School and Workshop on New Trends in Higgs Bundle Theory

Research Programme on Moduli Spaces

Madrid, 12-16 November 2018

ICMAT
INSTITUTO DE CIENCIAS MATEMÁTICAS

Instituto de Ciencias Matemáticas (ICMAT)

Contents

This meeting is devoted to several active fields of current research in the theory of Higgs bundles and their interplay with geometry, topology and theoretical physics. It will combine research lectures with a 3 hour mini-course, run in parallel so as to give a balance of research and training. The event is organized within the ICMAT **Research Programme on Moduli Spaces** (15 September – 15 December 2018). This activity also ties in with the **ICMAT Severo Ochoa Donaldson–Hitchin Laboratory** chaired jointly by the members of the scientific committee. The organizing committee wishes to thank you for your interest and participation in this conference.

Web page: https://www.icmat.es/RT/2018/RPMS/hbt.php

1 Venue

All **Lectures** will take place at **AULA NARANJA** (Orange Lecture Room) of ICMAT. This lecture room is equipped with blackboards and beamer projector.

The **Instituto de Ciencias Matemáticas** – ICMAT (Institute for Mathematical Sciences) — is a joint research institute of the Consejo Superior de Investigaciones Científicas – CSIC (Spanish National Research Council) and three Madrid universities: the Universidad Autónoma de Madrid (UAM), the Universidad Carlos III de Madrid, and the Universidad Complutense de Madrid.

ICMAT is in Cantoblanco Universidad, which is the main campus of **UAM** (Universidad Autónoma de Madrid). Its address is: ICMAT, Calle Nicolás Cabrera, number 13–15, 28049 Madrid; telephone +34 91 2999700.

2 Sponsors

ICMAT Severo Ochoa Programme, a research project of the Spanish Ministerio de Ciencia, Innovación y Universidades

USA National Science Foundation

The GEAR Network — GEometric structures And Representation varieties

3 Scientific committee

Simon Donaldson (Imperial College & Stony Brook)

Nigel Hitchin (Oxford)

4 Organizing committee

Luis Álvarez-Cónsul (ICMAT-CSIC, Madrid)

Steven Bradlow (Illinois, Urbana-Champaign)

Xiuxiong Chen (Stony Brook)

Mario García-Fernández (ICMAT-UAM, Madrid)

Oscar García-Prada (ICMAT-CSIC, Madrid)

Tomás L. Gómez (ICMAT-CSIC, Madrid)

5 Speakers

Olivier Biquard (ÉNS, Paris)

Lucas Branco (MPIM Bonn)

Ugo Bruzzo (SISSA, Trieste)

Brian Collier (Maryland)

Emilio Franco (Porto)

Sergei Gukov (Caltech)

Nigel Hitchin (Oxford)

Georgios Kydonakis (Strasbourg)

Qiongling Li (Nankai)

André Oliveira (Porto & UTAD)

Ana Peón-Nieto (Genève)

Carlos Simpson (Nice)

Jan Swoboda (München)

Alexander Thomas (Strasbourg)

Junho Peter Whang (MIT)

6 Programme

Mini-course by **Nigel Hitchin**: *Higgs bundles and mirror symmetry*

Workshop lectures:

Olivier Biquard: *From HyperKähler metrics on Higgs moduli spaces to Ricci flat Kähler metrics on complexified symmetric spaces*

Lucas Branco: *Higgs bundles and mirror symmetry — examples via isogenies*

Ugo Bruzzo: *A conjecture about Higgs bundles*

Brian Collier: *Generalized opers, conformal limits and maximal variations of Hodge structure*

Emilio Franco: *Branes on the singular locus of the Hitchin system via Borel and other parabolic subgroups*

Sergei Gukov: *Modular tensor categories from wild Higgs bundles*

Georgios Kydonakis: *Model Higgs bundles in exceptional components of character varieties*

Qiongling Li: *Dominations in moduli space of Higgs bundles*

André Oliveira: *Generalized Cayley correspondence and higher Teichmüller components for* $\mathrm{SO}(p,q)$*-Higgs bundles*

Ana Peón-Nieto: *Branes on the Hitchin system from torsion line bundles*

Carlos Simpson: *Higgs bundles in the geometric Langlands correspondence for genus 2 curves*

Jan Swoboda: *The large scale geometry of Higgs bundle moduli spaces*

Alexander Thomas: *Geometric higher Teichmüller theory — punctual Hilbert schemes in Higgs theory*

