Topological chiral spin liquids with PEPS ?

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- Spin liquids: trivial versus topological
- Chiral spin liquids, analogs of the Fractional Quantum Hall states: key features
- Some exemples / simple models hosting SU(2) / SU(N) CSL
- PEPS no-go theorem: no real "obstruction" to describe CSL with PEPS

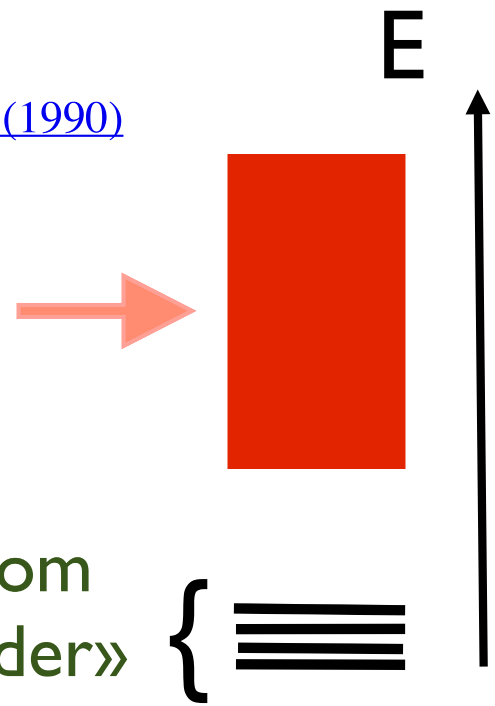
Exotic «topological liquids» beyond the «order parameter» paradigm

* no spontaneous broken symmetry
* no local order but...
* Topological order

X. G. Wen, International Journal of Modern Physics B4, pp. 239-271 (1990)

Excitations are fractionalized anyons

Degeneracy from «topological order»



TWO TYPES OF SPIN LIQUIDS:

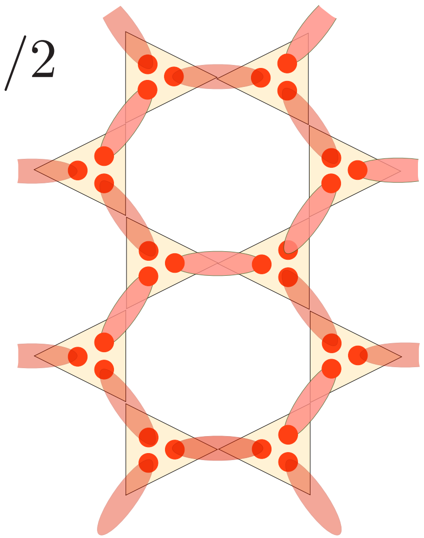
spin-S AKLT

$Bi_3Mn_4O_{12}(NO_3)$ material

J. Lavoie et al., Nat. Phys. 6, 850 (2010)

M. Matsuda et al., Phys. Rev. Lett. 105, 187201 (2010)

$S = z/2$



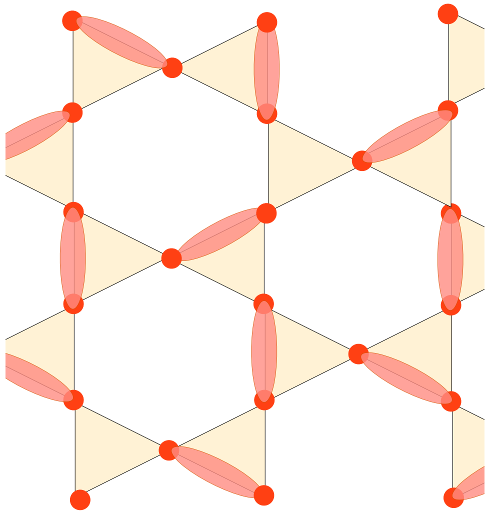
$S = 0$

«Trivial» liquid

spin-1/2 RVB

P. Fazekas and P.W. Anderson

Philosophical Magazine 30, 423-440 (1974)



Equal-weight superposition of NN singlet coverings

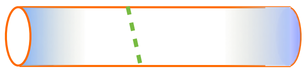
Topological liquid

Hasting-Oshikawa-LSM theorem

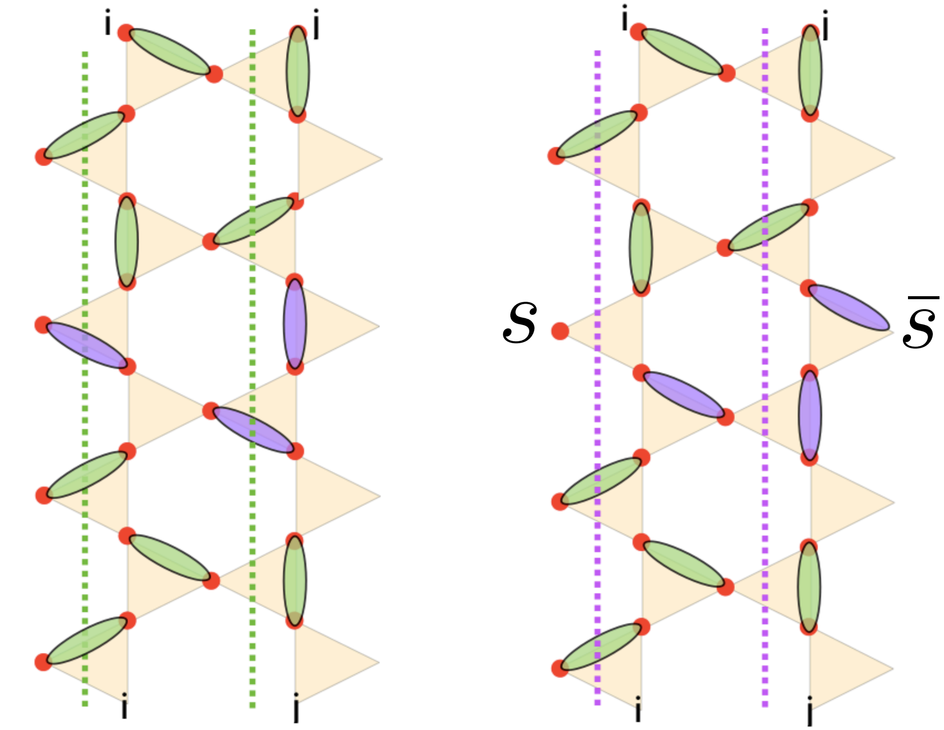
PS: can be written as a tensor network !

DP, N. Schuch, D. Perez-Garcia, I. Cirac, PRB 86, 014404 (2012)

Topological sectors



cylinder geometry



«even» $G_v = +1$

«odd» $G_v = -1$

Chiral topological spin liquids

- Topological (chiral) states are genuine in the field of the Fractional Quantum Hall effect

- Spin analogs on the lattice ?

Chiral SL (CSL) analogs of FQH states

1. Abelian CSL analog of Laughlin FQH state
2. Non-Abelian CSL analogs of non-Abelian FQH states (Moore-Read, Read-Rezayi, etc…)

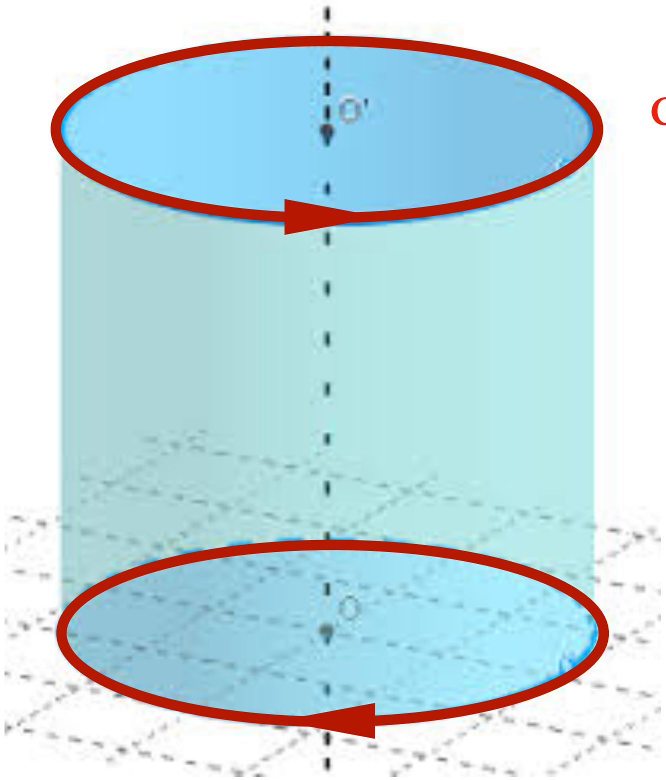
$\nu = \frac{1}{2}$ FQHS on a lattice (Kalmeyer-Laughlin, 1987):

(N=Nsites/2)

→ Paradigmatic "*Abelian* chiral spin liquid" if T & P are broken :

chiral spin liquids

analogs of FQH states



Protected edge modes described by Wess-Zumino-Witten (WZW) $SU(N)_k$ CFTs

"Long range entanglement"

Chiral spin-1/2 frustrated Heisenberg model on the square lattice

A. Nielsen, G. Sierra, J.I. Cirac, Nature Com. 4, 2864 (2013)

$$\psi_{\mathrm{P0}}^{\mathrm{CFT}}(s_1, s_2, ..., s_N) \propto \left\langle \phi_{s_1}(z_1)\phi_{s_2}(z_2)...\phi_{s_N}(z_N) \right\rangle$$

