Group Actions in Algebraic and Symplectic Geometry

Scientific Programme

Dates: 2–3 October 2018
Venue: ICMAT (Madrid)
Lectures will take place in the ICMAT orange lecture room.

Schedule

Tuesday, 2 October 2018:

10:30-11:30: **Frances Kirwan** (Oxford)
Moment maps and non-reductive geometric invariant theory

12:00-13:00: **Artur de Araujo** (ICMAT-CSIC)
Representations of generalized quivers

14:30-15:30: **Eva Miranda** (UPC Barcelona & Observatoire de Paris)
The geometry of the moment map for singular symplectic manifolds

16:00-17:00: **Claudio Bartocci** (Genova)
Moduli spaces of sheaves on algebraic surfaces as Poisson quiver varieties

Wednesday, 3 October 2018:

10:00-11:00: **David Alfaya** (ICMAT-CSIC)
Automorphism group of the moduli space of parabolic vector bundles

11:15-12:15: **Ignasi Mundet** (UB Barcelona)
Maximal compact subgroups of diffeomorphism and Hamiltonian diffeomorphism groups

12:30-13:30: **Frances Kirwan** (Oxford)
Special Colloquium: *Moduli spaces of unstable curves*

Titles and abstracts

David Alfaya: ***Automorphism group of the moduli space of parabolic vector bundles***

We find the automorphism group of the moduli space of parabolic bundles on a smooth curve (with fixed determinant and system of weights). This group is generated by: automorphisms of the marked curve, tensoring with a line bundle, taking the dual, and Hecke transforms (using the filtrations given by the parabolic structure). A Torelli theorem for parabolic bundles with arbitrary rank and generic weights is also obtained. These results are extended to the classification of birational equivalences which are defined over big open subsets (3-birational maps). Joint work with Tomas Gómez.

Artur de Araujo: ***Representations of generalized quivers***

We explain the definition of generalized quiver, and formulate the moduli problem of their representations, both in GIT and symplectic terms. We the explain how an explicit use of standard techniques tracing back to Kirwan one can prove a formula for the Poincare polynomials of the moduli space that is inductive in the semisimple rank of certain Levi sibgroups. This result generalizes a known result for the usual quiver representations. We finish by introducing the notion of generalized quiver bundle, finishing with a Hitchin–Kobayashi correspondence for the particular case of orthogonal generalized quiver bundles.

Claudio Bartocci: ***Moduli spaces of sheaves on algebraic surfaces as Poisson quiver varieties***

I shall present some examples of moduli spaces of framed sheaves on Hirzebruch surfaces which can be realised as quiver varieties. Such quiver varieties arise as spaces of isomorphism classes of representations of quivers with relations and, differently from Nakajima's quiver varieties, do not carry, in general, a symplectic structure. The existence of natural Poisson structures will be discussed.

Frances Kirwan: ***Moment maps and non-reductive geometric invariant theory***

When a complex reductive group acts linearly on a projective variety the GIT quotient can be identified with an appropriate symplectic quotient. The aim of this talk is to discuss an analogue of this description for GIT quotients by suitable non-reductive actions.

In general GIT for non-reductive linear algebraic group actions is much less well behaved than for reductive actions. However when the unipotent radical U of a linear algebraic group is graded, in the sense that a Levi subgroup has a central one-parameter subgroup which acts by conjugation on U with all weights strictly positive, then GIT for a linear action of the group on a projective variety is almost as well behaved as in the reductive setting, provided that we are willing to multiply the linearisation by an appropriate rational character. In this situation we can ask for a moment map description of the quotient.

Frances Kirwan: ***Moduli spaces of unstable curves*** (Special Colloquium)

Moduli spaces arise naturally in classification problems in geometry. The study of the moduli spaces of nonsingular complex projective curves (or equivalently of compact Riemann surfaces) goes back to Riemann himself in the nineteenth century.

The construction of the moduli spaces of stable curves of fixed genus is one of the classical applications of Mumford's geometric invariant theory (GIT), developed in the 1960s. Here a projective curve is stable if it has only nodes as singularities and its automorphism group is finite. The aim of this talk is to describe these moduli spaces and outline their GIT construction, and then explain how recent methods from non-reductive GIT can help us to classify the singularities of unstable curves in such a way that we can construct moduli spaces of unstable curves (of fixed singularity type).

Eva Miranda: ***The geometry of the moment map for singular symplectic manifolds***

A b-Poisson manifold is an even dimensional manifold endowed with a Poisson structure which is symplectic away from an hypersurface determined by a transversality condition. It is possible to associate a **b-symplectic form** to a b-Poisson structure by considering a closed 2-form of an enlarged complex (the b-complex) built up over the b-cotangent bundle. In this way, broadly speaking, we may do **"symplectic geometry over an algebroid"** to understand these Poisson manifolds. With these symplectic glasses on, we can consider Hamiltonian actions in this setting by enlarging the set of admissible functions and obtain a Delzant and convexity theorem for torus actions on b-Poisson manifolds. These theorems have a strong symplectic flavour but also encode some of the non-trivial Poisson invariants (like the periods of a modular vector field).

In this talk I will recall these results and look at them from a more general perspective: A desingularization procedure (**deblogging**) for more general singular structures called b^m-symplectic structures associates a family of symplectic forms or folded symplectic manifolds (depending on the parity of m) and has good convergence properties. This procedure puts in the same picture several geometries: symplectic, folded-symplectic, and Poisson geometry. Using this family of (folded) symplectic structures we can compare the geometry of the moment maps for b^m-symplectic manifolds, symplectic manifolds and folded symplectic manifolds and look at the **"limit" geometry**.

This is joint work with Victor Guillemin and Jonathan Weitsman.

Ignasi Mundet: ***Maximal compact subgroups of diffeomorphism and Hamiltonian diffeomorphism groups***

For a topological group G we consider the following properties:

(1) G is compact bounded (CB) if there exists a compact Lie group K such that any compact subgroup of G is isomorphic as a topological group to a closed subgroup of K.

(2) G is inner compact bounded (ICB) if there is a compact subgroup $K \leq G$ such that for every compact subgroup $H \leq G$ there is some $g \in G$ such that $gHg^{-1} \leq K$.

Clearly ICB $\Longrightarrow$ CB. Let $\mathcal{M}$ be the collection of all closed smooth manifolds. We prove that each of these collections of manifolds contains infinitely many diffeomorphism classes:

$$\mathcal{M}_0 = \{M \in \mathcal{M} \mid \mathrm{Diff}(M) \text{ is ICB}\},\ \mathcal{M}_1 = \{M \in \mathcal{M}\backslash\mathcal{M}_0 \mid \mathrm{Diff}(M) \text{ is CB}\},\ \mathcal{M}\backslash\mathcal{M}_1.$$

Let $\mathcal{S}$ be the collection of all closed symplectic manifolds. We prove that for every $M \in \mathcal{S}$ the Hamiltonian diffeomorphism group $Ham(M)$ is CB, and that both

$$\mathcal{S}_0 = \{M \in \mathcal{S} \mid Ham(M) \text{ is ICB}\} \text{ and } \mathcal{S} \setminus \mathcal{S}_0$$

contain infinitely many symplectomorphism classes. (In all the statements the topology is assumed to be C^r for an arbitrary r.)

These results depend on the relation between the CB property and Jordan's property.