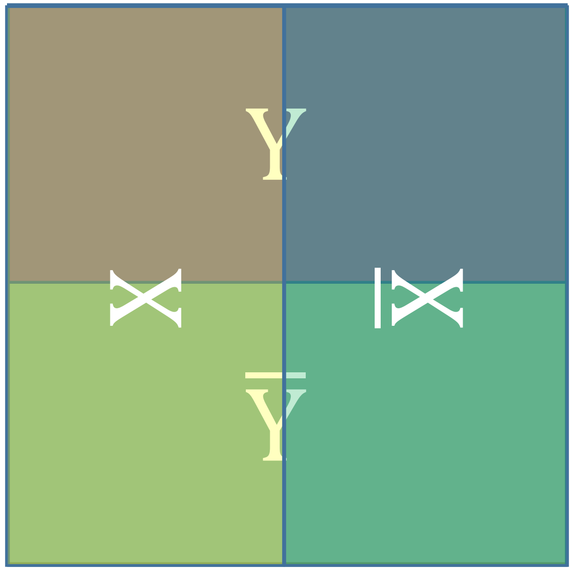
Build currents K by "smearing" (i.e., quasi-adiabatically evolving) the currents J:

$$\mathcal{H}^{-1}(\cdot) = i \sum_{a \neq b} \frac{|a\rangle\langle a|\,(\cdot)\,|b\rangle\langle b|}{E_a - E_b}$$

$$K_{\mathsf{jk}} = \mathcal{H}^{-1}\left(J_{\mathsf{jk}}\right)$$

Hall conductivity is expectation value of a **commutator** between K:

$$A \equiv i\left[K_{\mathsf{X}\overline{\mathsf{X}}}, K_{\mathsf{Y}\overline{\mathsf{Y}}}\right] = i \sum_{\mathsf{j}\in\mathsf{X}} \sum_{\mathsf{k}\in\overline{\mathsf{X}}} \sum_{\mathsf{l}\in\mathsf{Y}} \sum_{\mathsf{m}\in\overline{\mathsf{Y}}} [K_{\mathsf{jk}}, K_{\mathsf{lm}}]$$



arXiv:2006.14151

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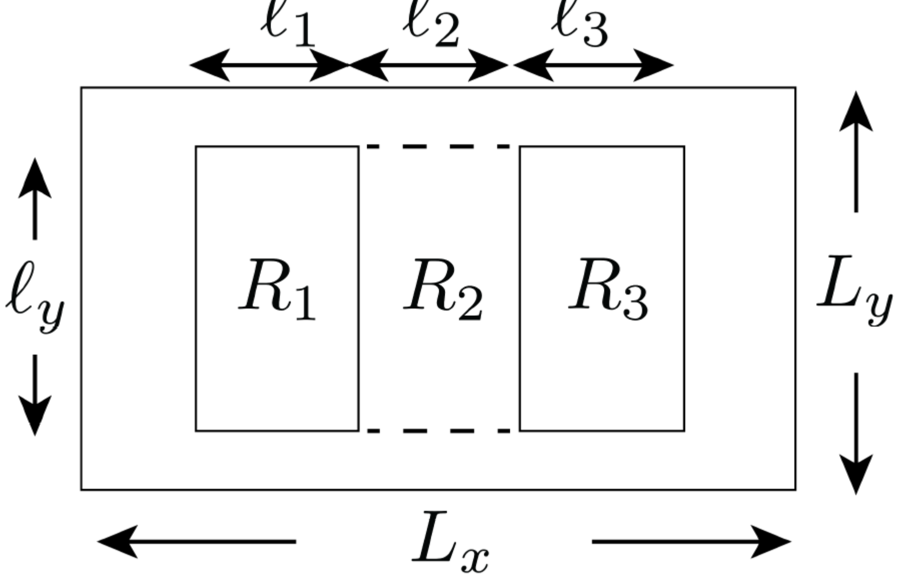
Chern insulator / fractional QH	$U(1)$ charge	1	SWAP/twist	×	∼	2005.13677
		2	”	✓	∼	”