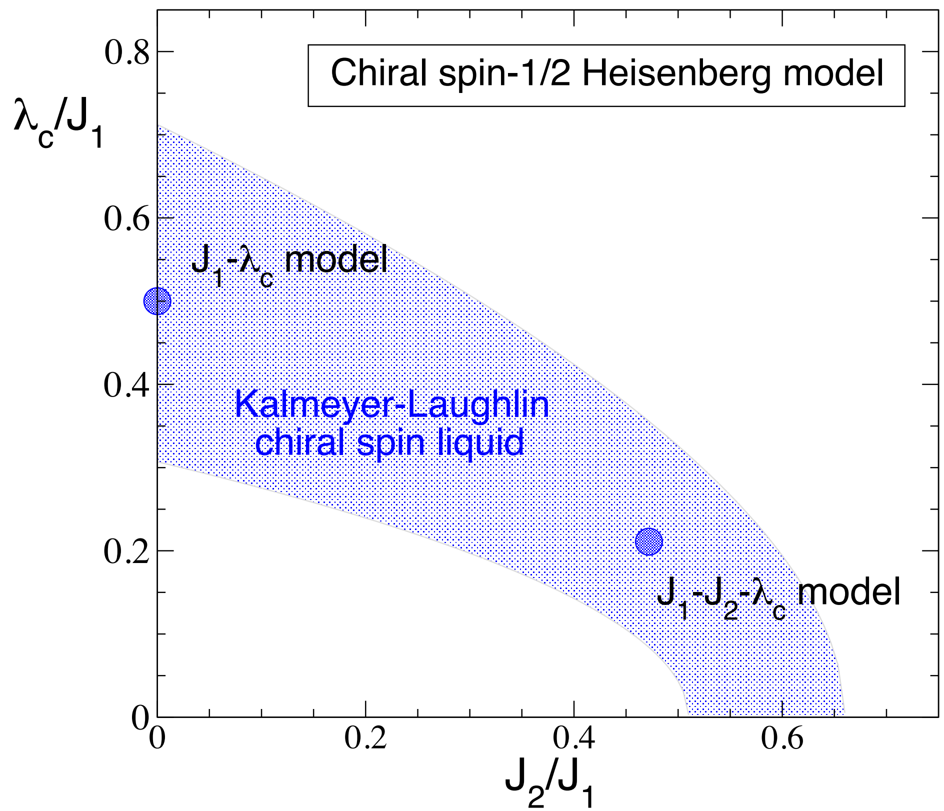
CFT correlator SU(2)$_1$ CFT

Parent Hamiltonien (long-range)

truncation

$$H = J_1 \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle\langle k,l \rangle\rangle} \mathbf{S}_k \cdot \mathbf{S}_l + \lambda_c \sum_{\square(ijkl)} i(P_{ijkl} - P_{ijkl}{}^{-1}),$$

Can be realized with synthetic gauge field

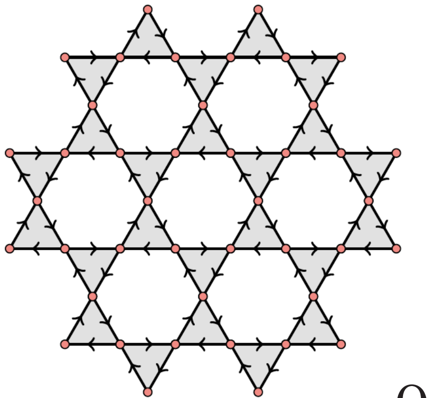


Schematic phase diagram based on their results

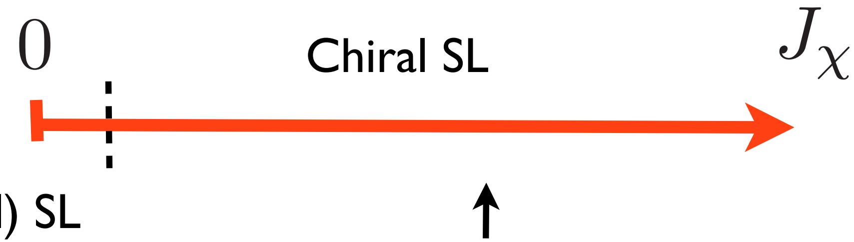
Abelian chiral SL in spin-1/2 chiral AFM (I)

B. Bauer, L. Cincio, B. P. Keller, M. Dolfi, G. Vidal, S. Trebst, A. W. W. Ludwig
Nature Communications 5, 5137 (2014)

Kagome lattice



$$H = J_{\rm HB} \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + J_\chi \sum_{i,j,k \in \triangle} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$$



Edge modes : chiral c=1 CFT (SU(2) $_1$ WZW)

More Abelian CSL in spin-1/2 AFM…

- Triangular lattice

  Shou-shu Gong et al., , Phys. Rev. B 96, xx (2017)

  A. Wietek, A.M. Lauchli, Phys. Rev. B 95, 035141 (2017)

- Spontaneous symmetry breaking in J_1-J_2-J_3 kagome AFM

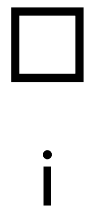
  S. Gong, W. Zhu, D.N. Sheng, Nat. Sci. Rep. 4, 6317 (2014)

  Yin-Chen He et al., PRL 112 (2014)

  A. Wietek, A. Sterdyniak, A.M. Lauchli, Phys. Rev. B 92, 125122 (2015)

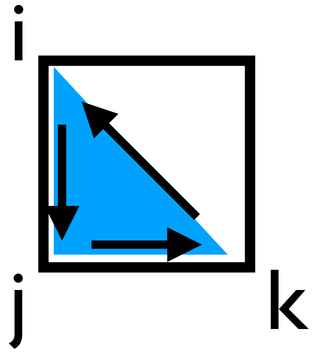
Extension to SU(N) Heisenberg models on the square lattice

N-dim fundamental irrep on every site :



$$H = J_1 \sum_{\langle i,j \rangle} P_{ij} + J_2 \sum_{\langle\langle k,l \rangle\rangle} P_{kl}$$

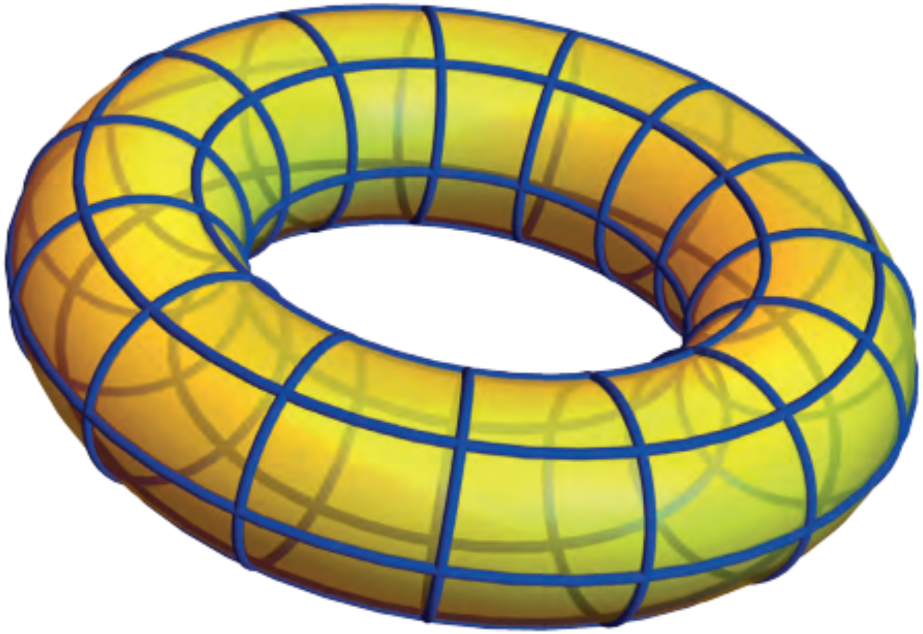
$$+ J_R \sum_{\triangle ijk} (P_{ijk} + P_{ijk}{}^{-1}) + iJ_{\mathrm{I}} \sum_{\triangle ijk} (P_{ijk} - P_{ijk}{}^{-1})$$



J.-Y. Chen, J.-W. Li, P. Nataf, S. Capponi, M. Mambrini, K. Totsuka, H.-H. Tu, A. Weichselbaum, J. von Delft & DP, Phys. Rev. B104, 235104 (2021)

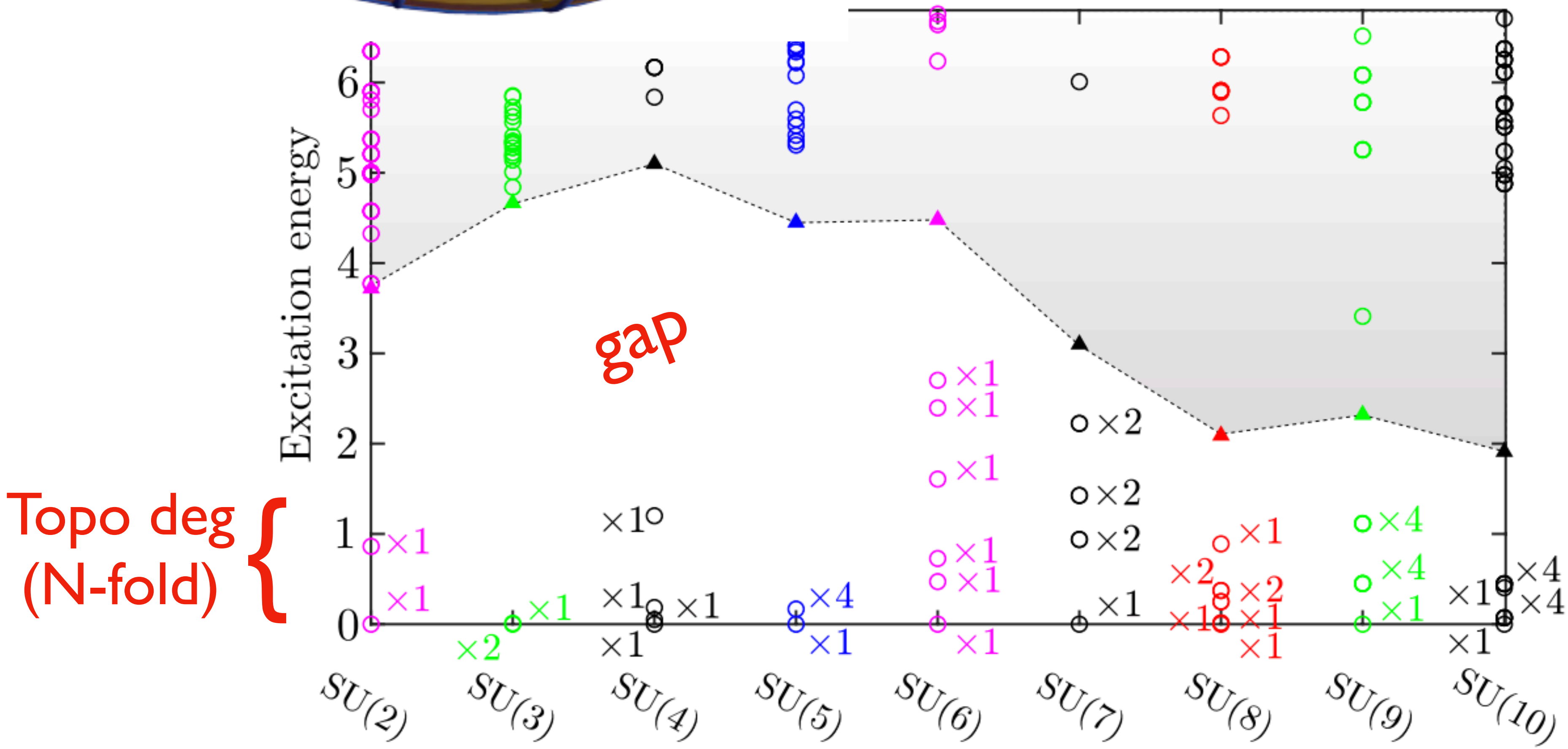
See also e.g. P. Nataf et al. PRL 2016 for triangular lattice

