# School and Workshop on New Trends in Higgs Bundle Theory ICMAT (Madrid), 12-16 November 2018

# **Scientific Programme**

# Mini-course:

Nigel Hitchin (Oxford): Higgs bundles and mirror symmetry

## Workshop lectures:

**Olivier Biquard** (ÉNS, Paris): From HyperKähler metrics on Higgs moduli spaces to Ricci flat Kähler metrics on complexified symmetric spaces

**Lucas Branco** (MPIM Bonn): *Higgs bundles and mirror symmetry* — *examples via isogenies* 

Ugo Bruzzo (SISSA, Trieste): A conjecture about Higgs bundles

**Brian Collier** (Maryland): Generalized opers, conformal limits and maximal variations of Hodge structure

**Emilio Franco** (Porto): Branes on the singular locus of the Hitchin system via Borel and other parabolic subgroups

Sergei Gukov (Caltech): Modular tensor categories from wild Higgs bundles

**Georgios Kydonakis** (Strasbourg): Model Higgs bundles in exceptional components of character varieties

Qiongling Li (Nankai): Dominations in moduli space of Higgs bundles

André Oliveira (Porto & UTAD): Generalized Cayley correspondence and higher Teichmüller components for SO(p,q)-Higgs bundles

Ana Peón-Nieto (Genève): Branes on the Hitchin system from torsion line bundles

**Carlos Simpson** (Nice): *Higgs bundles in the geometric Langlands correspondence for genus 2 curves* 

Jan Swoboda (München): The large scale geometry of Higgs bundle moduli spaces

**Alexander Thomas** (Strasbourg): Geometric higher Teichmüller theory — punctual Hilbert schemes in Higgs theory

Junho Peter Whang (MIT): Diophantine analysis on moduli of local systems

# Mini-course syllabus:

Nigel Hitchin (Oxford): Higgs bundles and mirror symmetry
Lecture 1. Hodge theory, abelian and non-abelian SYZ mirror symmetry and semiflat metric for a Calabi-Yau The integrable system and semiflat metric for Higgs bundles
Lecture 2. Mirror symmetry and Langlands duality Duals of abelian varieties, examples BAA-branes and BBB-branes Invariant Lagrangian submanifolds Mirror symmetry in rank 2
Lecture 3. Hyperholomorphic bundles The Dirac bundle, the universal bundle The Hecke transform

Lagrangians and hyperholomorphic bundles

## Workshop abstracts:

**Olivier Biquard** (ÉNS, Paris): From HyperKähler metrics on Higgs moduli spaces to Ricci flat Kähler metrics on complexified symmetric spaces

Using Higgs moduli spaces in the 90's I had constructed hyperKhler metrics on cotangent spaces of all generalized flag manifolds of complex semisimple groups. Studying the asymptotics of this metric leads to interesting generalizations on complexified symmetric spaces. Joint work with Thibaut Delcroix.

Lucas Branco (MPIM Bonn): Higgs bundles and mirror symmetry — examples via isogenies

According to mirror symmetry, complex Lagrangians in the Higgs bundle moduli space for a complex group are related to hyperkähler subvarieties of the Higgs bundle moduli space for the Langlands dual group. After discussing some general constructions, we focus on this duality for complex Lagrangians arising from two real forms of  $SO(4,\mathbb{C})$  and explain how our results relate to the conjectural picture.

#### Ugo Bruzzo (SISSA, Trieste): A conjecture about Higgs bundles

My talk will be about a class of Higgs bundles that we call "curve semistable"; a Higgs bundle E on a smooth projective variety X is said to be curve semistable if for every morphism  $f: C \to X$ , where C is a projective curve, the pullback  $f^*E$  is semistable. For ordinary bundles it is known that a bundle is curve semistable if and only if it is semistable with respect to some polarization and its discriminant vanishes. The analogous statement for Higgs bundles is still conjectural. We shall discuss some partial progress towards the proof of this conjecture.

**Brian Collier** (Maryland): Generalized opers, conformal limits and maximal variations of Hodge structure

For a complex semisimple Lie group G, the space of opers is an affine holomorphic Lagrangian submanifold of the moduli space of flat G connections which is biholomorphic to the base of the Hitchin fibration. These objects are related to the interaction of certain holomorphic connections with structure group reductions to a Borel subgroup. In this talk we will introduce a notion of opers for other parabolic subgroups and parameterize these objects. Furthermore, we will see that the so called conformal lift identifies these generalized opers with the stable manifold of certain  $\mathbb{C}^*$ -fixed points in the Higgs bundle moduli space.

**Emilio Franco** (Porto): Branes on the singular locus of the Hitchin system via Borel and other parabolic subgroups

The moduli space of Higgs bundles has an extremely rich geometry, it is a hyperKaehler variety and fibrates over a vector space becoming an integrable system named the Hitchin system. Its importance in theoretical physics comes from the fact that a dimensional reduction of an N = 4 Super Yang–Mills gauge theory can be rewritten as a 2-dimensional sigma-model with the Hitchin system as a target. In this context, Kapustin and Witten reinterpreted the classical limit of S-duality of the original SYM gauge theory as mirror symmetry in the target (the Hitchin system). They also introduced the appropriate notion of branes in the Hitchin system respecting the hyperKaehler structure. In this talk I will study the behaviour under mirror symmetry of a family of branes living on the singular locus of the Hitchin system. I will also describe their geometry and the role of the Borel subgroup. The picture can be generalized to other parabolic subgroups. This is joint work with Ana Peón-Nieto.