CHERN NUMBER

Given U(1)-symmetric state, can define **Chern-number observable**:

$$A = W^{\dagger}_{R_1}(\phi)\mathbb{S}_{1,3}W_{R_1}(\phi)V^s_{R_1\cup R_2}$$



Ingredients similar to Tasaki's formulation:

$$W_R(\phi) = \prod_{(x,y)\in R} e^{i\hat{n}(x,y)\phi}$$ $U(1)$ symmetry

$$V_R = \prod_{(x,y)\in R} e^{i\frac{2\pi y}{\ell_y}\hat{n}(x,y)}$$ similar to twist

$$\mathbb{S}_{1,3}$$ SWAP operator; relates expectation value to TQFT path integrals (don't ask)

arXiv:2005.13677

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2D topological	any	∞	entanglement spectrum	✓	~	0805.0332
		∞	top. entanglement entropy	✓	~	hep-th/0510092, cond-mat/0510613
		∞	modular commutator	✓	~	2110.06932

MODULAR COMMUTATOR → CHIRAL CENTRAL CHARGE

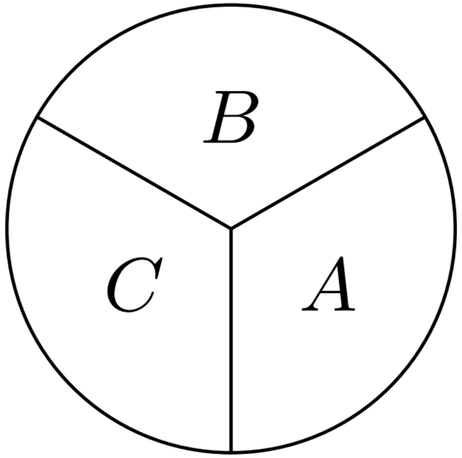
Given Hamiltonian H, define **energy currents** J: $J_{\mathsf{jk}} \equiv -i\,[H_{\mathsf{j}}, H_{\mathsf{k}}]$ $\quad H = \sum_{\mathsf{j}} H_{\mathsf{j}}$

But what if we **only have state** $\rho = |\psi\rangle\langle\psi|$ and **not** its Hamiltonian?

- Can use **modular Hamiltonian** instead:

$$H^{\mathrm{mod}}_{\mathsf{X}} \equiv -\ln \rho_{\mathsf{X}} \qquad J^{\mathrm{mod}}_{\mathsf{jk}} = -i\left[H^{\mathrm{mod}}_{\mathsf{j}}, H^{\mathrm{mod}}_{\mathsf{k}}\right]$$

For sufficiently coarse-grained system, expectation of **modular commutator** yields **chiral central charge** c_{-}:



$$A = i[H^{\mathrm{mod}}_{\mathsf{AB}}, H^{\mathrm{mod}}_{\mathsf{BC}}]$$

First signature of **chiral** many-body entanglement:

$$\langle\psi|A|\psi\rangle \to -\langle\psi|A|\psi\rangle \quad \text{when} \quad \psi \to \psi^{\star}$$

arXiv:2110.06932, 2110.10400

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LOCAL INVARIANTS → ML ALGORITHMS DISTINGUISH PHASES

✓ Obtain **classical shadows** $S(\psi)$ --- efficient snapshots of some quantum state $|\psi\rangle$.