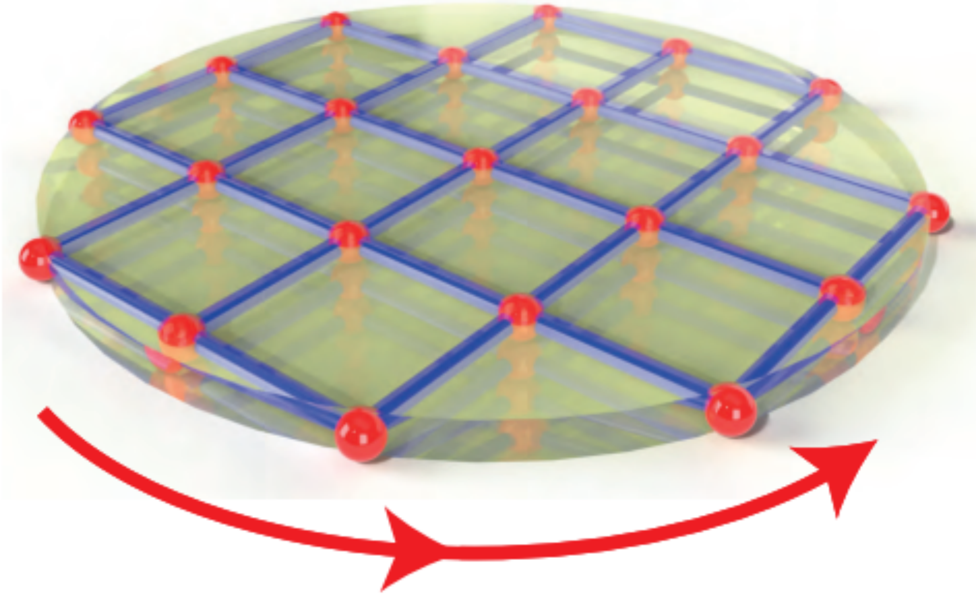
ED on torus geometry



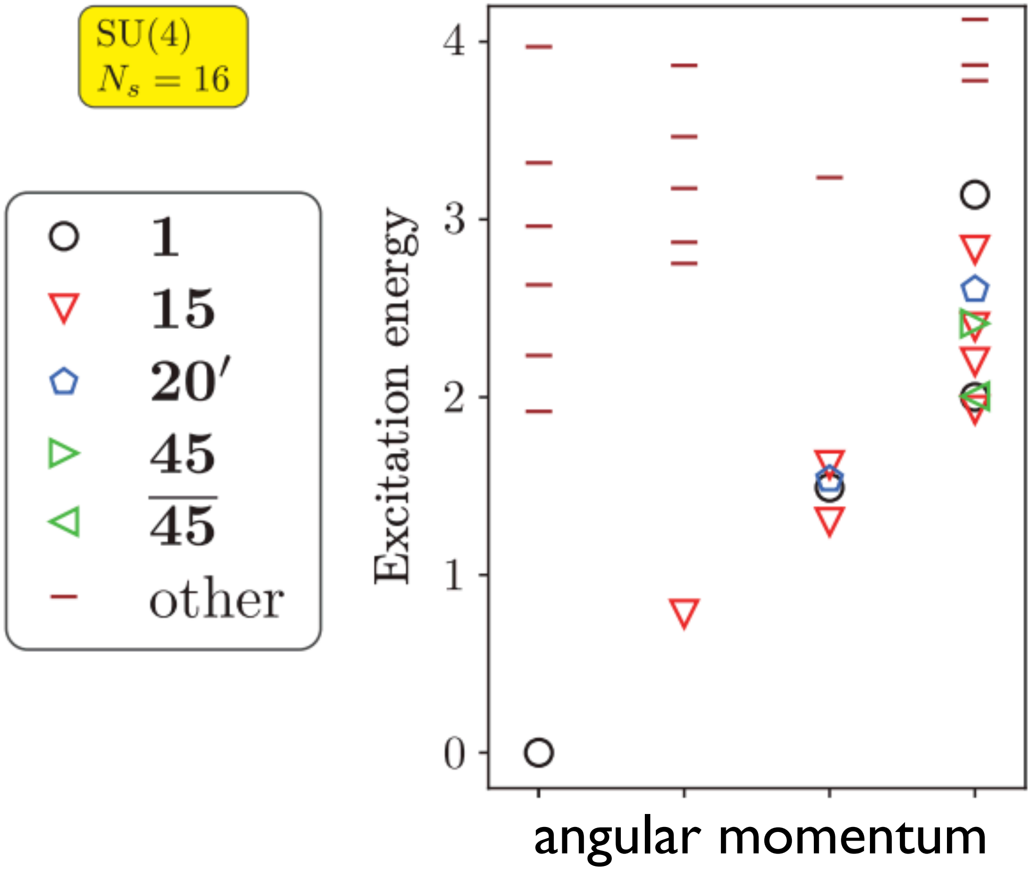
S. Capponi

P. Nataf

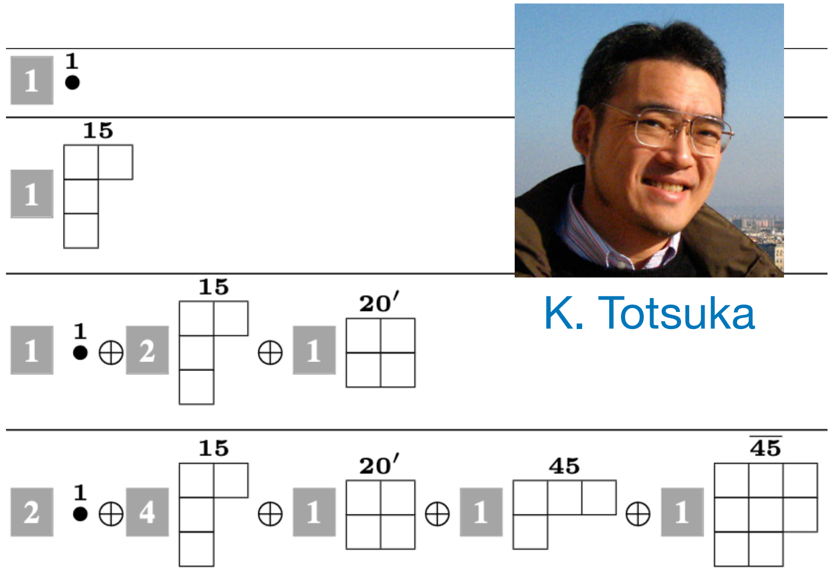




ED on open systems



WZW SU(4)$_1$ CFT



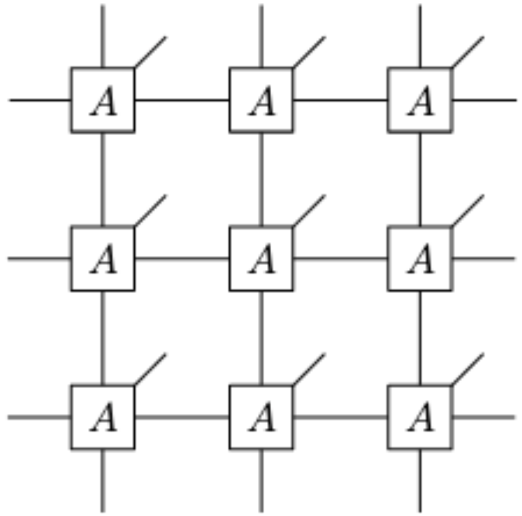
K. Totsuka

PEPS tensor networks as variational ansatz

$$|\Psi\rangle = \sum C_{i_1,i_2,\ldots,i_N}|i_1, i_2, \ldots, i_{N_s}\rangle$$

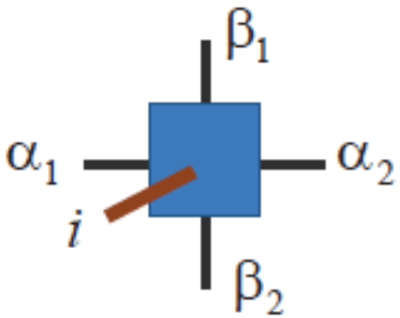
$$i = \{1, \cdots, d_{\text{phys}}\}$$

$$|\Psi(A)\rangle =$$



I. Cirac
F. Verstraete
G. Vidal

$$A^{i}_{\alpha_1,\alpha_2;\beta_1,\beta_2}$$



dimension of auxilary (or virtual) space

$$\alpha, \beta = \{1, \ldots, D\}$$

infinite-PEPS or "iPEPS" in thermodynamic limit

iPEPS for frustrated chiral spin-1/2 models

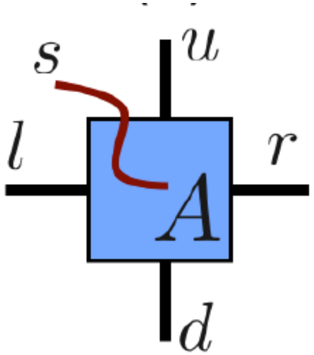
Symmetric PEPS

DP, Phys. Rev. B **96**, 121118 (2017)