Junho Peter Whang: *Diophantine analysis on moduli of local systems*

7 Schedule

	Monday 12 Nov	Tuesday 13 Nov	Wednesday 14 Nov	Thursday 15 Nov	Friday 16 Nov	
					U. Bruzzo	10:00-11:00
10:30-11:30	N. Hitchin	C. Simpson	O. Biquard	N. Hitchin	Coffee Break	11:00-11:20
11:30-12:00	Coffee break	Coffee break	Coffee break	Coffee break	J. P. Whang	11:20-12:20
12:00-13:00	A. Oliveira	B. Collier	J. Swoboda	L. Branco	S. Gukov	12:30-13:30
13:00-15:00	Lunch	Lunch	Lunch	Lunch		
15:00-16:00	A. Thomas	Qiongling Li		E. Franco		
16:00-16:30	Coffee break	Coffee break		Coffee break		
16:30-17:30	G. Kydonakis	N. Hitchin		A. Peón-Nieto		

8 Abstracts

Mini-course syllabus:

Nigel Hitchin: *Higgs bundles and mirror symmetry*

Lecture 1. Hodge theory, abelian and non-abelian

SYZ mirror symmetry and semiflat metric for a Calabi-Yau
The integrable system and semiflat metric for Higgs bundles

Lecture 2. Mirror symmetry and Langlands duality

Duals of abelian varieties, examples
BAA-branes and BBB-branes
Invariant Lagrangian submanifolds
Mirror symmetry in rank 2

Lecture 3. Hyperholomorphic bundles

The Dirac bundle, the universal bundle
The Hecke transform
Lagrangians and hyperholomorphic bundles

Workshop abstracts:

Olivier Biquard: *From HyperKähler metrics on Higgs moduli spaces to Ricci flat Kähler metrics on complexified symmetric spaces*

Using Higgs moduli spaces in the 90's I had constructed hyperKhler metrics on cotangent spaces of all generalized flag manifolds of complex semisimple groups. Studying the asymptotics of this metric leads to interesting generalizations on complexified symmetric spaces. Joint work with Thibaut Delcroix.

Lucas Branco: *Higgs bundles and mirror symmetry — examples via isogenies*

According to mirror symmetry, complex Lagrangians in the Higgs bundle moduli space for a complex group are related to hyperkähler subvarieties of the Higgs bundle moduli space for the Langlands dual group. After discussing some general constructions, we focus on this duality for complex Lagrangians arising from two real forms of $\mathrm{SO}(4,\mathbb{C})$ and explain how our results relate to the conjectural picture.

Ugo Bruzzo: *A conjecture about Higgs bundles*

My talk will be about a class of Higgs bundles that we call "curve semistable"; a Higgs bundle E on a smooth projective variety X is said to be curve semistable if for every morphism $f\colon C \to X$, where C is a projective curve, the pullback f^*E is semistable. For ordinary bundles it is known that a bundle is curve semistable if and only if it is semistable with respect to some polarization and its discriminant vanishes. The analogous statement for Higgs bundles is still conjectural. We shall discuss some partial progress towards the proof of this conjecture.

Brian Collier: *Generalized opers, conformal limits and maximal variations of Hodge structure*

For a complex semisimple Lie group G, the space of opers is an affine holomorphic Lagrangian submanifold of the moduli space of flat G connections which is biholomorphic to the base of the Hitchin fibration. These objects are related to the interaction of certain holomorphic connections with structure group reductions to a Borel subgroup. In this talk we will introduce a notion of opers for other parabolic subgroups and parameterize these objects. Furthermore, we will see that the so called conformal lift identifies these generalized opers with the stable manifold of certain $\mathbb{C}^*$-fixed points in the Higgs bundle moduli space.

Emilio Franco: *Branes on the singular locus of the Hitchin system via Borel and other parabolic subgroups*

The moduli space of Higgs bundles has an extremely rich geometry, it is a hyper-Kaehler variety and fibrates over a vector space becoming an integrable system named the Hitchin system. Its importance in theoretical physics comes from the fact that a dimensional reduction of an $N = 4$ Super Yang–Mills gauge theory can be rewritten as a 2-dimensional sigma-model with the Hitchin system as a target. In this context, Kapustin and Witten reinterpreted the classical limit of S-duality of the original SYM gauge theory as mirror symmetry in the target (the Hitchin system). They also introduced the appropriate notion of branes in the Hitchin system respecting the hyperKaehler structure. In this talk I will study the behaviour under mirror symmetry of a family of branes living on the singular locus of the Hitchin system. I will also describe their geometry and the role of the Borel subgroup. The picture can be generalized to other parabolic subgroups. This is joint work with Ana Peón-Nieto.