$|\psi\rangle \to \{S(\psi)\}$

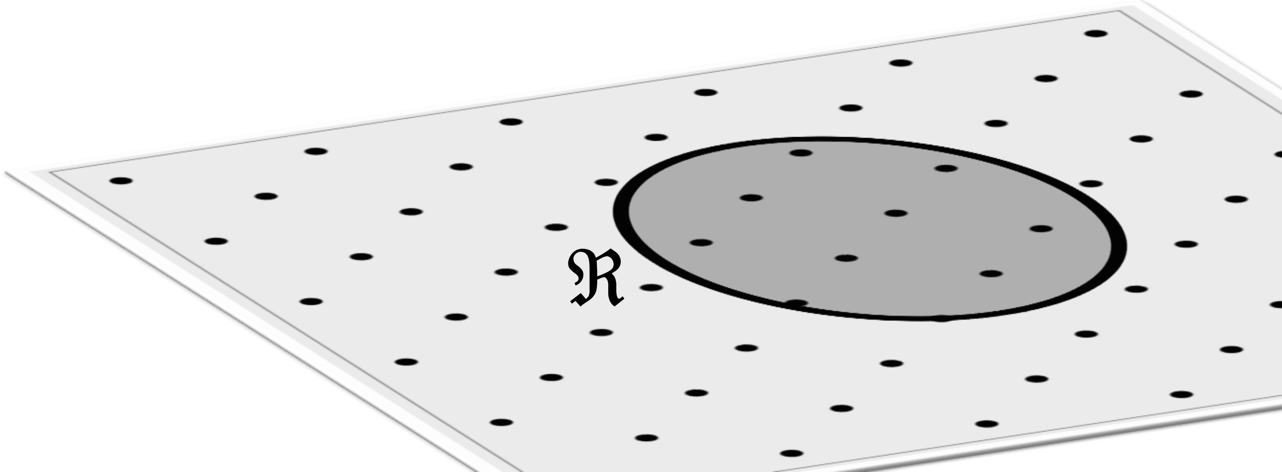
arXiv:2002.08953

✓ Convert into **feature vector v** that linearizes all powers p of reduced density matrices on all regions $\mathfrak{R}$ for all supports w.

$$\mathbf{v}[S(\psi)] = \bigoplus_{p=0}^{\infty} \frac{1}{\sqrt{p!}} \left(\bigoplus_{w=0}^{\infty} \frac{1}{\sqrt{w!}} \bigoplus_{|\mathrm{supp}\mathfrak{R}|=w} S_{\mathfrak{R}}(\psi) \right)^{\otimes p}$$

✓ Any function of state that can be written as a power series in reduced density matrices becomes a **linear function** in feature space!

$$f(\rho) \to \langle f, \mathbf{v} \rangle$$

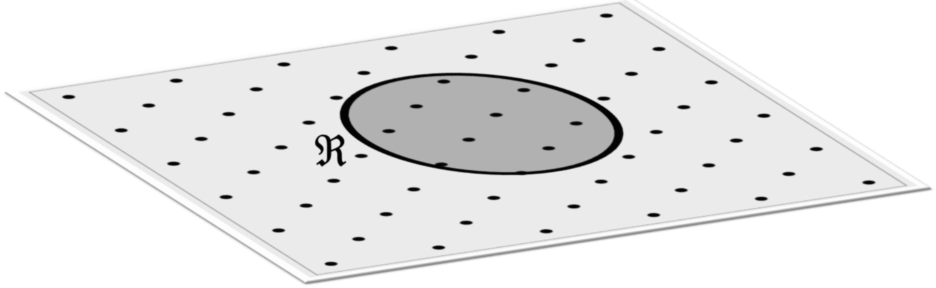


arXiv:2106.12627, Appx. J

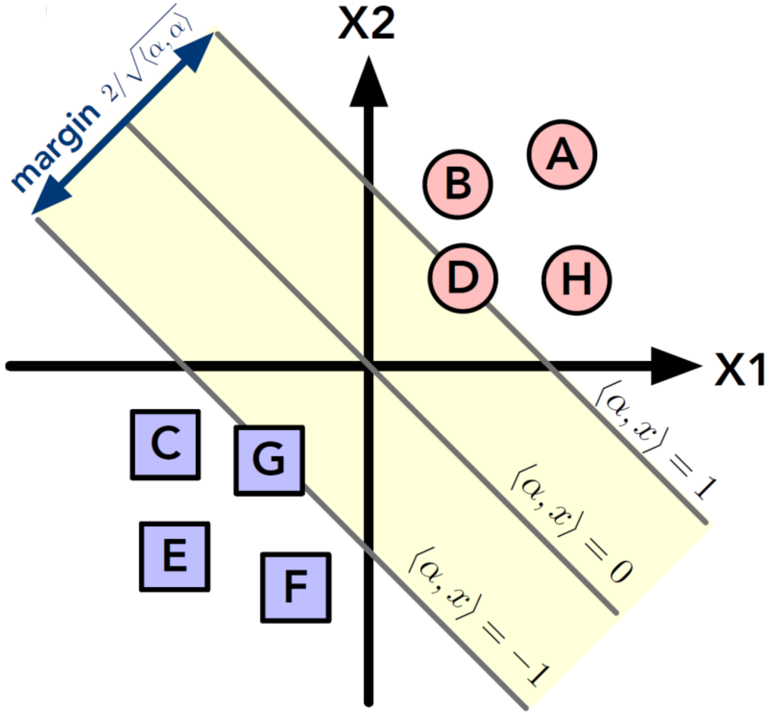
LOCAL INVARIANTS → ML ALGORITHMS DISTINGUISH PHASES