J. Hasik, M. Van Damme, DP, L. Vanderstraeten,
Phys. Rev. Lett. **129**, 1772001 (2022).

Non-symmetric PEPS

PEPS with continuous U(1) or SU(N) symmetries



First, introduced for SU(2)-invariant PEPS

M. Mambrini, R. Orus & DP, Phys. Rev. B 94, 205124 (2016)

* virtual space : $V = S_1 \oplus S_2 \oplus \cdots S_p$

Can be adapted to SU(N) (Ji-Yao Chen)

Add discrete point group symmetry

Chiral PEPS ansatz: $A = A_R + iA_I$

$$A_R = \sum_\alpha \lambda_\alpha A_\alpha^{(A_1)} \qquad A_I = \sum_\beta \gamma_\beta A_\beta^{(A_2)}$$

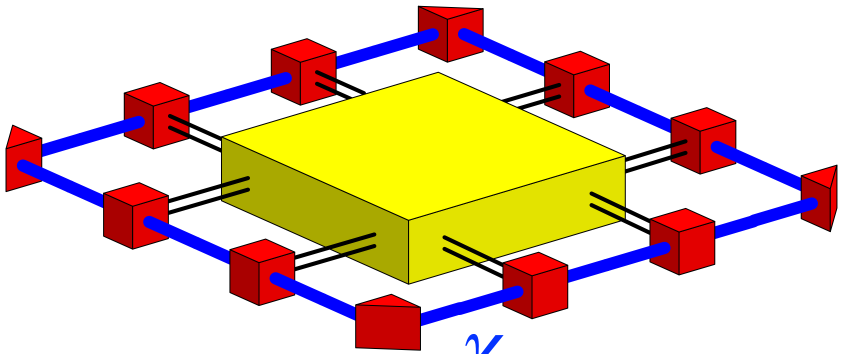
Different irreps !

DP, J. Ignacio Cirac & Norbert Schuch, Phys. Rev. B 91, 224431 (2015)

iPEPS method

$$\langle \Psi_{\rm PEPS} | H_{\Box} | \Psi_{\rm PEPS} \rangle$$

$$=$$



- Environment constructed by renormalization of the corner transfer matrix (CTM)

T. Nishino & K. Okunichi, J. Phys. Soc. J. **65**, 891 (1996)
R. Orus & G. Vidal, Phys. Rev. B **80**, 094403 (2009)

- Variational optimisation scheme based on a conjugate gradient method

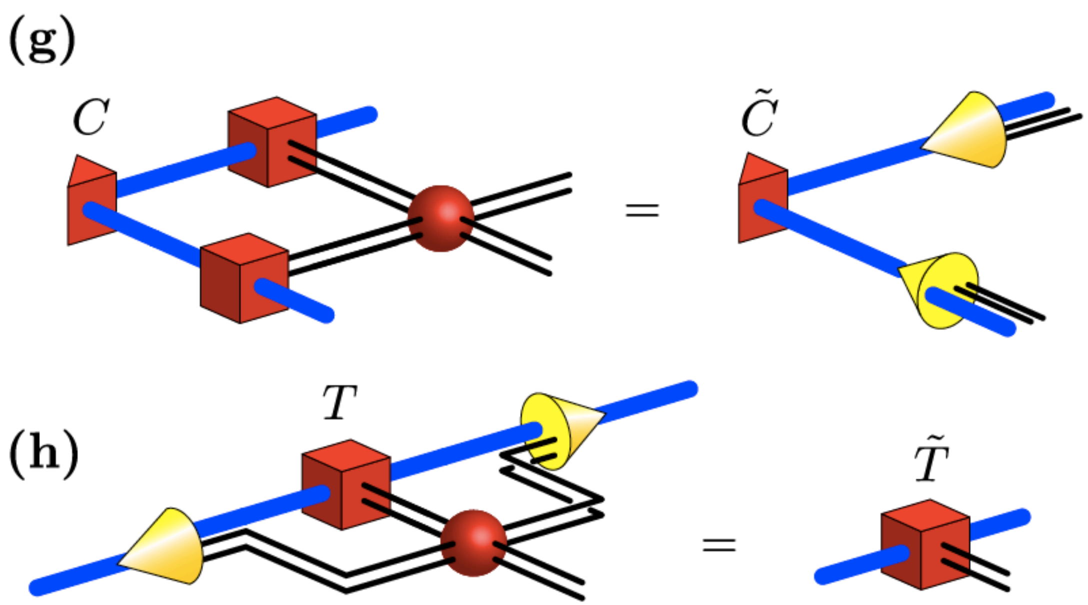
used for non-chiral AFM: L. Vanderstraeten, J. Haegeman, P. Corboz, F. Verstraete, Phys. Rev. B **94**, 155123 (2016)
DP & M. Mambrini, Phys. Rev. B **96**, 014414 (2017)

used for spin-1/2 chiral AFM: DP, Phys. Rev. B **96**, 121118 (2017)

- If non-symmetric PEPS: use automatic differenciation

J. Hasik (2019)

CTM Renormalization Group algorithm



- Specific features for symmetric PEPS providing better stability :
    - The CTM is Hermitian -> SVD replaced by ED (better stability)
    - All C (corner) and T (edges) matrices are the same
    - SU(2) symmetry preserved in the CTM fixed point.

No-go theorem for chiral PEPS

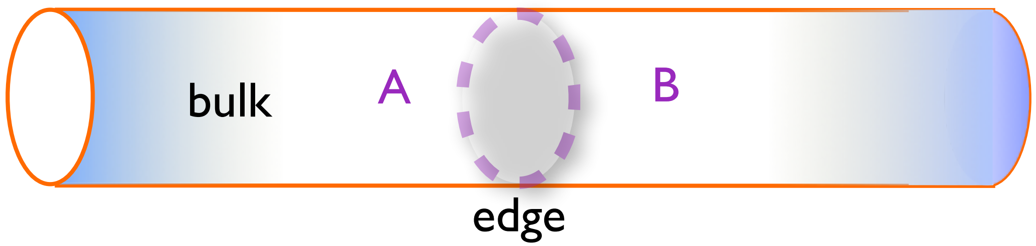
Chiral TNS of free fermions (Gaussian PEPS) have no gapped local parent Hamiltonians

T.B. Wahl, H.-H. Tu, N. Schuch, J.I. Cirac
Phys. Rev. Lett. 111, 236805 (2013)

J. Dubail, N. Read
Phys. Rev. B 92, 205307 (2015)

For interacting spins,
likely such a no-go theorem applies
(see argument next)

Hand-waiving argument from PEPS bulk-edge correspondence



$$\rho_A = \mathrm{Tr}_B |\Psi><\Psi| = \exp\left(-H_b\right)$$

$\xi_{\text{bulk}} \sim \lambda$ range of boundary H

J. Ignacio Cirac, DP, Norbert Schuch, Frank Verstraete, Phys. Rev. B 83, 245134 (2011)

If chiral edge mode (with discontinuous dispersion)

↓

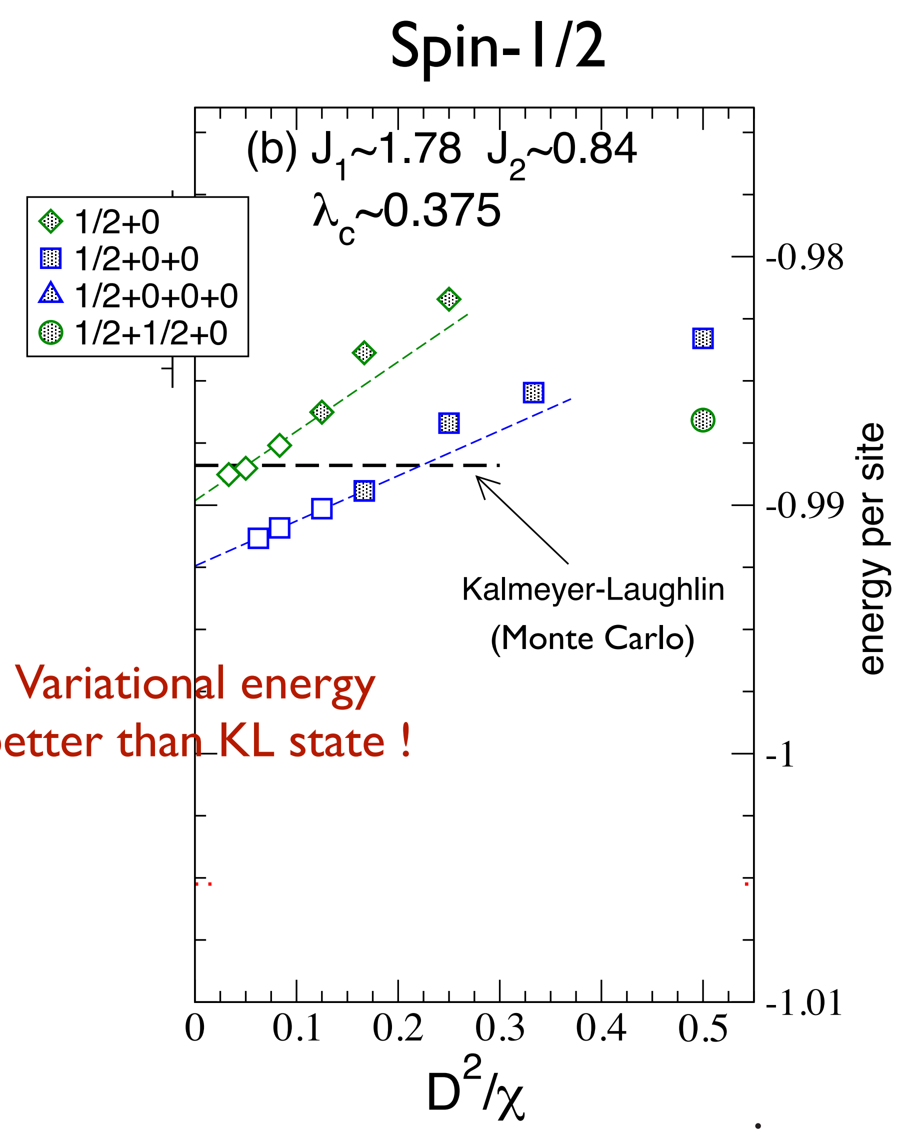
Boundary H long-range

↓ bulk-edge correspondence

Bulk correlation length (strictly speaking) infinite

Nevertheless, is there a real obstruction to construct "Physically relevant" chiral topological PEPS ?