Sergei Gukov (Caltech): Modular tensor categories from wild Higgs bundles

**Georgios Kydonakis** (Strasbourg): Model Higgs bundles in exceptional components of character varieties

Gluing ASD connections over a complex connected sum of manifolds is a wellknown technique in Yang–Mills theory and has been effectively modified for several types of equations in gauge theory. An adaptation of this technique in the context of Higgs bundles involves the utilization of the linearization of a relevant elliptic operator coming from the Hitchin equations. The construction can be used to build model Higgs bundles in exceptional components of character varieties  $\mathcal{R}(G)$  for which good models have not been known before, as for the case  $G = \text{Sp}(4, \mathbb{R})$  and G = SO(n, n + 1).

#### Qiongling Li (Nankai): Dominations in moduli space of Higgs bundles

In this talk, we discuss some progress towards two "domination" conjectures made on the moduli space of Higgs bundles. First, we show for a Hitchin representation in  $PSL(n,\mathbb{R})$ , every equivariant minimal immersion from a hyperbolic plane into the corresponding symmetric space is distance-increasing. Secondly, consider Hitchin fibers at  $(q_2, 0, ..., 0)$ , that is, the fibers containing Fuchsian locus. We show a comparison theorem on the length spectrum between surface group representations in such fibers with Fuchsian ones, as a generalization of the  $SL(2,\mathbb{C})$  case shown by Deroin and Tholozan.

#### André Oliveira (Porto & UTAD): Generalized Cayley correspondence and higher Teichmüller components for SO(p,q)-Higgs bundles

For many classes of Lie groups G, natural topological invariants completely label connected components of the moduli spaces G-Higgs bundles in a curve. Hitchin components in the split real form case, and maximal components in the Hermitian (tube type) case, were the only previously know cases where these natural invariants do not fully distinguish connected components. In this talk we will explain the existence of new such higher Teichmüller components in the moduli spaces of SO(p, q)-Higgs bundles, via a generalized Cayley correspondence. In general, these groups lie outside the above mentioned classes of real forms, but fit in a conjectural natural framework which seems to be associated to the existence of such components.

#### Ana Peón-Nieto (Genève): Branes on the Hitchin system from torsion line bundles

In this talk I will explain how mirror symmetry operates on some natural hyperholomorphic branes on the Hitchin system, given by fixed points by tensorisation with a torsion line bundle. Generically, they are supported over the locus of integral spectral curves. Many aspects of their geometry are however more easily understood through branes supported on the most singular locus of a related Hitchin system. I will refer to this interplay during the exposition. This is joint work with E. Franco, P. Gothen and A. Oliveira.

# **Carlos Simpson** (Nice): *Higgs bundles in the geometric Langlands correspondence for genus 2 curves*

This is joint work with Ron Donagi and Tony Pantev. They propose to look at the local systems predicted by geometric Langlands through the lens of the nonabelian Hodge correspondence. The Higgs bundle over  $\operatorname{Bun}_G$  would have as spectral variety the appropriate fiber of the Hitchin fibration, sitting inside  $T^*\operatorname{Bun}_G$ . Here, we look at the case of rank 2 bundles on a compact genus 2 curve. The moduli spaces are  $\mathbb{P}^3$  and an intersection of two quadrics in  $\mathbb{P}^5$ . For the Higgs bundle obtained by pushforward from a general fiber of the Hitchin fibration, we show how to adjust the parabolic structure to obtain vanishing of the first and second parabolic Chern classes over  $\operatorname{Bun}_G$ , thus giving a local system on the complement of the wobbly locus.

#### Jan Swoboda (München): The large scale geometry of Higgs bundle moduli spaces

In this talk I will explain recent joint work with Rafe Mazzeo, Hartmut Weiß and Frederik Witt on the asymptotics of the natural  $L^2$ -metric  $G_{L^2}$  on the moduli space  $\mathcal{M}$  of rank-2 Higgs bundles over a Riemann surface  $\Sigma$  as given by the set of solutions to Hitchin's self-duality equations. I will show that on the regular part of the Hitchin fibration this metric is well-approximated by the semiflat metric  $G_{\rm sf}$  coming from the completely integrable system on  $\mathcal{M}$ . This also reveals the asymptotically conic structure of  $G_{L^2}$ , with (generic) fibres of the above fibration being asymptotically flat tori. This result confirms some aspects of a far reaching conjectural picture suggested by Gaiotto, Moore and Neitzke. Its proof is based on a detailed understanding of the ends structure of  $\mathcal{M}$ .

**Alexander Thomas** (Strasbourg): Geometric higher Teichmüller theory — punctual Hilbert schemes in Higgs theory

The construction of Hitchin's component, a generalization of Teichmüller space, via Higgs bundles does not shed light on its geometric origin. We will construct and analyze a geometric structure on surfaces, generalizing the complex structure, whose moduli space is conjecturally Hitchin's component. These higher complex structures are defined using the punctual Hilbert scheme of the plane. Joint work with Vladimir Fock.

#### Junho Peter Whang (MIT): Diophantine analysis on moduli of local systems

We consider the Diophantine geometry of moduli spaces for special linear rank two local systems on topological surfaces. After motivating their Diophantine study, we use differential geometric tools to obtain a finiteness theorem for their integral orbits under mapping class group dynamics, generalizing a classical work of Markoff (1880). We also discuss effective finiteness results for integral points on curves in these moduli spaces, as well as more recent work if time permits.