Sergei Gukov: *Modular tensor categories from wild Higgs bundles*

Georgios Kydonakis (Strasbourg): *Model Higgs bundles in exceptional components of character varieties*

Gluing ASD connections over a complex connected sum of manifolds is a well-known technique in Yang–Mills theory and has been effectively modified for several types of equations in gauge theory. An adaptation of this technique in the context of Higgs bundles involves the utilization of the linearization of a relevant elliptic operator coming from the Hitchin equations. The construction can be used to build model Higgs bundles in exceptional components of character varieties $\mathcal{R}(G)$ for which good models have not been known before, as for the case $G = \mathrm{Sp}(4, \mathbb{R})$ and $G = \mathrm{SO}(n, n+1)$.

Qiongling Li: *Dominations in moduli space of Higgs bundles*

In this talk, we discuss some progress towards two "domination" conjectures made on the moduli space of Higgs bundles. First, we show for a Hitchin representation in PSL(n,$\mathbb{R}$), every equivariant minimal immersion from a hyperbolic plane into the corresponding symmetric space is distance-increasing. Secondly, consider Hitchin fibers at $(q_2, 0, ..., 0)$, that is, the fibers containing Fuchsian locus. We show a

comparison theorem on the length spectrum between surface group representations in such fibers with Fuchsian ones, as a generalization of the SL(2,$\mathbb{C}$) case shown by Deroin and Tholozan.

André Oliveira: *Generalized Cayley correspondence and higher Teichmüller components for* $\mathrm{SO}(p,q)$*-Higgs bundles*

For many classes of Lie groups G, natural topological invariants completely label connected components of the moduli spaces G-Higgs bundles in a curve. Hitchin components in the split real form case, and maximal components in the Hermitian (tube type) case, were the only previously know cases where these natural invariants do not fully distinguish connected components. In this talk we will explain the existence of new such higher Teichmüller components in the moduli spaces of $\mathrm{SO}(p,q)$-Higgs bundles, via a generalized Cayley correspondence. In general, these groups lie outside the above mentioned classes of real forms, but fit in a conjectural natural framework which seems to be associated to the existence of such components.

Ana Peón-Nieto: *Branes on the Hitchin system from torsion line bundles*

In this talk I will explain how mirror symmetry operates on some natural hyperholomorphic branes on the Hitchin system, given by fixed points by tensorisation with a torsion line bundle. Generically, they are supported over the locus of integral spectral curves. Many aspects of their geometry are however more easily understood through branes supported on the most singular locus of a related Hitchin system. I will refer to this interplay during the exposition. This is joint work with E. Franco, P. Gothen and A. Oliveira.

Carlos Simpson: *Higgs bundles in the geometric Langlands correspondence for genus 2 curves*

This is joint work with Ron Donagi and Tony Pantev. They propose to look at the local systems predicted by geometric Langlands through the lens of the nonabelian Hodge correspondence. The Higgs bundle over Bun_G would have as spectral variety the appropriate fiber of the Hitchin fibration, sitting inside $T^*\mathrm{Bun}_G$. Here, we look at the case of rank 2 bundles on a compact genus 2 curve. The moduli spaces are $\mathbb{P}^3$ and an intersection of two quadrics in $\mathbb{P}^5$. For the Higgs bundle obtained by pushforward from a general fiber of the Hitchin fibration, we show how to adjust the parabolic structure to obtain vanishing of the first and second parabolic Chern classes over Bun_G, thus giving a local system on the complement of the wobbly locus.

Jan Swoboda: *The large scale geometry of Higgs bundle moduli spaces*

In this talk I will explain recent joint work with Rafe Mazzeo, Hartmut Weiß and Frederik Witt on the asymptotics of the natural L^2-metric G_{L^2} on the moduli space $\mathcal{M}$ of rank-2 Higgs bundles over a Riemann surface Σ as given by the set of solutions to Hitchin's self-duality equations. I will show that on the regular part of the Hitchin fibration this metric is well-approximated by the semiflat metric G_{sf} coming from the completely integrable system on $\mathcal{M}$. This also reveals the asymptotically

conic structure of G_{L^2}, with (generic) fibres of the above fibration being asymptotically flat tori. This result confirms some aspects of a far reaching conjectural picture suggested by Gaiotto, Moore and Neitzke. Its proof is based on a detailed understanding of the ends structure of $\mathcal{M}$.