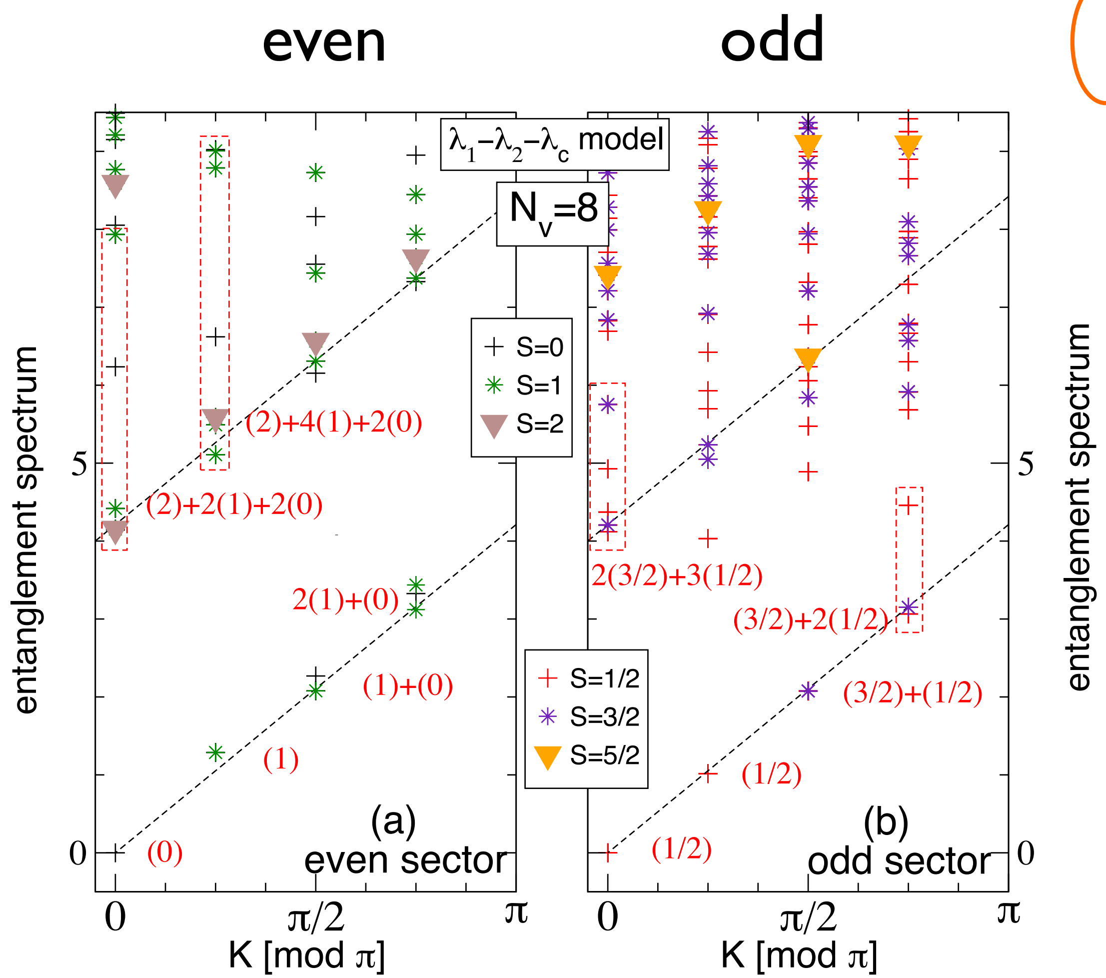
Variational energy

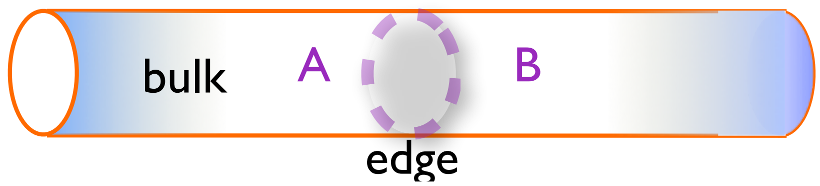


Variational energy better than KL state !

$\chi = \text{environment dimension}$

Entanglement spectrum SU(2)-symmetric PEPS



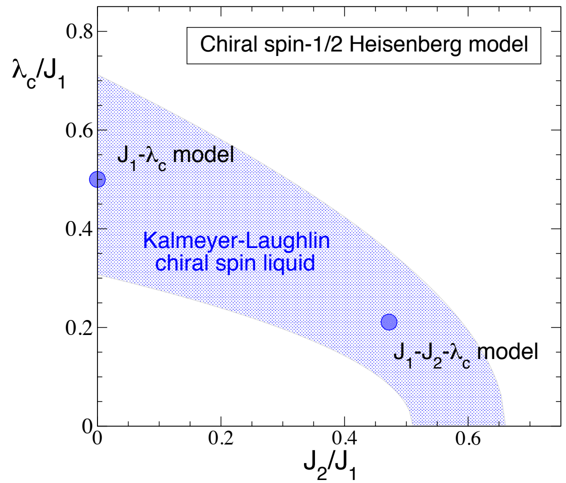


"tower of states" of chiral $SU(2)_1$ CFT

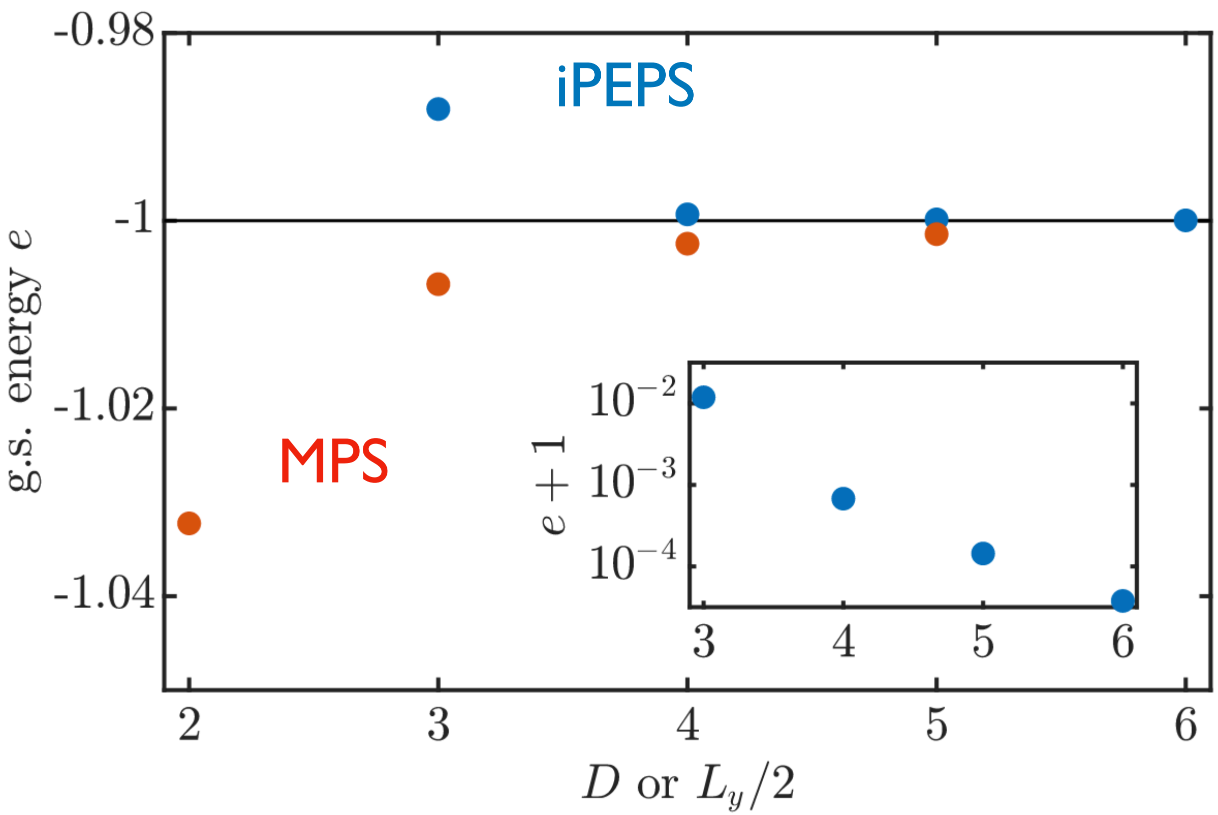
Abelian Laughlin Chiral SL

DP, Phys. Rev. B **93**, 121118 (2017)