1. **Inner product** between features vectors can be efficiently:
   - ➢ Shadows are tensor products, so combinatorics of different regions $\mathfrak{R}$ simplifies during inner product.
   - ➢ Direct sums over all powers p and all supports w each wrapped into exponentials during inner product.

$$\mathbf{v}[S(\psi)] = \bigoplus_{p=0}^{\infty} \frac{1}{\sqrt{p!}} \left(\bigoplus_{w=0}^{\infty} \frac{1}{\sqrt{w!}} \bigoplus_{|\mathrm{supp}\mathfrak{R}|=w} S_{\mathfrak{R}}(\psi) \right)^{\otimes p}$$



2. **Separating hyperplanes** naturally exist between states in different phases in feature space.
   - ➢ If there exists a local invariant A, then the support-vector machine (SVM) algorithm **will find** the corresponding hyperplane.

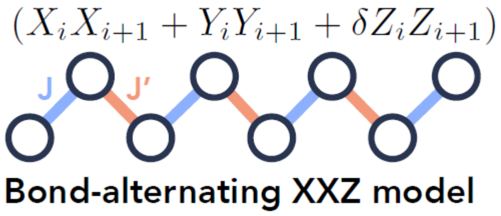


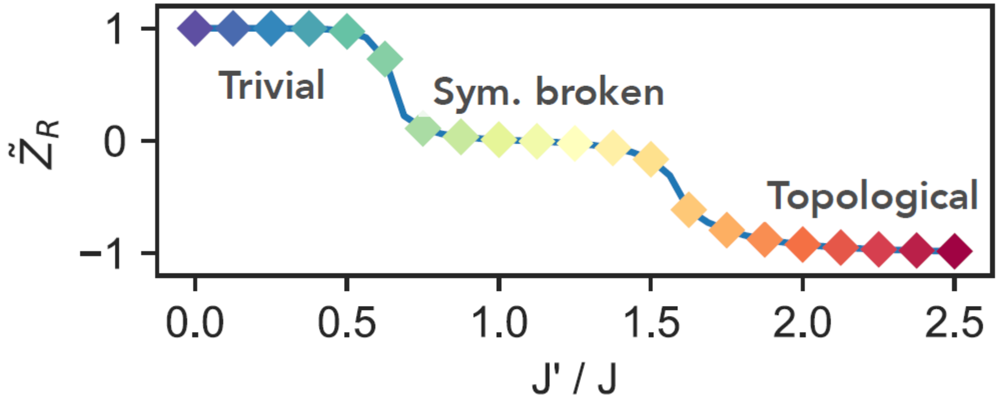
arXiv:2106.12627, Appx. J

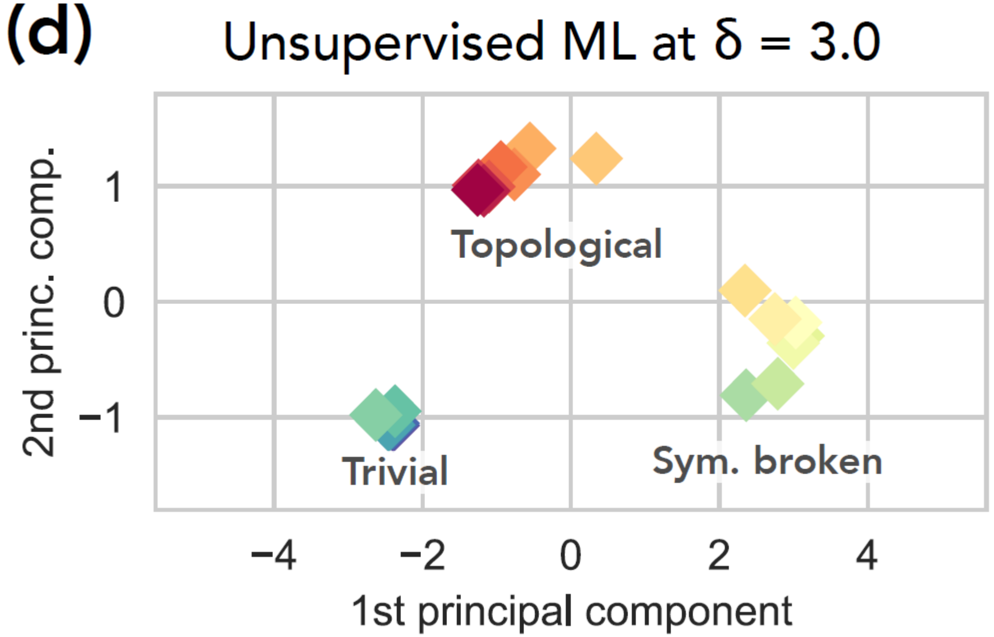
LOCAL INVARIANTS → ML ALGORITHMS DISTINGUISH PHASES