Alexander Thomas: *Geometric higher Teichmüller theory — punctual Hilbert schemes in Higgs theory*

The construction of Hitchin's component, a generalization of Teichmüller space, via Higgs bundles does not shed light on its geometric origin. We will construct and analyze a geometric structure on surfaces, generalizing the complex structure, whose moduli space is conjecturally Hitchin's component. These higher complex structures are defined using the punctual Hilbert scheme of the plane. Joint work with Vladimir Fock.

Junho Peter Whang: *Diophantine analysis on moduli of local systems*

We consider the Diophantine geometry of moduli spaces for special linear rank two local systems on topological surfaces. After motivating their Diophantine study, we use differential geometric tools to obtain a finiteness theorem for their integral orbits under mapping class group dynamics, generalizing a classical work of Markoff (1880). We also discuss effective finiteness results for integral points on curves in these moduli spaces, as well as more recent work if time permits.

9 Meals, coffee breaks, and social dinner

Lunch has been arranged for registered participants at Restaurante Autoservicio-Minimarket in Plaza Mayor (ground floor), UAM, from Monday until Friday. You will find **lunch tickets** inside your badge holder.

There will be **coffee breaks** for registered participants in the morning and afternoon sessions.

Social dinner: Wednesday, at 20:15, in Restaurante Mesón del Cid.
Address: Calle Fernández de la Hoz, 57, Madrid (*57, Fernández de la Hoz Street, Madrid*).
Phone: +34 914420755/+34 616953494. Web: `https://www.mesondelcidmadrid.com/`
The closest metro station is *Gregorio Marañón*. Other metro stations nearby are *Alonso Cano* and *Ríos Rosas*. The *cercanías* train station *Nuevos Ministerios* is 1 km away.

10 List of participants

1. David Alfaya (ICMAT-CSIC)
2. Luis Álvarez-Cónsul (ICMAT-CSIC)
3. Olivier Biquard (ÉNS, Paris)
4. Steven Bradlow (Urbana-Champaign)
5. Lucas Branco (MPIM Bonn)
6. Ugo Bruzzo (SISSA, Trieste)
7. Luis Ángel Calvo (ICMAT-CSIC)

8. Raffaele Carbone (Roma 3)
9. Brian Collier (Maryland)
10. Artur de Araujo (ICMAT-CSIC)
11. Andoni de Arriba de la Hera (ICMAT-CSIC)
12. Udhav Fowdar (UCL London & LSGNT)
13. Daniel Fox (UPM Madrid)
14. Emilio Franco (Porto)
15. Guillermo Gallego Sánchez (UCM)
16. Mario Garcia-Fernandez (ICMAT-UAM)
17. Oscar García-Prada (ICMAT-CSIC)
18. Tomás Gómez (ICMAT-CSIC)
19. Beatriz Graa (Salamanca) (viernes)
20. Sergei Gukov (Caltech)
21. Nigel Hitchin (Oxford)
22. Johannes Horn (Heidelberg)
23. Inder Kaur (IMPA)
24. Georgios Kydonakis (Strasbourg)
25. Claude LeBrun (Stony Brook)
26. Qiongling Li (Nankai)
27. Elena Lukzen (SISSA, Trieste)
28. Challenger Mishra (ICMAT & Oxford)
29. Ismael Morales (UAM, Madrid)
30. Miguel Ángel Moya Berlanga (ICMAT-CSIC)
31. André Oliveira (Porto & UTAD)
32. Ana Peón-Nieto (Genève)
33. Manuel Jesús Pérez García (ICMAT-CSIC)
34. Mohammad Reza Rahmati (Grenoble)
35. Carlos Simpson (Nice)
36. Ricardo Suárez (ICMAT-CSIC)
37. Jan Swoboda (München)
38. Alexander Thomas (Strasbourg)
39. Junho Peter Whang (MIT)
40. Alfonso Zamora (San Pablo CEU Madrid)
41. Yingying Zhang (Tsinghua)

11 Wifi

Internet is available at the venue of the conference using the wireless network **ICMAT**.