Moving to non-symmetric PEPS…



Energetics

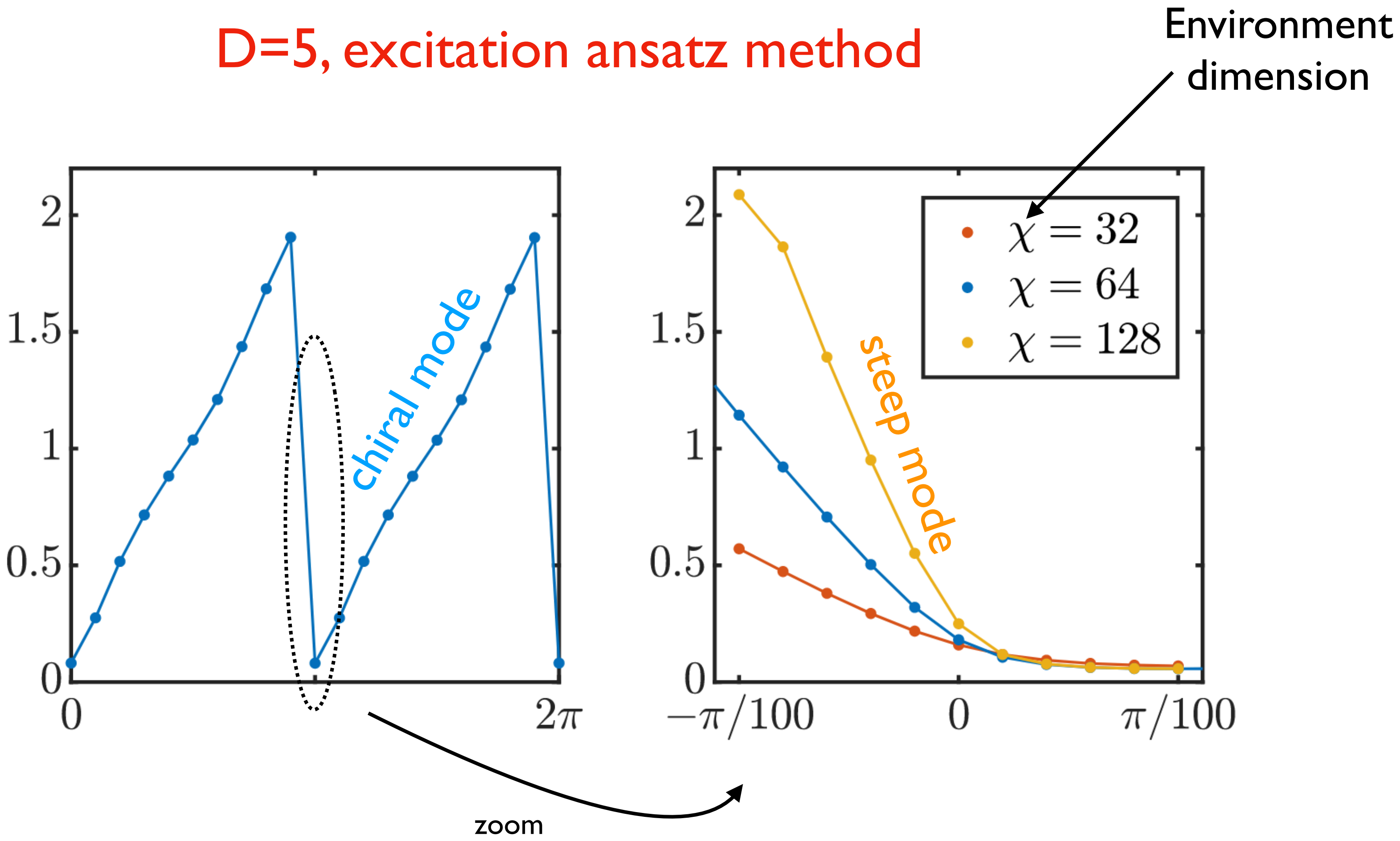


Faster convergence than state-of-the-art MPS on cylinders !

Entanglement spectrum

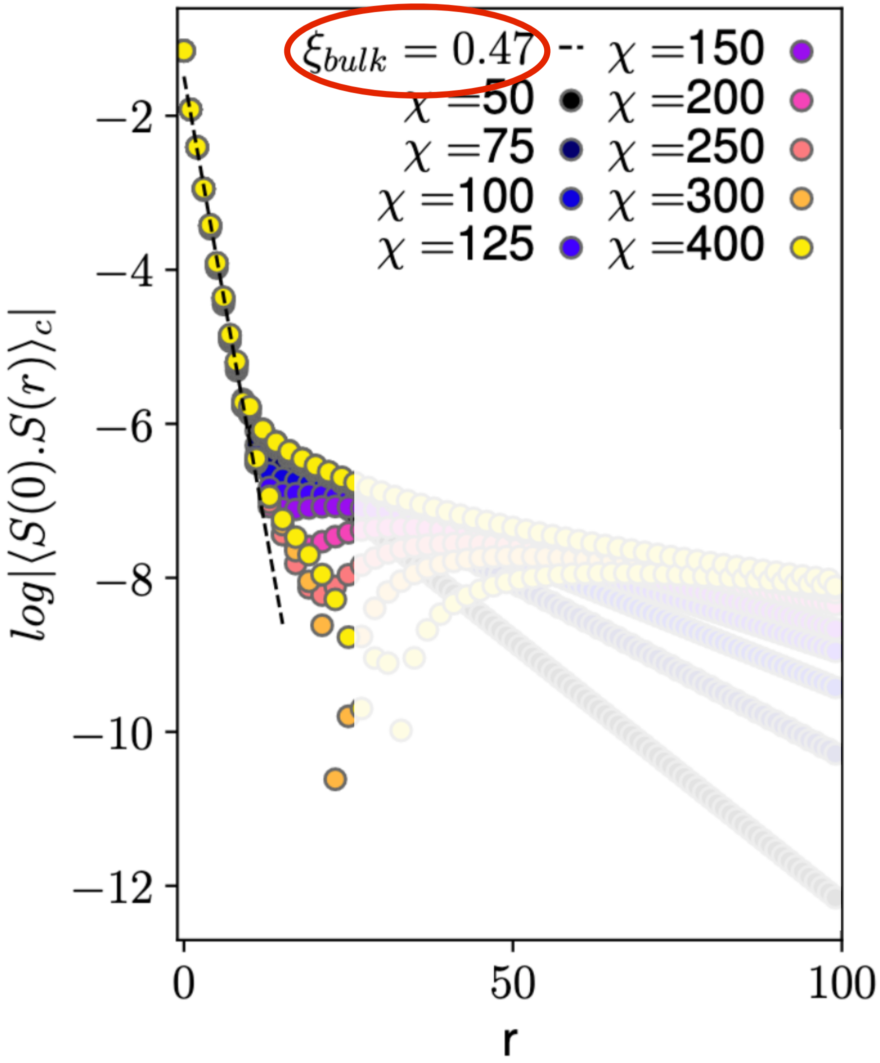
(infinitely-long edge)

D=5, excitation ansatz method



Perfectly chiral when $\chi \to \infty$

Very fast decay of spin-spin correlations

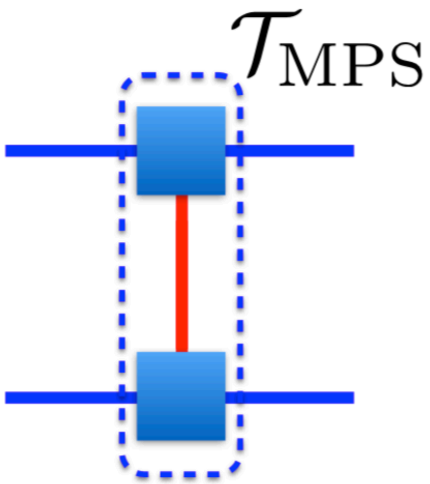


D=5 iPEPS

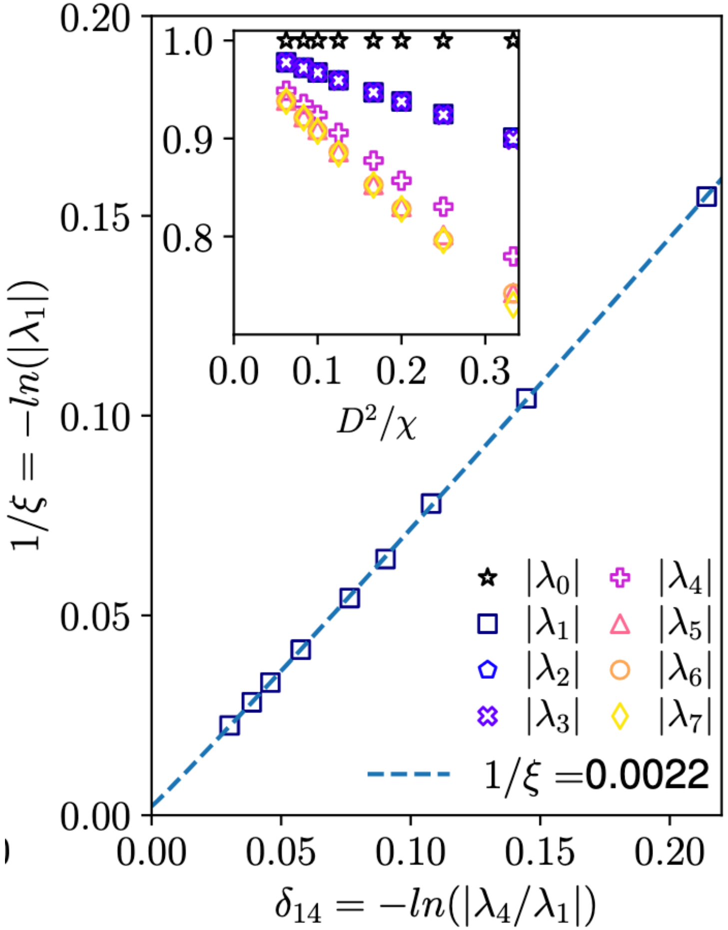
Consistent with bulk gap

But… diverging correlation length !