1D SPT

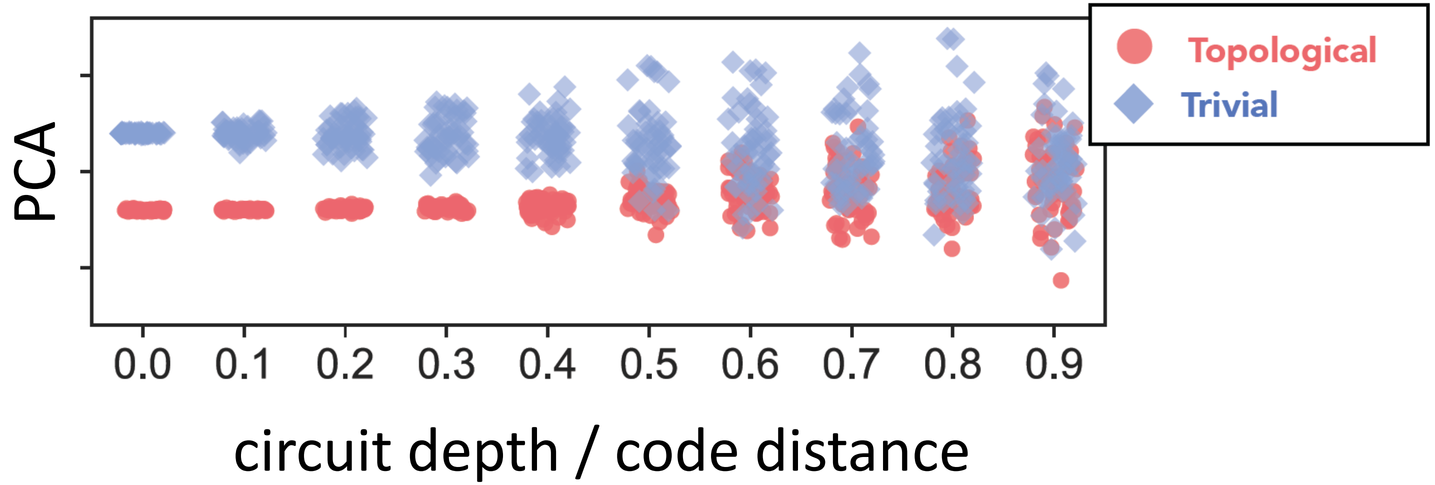
Non-local order par.







Surface code



arXiv:2106.12627, Appx. D

DISCUSSION & CONCLUSION

- In the not-so-distant future, we should have access to devices that would allow us to **simulate phases of matter** not readily available in natural materials.

- We need to design engineering protocols to verify that a state created on a device is a representative of a desired phase.
  - Convert **condensed-matter intuition** into **rigorous engineering**.
  - Learn something about quantum phases along the way.

- We have made progress in this direction (see table), **more systematic efforts** would be useful.

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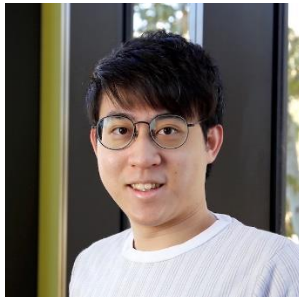
ACKNOWLEDGEMENTS

Provably efficient machine learning for quantum many-body problems
arXiv:2106.12627


Hsin-Yuan (Robert) Huang


Richard Kueng


Giacomo Torlai

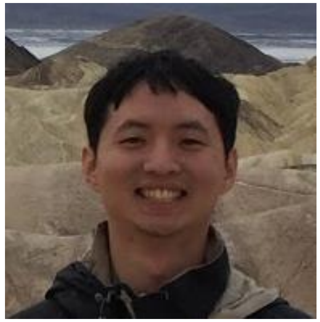

John Preskill

Chiral central charge from a single bulk wave function
arXiv:2110.06932

Modular commutator in gapped quantum many-body systems
arXiv:2110.10400


Isaac Kim


Bowen Shi


Kohtaro Kato

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University of Maryland

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~50 experimentalists

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