Quantum log-Sobolev inequalities and conditional relative entropy

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The evolution of open quantum systems which weakly interact with a memory-less bath is modeled by continuous semigroups of trace-preserving completely positive maps, or quantum Markov semigroups (QMS). One way to obtain strong bounds on the mixing time of such semigroups is via a group of functional inequalities named log-Sobolev inequalities, which are related to hypercontractivity properties of the semigroup.

Inspired by a series of results for classical spin systems, one can hope that, for certain classes of many-body QMSs on quantum spin systems, it is possible to determine whether the semigroup satisfies a log-Sobolev inequality just by looking at proprieties of the fixed point of the evolution. In this talk I will present some recent results in this direction. In particular, I will introduce a *quantum conditional relative entropy* and show how it can be used to prove quasi-factorization properties of the quantum relative entropy. These can be then used to bound log-Sobolev constants for product semigroups with heath-bath generators, and under stronger assumptions in more general situations.

(joint work with Á. Capel and D. Pérez-García).