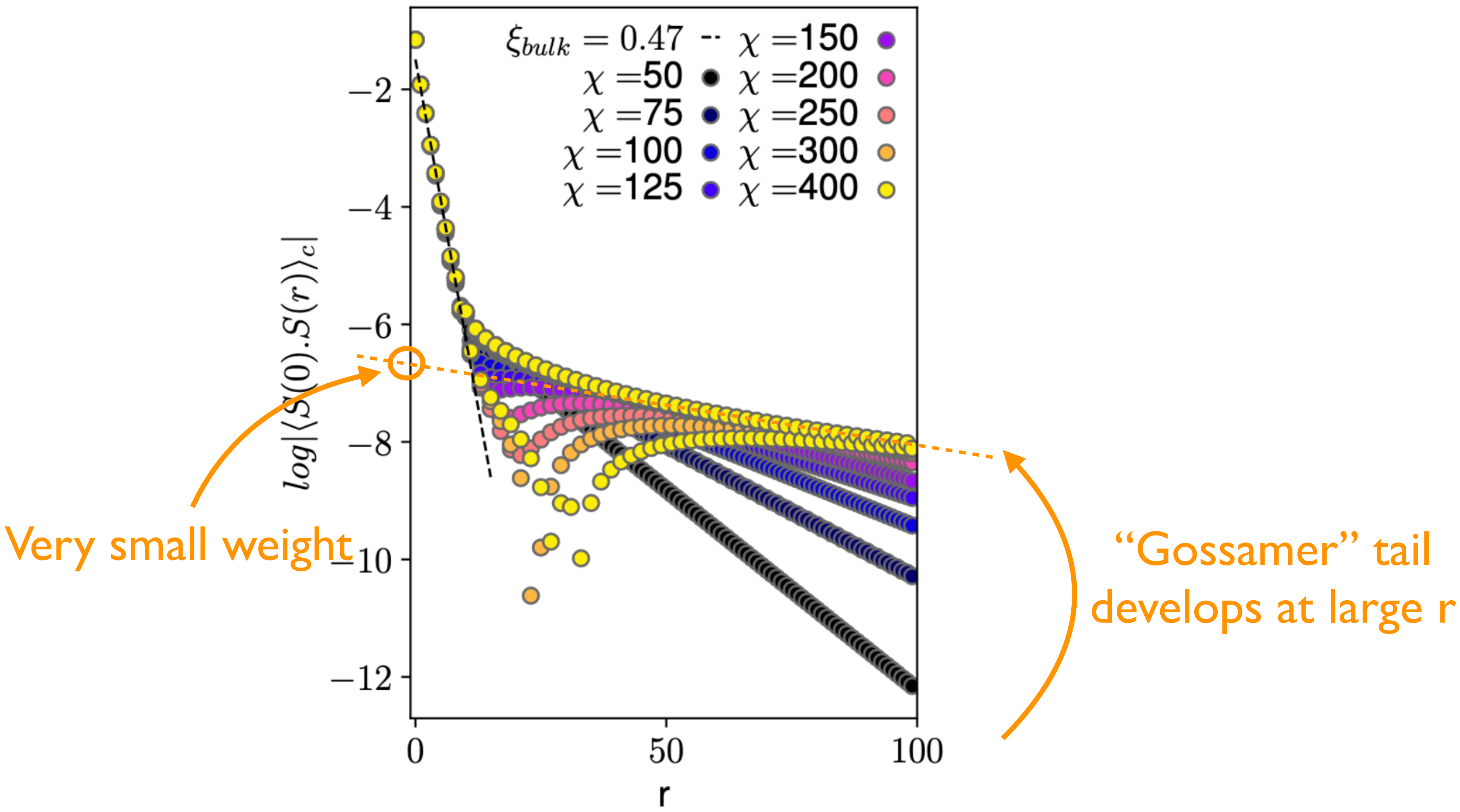
Transfer matrix



$$\xi_n = -1/\ln(\lambda_n/\lambda_{\max})$$



Closer look at spin-spin correlations…



$$C(d) = C_{\text{bulk}}(d) + C_{\text{tail}}(d)$$

$$C_{\text{tail}}(d) = \sum_{i>i_{\text{tail}}} w(\xi_i)\, \exp(-d/\xi_i)$$

Coming back to Gaussian PEPS for Chern insulators…

Qi-Wu-Zhang (QWZ) model with Chern number C=1

$$H(k) = h_x\sigma_x + h_y\sigma_y + h_z\sigma_z$$
$$= 2t'\sin k_x\sigma_x + 2t'\sin k_y\sigma_y + (\Delta - 2t\cos k_x - 2t\cos k_y)\sigma_z.$$

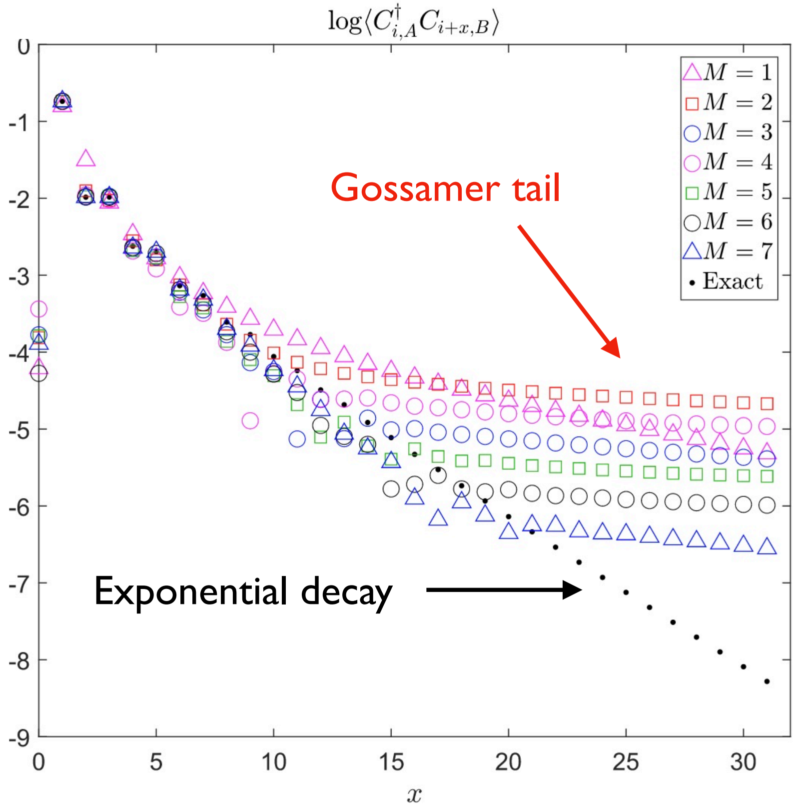
σ represent sublattice

Gaussian PEPS (approximation):

M fermion modes => $D=4^M$

T.B. Wahl, H.-H. Tu, N. Schuch, J.I. Cirac
PRL 111, 236805 (2013)

J.W. Li, J. von Delft, H.-H. Tu,
PRB 107, 085148 (2023)

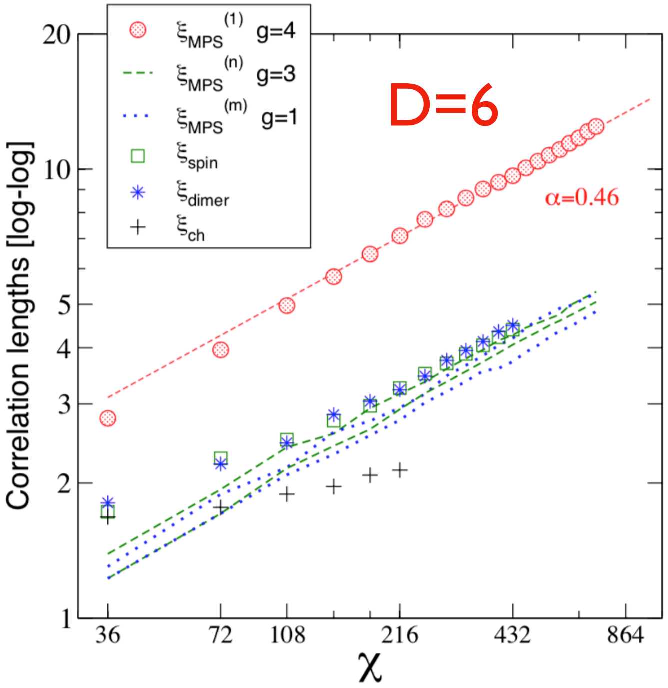


Spin-spin correlation function for
Gutzwiller-projected Gaussian PEPS ?

Sen Niu et al.
Work in progress — see poster

Discussion (Ia)

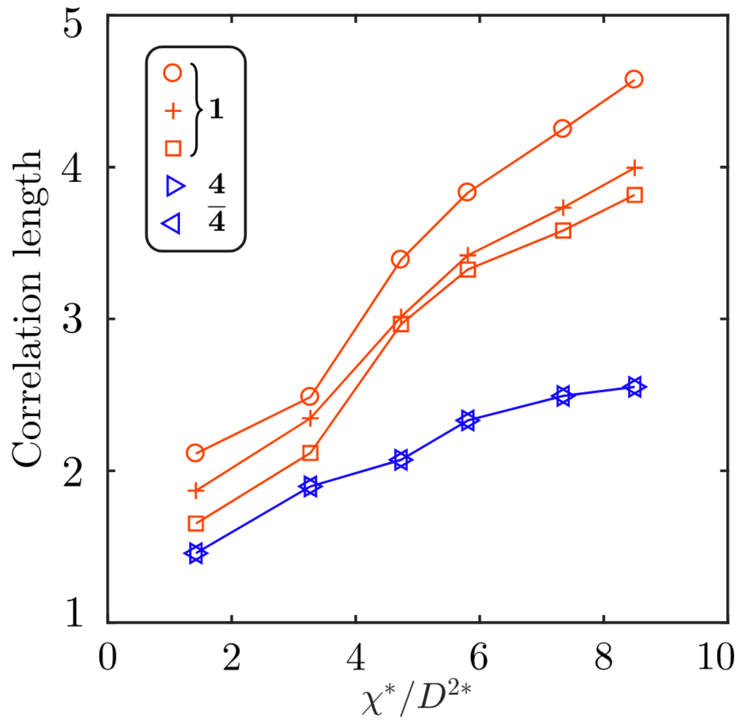
- Similar features in higher-S SU(2) and SU(N) CSL (square lattice) :



Spin-1 $SU(2)_2$ non-Abelian CSL

Ji-Yao Chen, Laurens Vanderstraeten, Sylvain Capponi & DP
Phys. Rev. B 98, 184409 (2018)

D=15



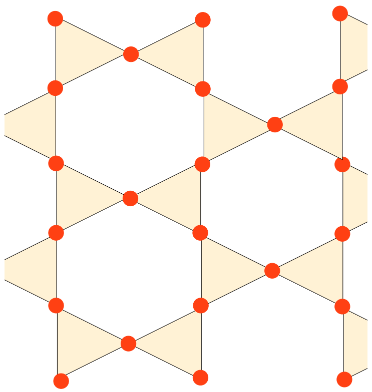
$SU(4)_1$ Abelian CSL

Ji-Yao Chen, Jheng-Wei Li, Pierre Nataf et al.
Phys. Rev. B 104, 235104 (2021)

