Strongly symmetric spectral convex bodies are Jordan algebra state spaces: Abstract

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In the 1930s, P. Jordan introduced the algebras now called Jordan algebras, as a potential algebraic setting for quantum theory, abstracting properties of the space of observables in quantum mechanics. P. Jordan, J. von Neumann, and E. Wigner then showed that the normalized state spaces of simple finite-dimensional Euclidean Jordan algebras are the convex compact sets of real, complex, and quaternionic density matrices, Euclidean balls in any dimension, and 3-dimensional octonionic density matrices. In this talk, I'll describe the results of `https://arxiv.org/1904.03753`, in which we show that these state spaces, and the finite-dimensional simplices (which are classical state spaces) are the unique finite-dimensional compact convex sets Ω that are spectral and strongly symmetric. Spectrality means that every state has a convex decomposition into perfectly distinguishable pure states, and strong symmetry means that the affine automorphism group of Ω (sometimes called "reversible transformation group") acts transitively on the set of lists (of each fixed length) of perfectly distinguishable states. Our work builds on results of [1], who characterized the same set of state spaces by spectrality, strong symmetry, and the absence of "higher-order interference" (interference irreducibly involving more than two paths). Because Jordan-algebraic state spaces do not exhibit higher-order interference ([2, 3]) our work implies that this last property follows from the first two.

By adding one or two additional assumptions, such as energy observability [1] (or the closely related *Connes orientation* [4, 5] or *dynamical correspondence* [5]) or the existence of tomographically local composites of any pair of state spaces in the theory [6] one can use this theorem to obtain simple characterizations of the usual mixed-state spaces of finite-dimensional quantum mechanics over the complex field, i.e. the sets of $n \times n$ Hermitian density matrices.

Important aspects of quantum and classical thermodynamics (including the majorization of the outcome probabilities of fine-grained measurements on a state, by the "spectrum" of a state, and the consequent equivalence of measurement entropies and preparation entropies defined as the values of Schur-concave functions on the spectrum) and of query complexity (including the order $\sqrt{N}$ lower bound on Grover's "search" problem) have been generalized to classes of probabilistic theories whose state spaces satisfy natural postulates including or implying spectrality and strong symmetry. Our result shows that these apply to a narrower class of theories than might have been hoped, already close to complex quantum theory since their state spaces are Jordan algebraic.

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