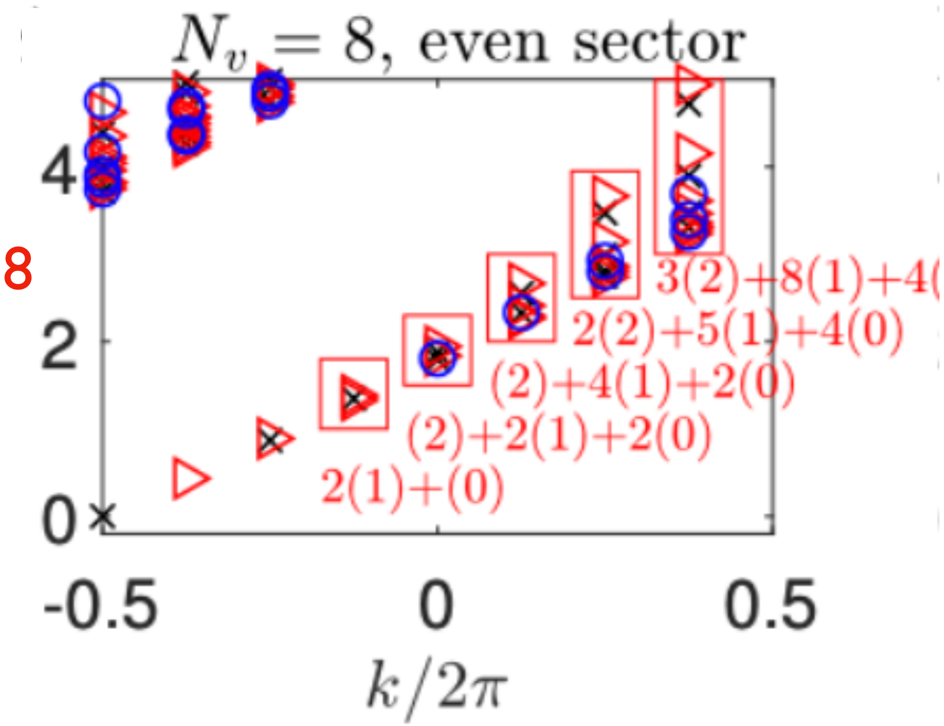
Discussion (Ib)

- Similar features in other lattices :

Kagome spin-1/2 $SU(2)_1$ CSL:



D=8



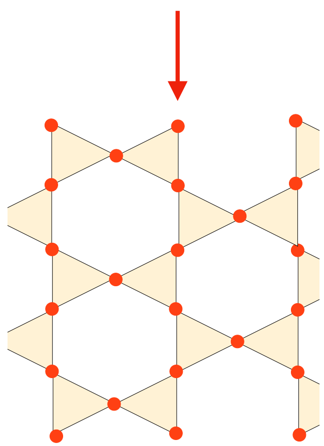
Sen Niu, Juraj Hasik, Ji-Yao Chen & DP
PRB 106, 245119 (2022)

Discussion (Ic)

- Pitfalls: non-chiral states breaking time-reversal symmetry

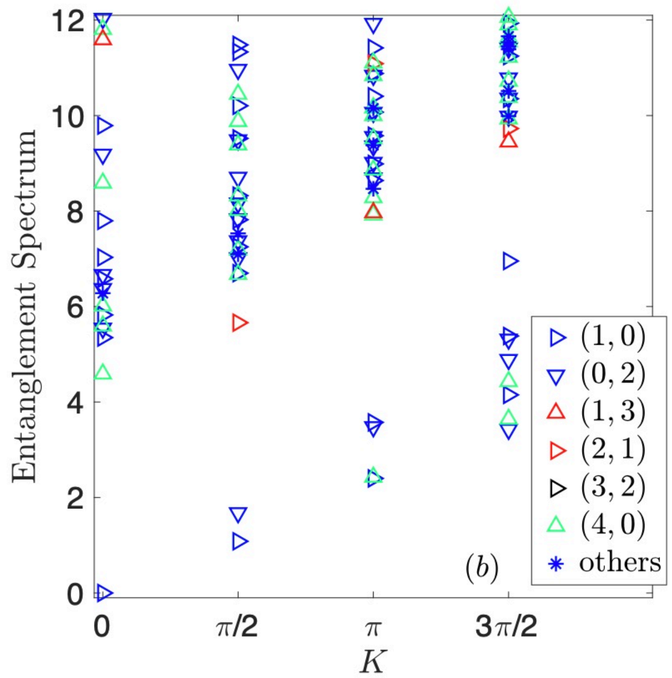
SU(3) symmetric topological spin liquid (D=7 PESS):
two counter-propagating edge modes $v_{fast} >> v_{slow}$

I. Kurecic, L. Vanderstraeten, N. Schuch, PRB 99, 045116 (2019)
M. J. Arildsen, N. Schuch, A. W. W. Ludwig, arXiv:2207.03246 (2022)



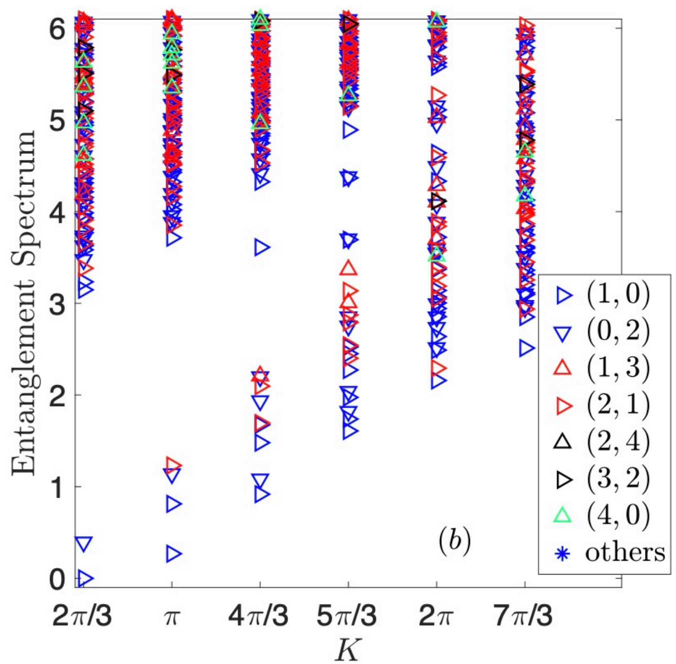
Could be realized in a simple chiral SU(3) Heisenberg model?
(paper in preparation)

Z3 charge=+1:



MPS (L_y=4 cylinder): consistent with $SU(3)_1$
L. Vanderstraeten

?



PESS (D=13, L_y=6 cylinder): consistent with $SU(3)_1 \times \overline{SU(3)_1}$
Ji-Yao Chen

Summary

- Chiral PEPS possess all features of top CSL. Topo order from Z_N gauge symmetry.
- The long-range correlation "tail" is an artifact of chiral PEPS needed to comply with PEPS bulk-edge correspondence
- NOT a practical limitation in PEPS descriptions of chiral SL
- TN techniques are variational approaches becoming as successful in 2D (and 3D ?) as DMRG in 1D.
- Can provide classification of topological phases of quantum spin systems

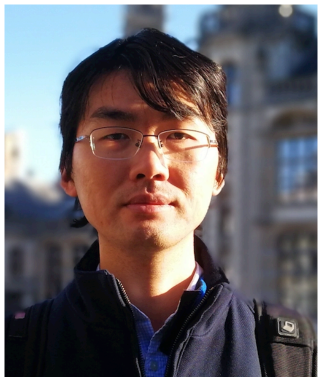
!! Acknowledgements:

Many thanks to :

L. Vanderstraeten

M. Mambrini

S. Capponi



Ji-Yao Chen

Juraj Hasik

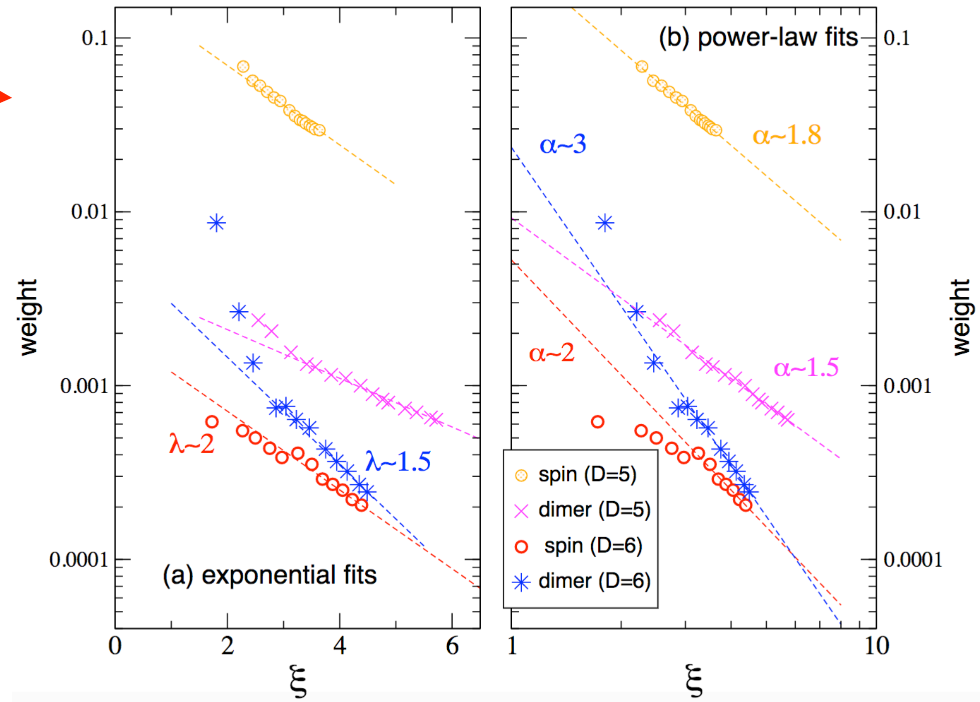
Sen Niu

+ many others….

Gossamer long-range correlations

$$C(d) = C_{\text{bulk}}(d) + C_{\text{tail}}(d)$$

$$C_{\text{tail}}(d) = \sum_{i > i_{\text{tail}}} w(\xi_i)\, \exp(-d/\xi_i)$$



(Laplace transform)

stretched exponential

power law